12.

C-48

University of Hyderabad,
Entrance Examination, 2004
Ph.D. (Statistics-OR)

Hall Ticket No.							

Time: **2** hours

Max. Marks: **75**
Part A: **25**
Part B: **50**

Instructions

1. Calculators are not allowed.
2. Part A carries 25 marks. Each correct answer carries **1 mark** and each wrong answer carries $-\frac{1}{4}$ **mark.** So do not gamble. If you want to change any answer, cross out the old one and circle the new one. Over written answers will be ignored.
3. Part B carries 50 marks. Instructions for answering Part B are given at the beginning of Part B.
4. Use a separate booklet for Part B.

Answer Part A by **circling** the correct letter in the array below:

1	a	b	c	d	e
2	a	b	c	d	e
3	a	b	c	d	e
4	a	b	c	d	e
5	a	b	c	d	e
6	a	b	c	d	e
7	a	b	c	d	e
8	a	b	c	d	e
9	a	b	c	d	e
10	a	b	c	d	e
11	a	b	c	d	e
12	a	b	c	d	e
13	a	b	c	d	e
14	a	b	c	d	e
15	a	b	c	d	e
16	a	b	c	d	e
17	a	b	c	d	e
18	a	b	c	d	e
19	a	b	c	d	e
20	a	b	c	d	e
21	a	b	c	d	e
22	a	b	c	d	e
23	a	b	c	d	e
24	a	b	c	d	e
25	a	b	c	d	e

C.48

Part A

Find the correct answer and mark it on the answer sheet on the **top page**. A correct answer gets **1 mark** and a wrong answer gets $-\frac{1}{4}$ **mark**.

1. The sum of the series $\sum_{1}^{\infty} \frac{1}{n(n+1)}$ is

   (a) $\pi^2/6$
   (b) $\pi^2/8$
   (c) 1
   (d) 2
   (e) None of the above

2. Suppose A_n is a sequence of sets defined by $A_n = [0,2]$ if n is odd and $A_n = [1,3]$ if n is even. Then

   (a) $lim\ sup\ A_n = \{3\}$ and $lim\ inf\ A_n = \{0\}$.
   (b) $lim\ sup\ A_n = \{2\}$ and $lim\ inf\ A_n = \{1\}$
   (c) $lim\ sup\ A_n = [2,3]$ and $lim\ inf\ A_n = [0,1]$
   (d) $lim\ sup\ A_n = [0,3]$ and $lim\ inf\ A_n = [1,2]$.
   (e) None of the above.

3. Let $x_n = 1 + \frac{(-1)^n}{n}$. Then

   (a) $\{x_n\}$ does not converge.
   (b) $\{x_n\}$ is convergent because $\{x_n\}$ is increasing and bounded above.
   (c) $\{x_n\}$ is convergent because $\{x_n\}$ is decreasing and bounded below.
   (d) $\{x_n\}$ is convergent because $\{x_n\}$ has finitely many distinct terms.
   (e) none of the above is true.

4. If A = (-3, 3] and B = [-2, 2), then A$\triangle$B is

   (a) an an open interval.
   (b) a closed interval
   (c) an open set
   (d) a closed set
   (e) none of the above.

C.48

5. $\lim_{n\to\infty}\left(1-\frac{a_n}{n}\right)^n$, where $a_n = \left(1+\frac{2}{n}\right)^n$ is equal to

   (a) 1.
   (b) e^{-e^2}.
   (c) e^{-2}.
   (d) e^{-1}.
   (e) none of the above.

6. If A, B, C are three events such that $P(A)P(B)P(C) > 0$ and $P(B) > P(A)$, which of the following is always true?

   (a) $A \subset B$.
   (b) $P(\bar{A}) > P(\bar{B})$.
   (c) $P(B|C) > P(A|C)$.
   (d) $P(A) > P(\bar{B})$.
   (e) None of the above.

7. If for a random variable X, $\mathrm{E}(X) = 0$ and $\mathrm{E}(X^2) = 0.6$, then

   (a) $P[-1 \leq X \leq 1] \leq 0.6$.
   (b) $P[-1 \leq X \leq 1] \geq 0.6$.
   (c) $P[-1 \leq X \leq 1] \leq 0.36$.
   (d) $P[-1 \leq X \leq 1] \geq 0.4$.
   (e) nothing can be said about $P[-1 \leq X \leq 1]$.

8. A random variable X denotes number of failures before getting 1st success in independent identical Bernoulli trials with probability of success θ. If $\mathrm{E}(X) = 2.5$ and $\mathrm{Var}(X) = 6.25$, then $P[X \geq 2]$ is equal to

   (a) 0.36.
   (b) 0.6.
   (c) 0.64.
   (d) 0.08.
   (e) none of the above.

9. $X_1, \cdots, X_n$ are i.i.d. continuous random variables and let R_1 be the rank of X_1. Then $\mathrm{E}(R_1)$ is

   (a) $\frac{n+1}{2}$.
   (b) $\frac{1}{n}$.

(c) 1.

(d) 0.

(e) none of the above.

10. In a simple linear regression of Y on X, if the correlation coefficient between the regressor X and $\hat{Y}$ which are the predicted values of Y is -1 then the regression between X and Y is

(a) -1.

(b) 1.

(c) -0.5.

(d) 0.5.

(e) none of the above.

11. Two samples of different sizes were independently drawn from two different populations which are to be compared. For sample 1, the mean is 10 and the variance is 4 and for sample 2, the mean is 145 and variance is 91.5. Which of the following measures will be most appropriate to make comparison of two populations?

(a) Kolmogorov-Smirnov statistic

(b) F statistic

(c) coefficient of variations

(d) two sample t statistic

(e) none of the above.

12. In SRSWOR of n units from a population of N units which are numbered, the probability that the (N-1)th and Nth population units are included in the sample is

(a) $\frac{N(N-1)}{N^2}$.

(b) $\frac{n(n-1)}{N(N-1)}$.

(c) $\frac{1}{\binom{n}{2}}$.

(d) $\frac{1}{\binom{N}{2}}$.

(e) none of the above.

13. To test certain hypotheses using some statistic T, a one sided right tailed critical region is considered.The p-value of the statistic based on the observed sample is 0.09 using Normal distribution under the null hypothesis. Hence its p-value using t-distribution with 15 degrees of freedom under the null hypothesis will be

   (a) greater than 0.09.
   (b) 0.05.
   (c) less than 0.09.
   (d) equal to 0.09.
   (e) none of the above.

14. The 95^{th} sample percentile of 20 observations is 27. A positive constant a was added to three largest observations and subtracted from the remaining observations. The 95^{th} sample percentile of the new observations will be

   (a) 27.
   (b) 27 + a.
   (c) 27 - a.
   (d) 27 + 0.95a.
   (e) none of the above.

15. If $\underline{X} \sim N_3 \left(\underline{0}, \begin{pmatrix} 1 & 0.5 & 0 \\ 0.5 & 4 & 0 \\ 0 & 0 & 1 \end{pmatrix} \right)$, $\text{Var}(X_2|X_1)$ is

   (a) 4.
   (b) 3.
   (c) 4/3.
   (d) 2.
   (e) none of the above.

16. If $X_1, X_2 \sim U(-1, 1)$ and are independent, then $X_1 + X_2$ and $X_1 - X_2$

   (a) have different expected values.
   (b) have same expected values but different variances.
   (c) are independent.
   (d) are equal almost everywhere.
   (e) are identically distributed.

C. 48

17. Suppose $\Omega = (-\infty, \infty)$, and $\mu((a,b]) = \Phi(b) - \Phi(a)$ where $\Phi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-z^2/2} dz$. For a sequence of random variables $\{X_n\}$ defined as

$$X_n(\omega) = \begin{cases} -1, & \omega \le -n \\ 0, & -n < \omega < n \\ 1, & n \le w. \end{cases}$$

Then which of the following is not true?

(a) $X_n \xrightarrow{P} 0$.
(b) $V(X_n) \to 0$.
(c) $X_n \nrightarrow 0$ almost surely.
(d) $E(X_n) \to 0$.
(e) $\Phi_n(t) \to 1$, where $\Phi_n(t)$ is the characteristic function of X.

18. If $\Phi(t)$ is the characteristic function of a random variable, which of the following is not a characteristic function?

(a) $\bar{\Phi}(t)$
(b) $(\Phi(t))^4$.
(c) $1 - \frac{3}{8}(1 - \Phi(t))$.
(d) $2\Phi(t) - 1$.
(e) $|\Phi(t)|^4$.

19. A is a 3× 3 nonsingular matrix with eigen values 1, 2, and 3. Hence the determinant of the matrix $B = A^2 - 2A$

(a) is 24.
(b) is 0.
(c) is 12.
(d) is 2.
(e) can not be computed based on the given information.

20. X_1, X_2, X_3 are i.i.d N(0,1) random variables. What should be the value of c so that $2X_1 - 3X_2 + 5X_3$ and $cX_1 + 2X_2 + 4X_3$ are independent?

(a) 3.
(b) -2.
(c) 7.
(d) -7.
(e) none of the above.

21. A finite state markov chain has following transition probability matrix.

$$\begin{pmatrix} 1/4 & 0 & 1/3 & 5/12 \\ 0 & 1/2 & 1/2 & 0 \\ 0 & 1/3 & 2/3 & 0 \\ 1/4 & 1/4 & 1/4 & 1/4 \end{pmatrix}$$

(a) The markov chain is irreducible.

(b) All states are recurrent.

(c) {1, 4} and {2, 3} are closed communication classes.

(d) {1, 4} is a closed communicating class but {2, 3} is not a closed communicating class.

(e) {2, 3} is a closed communicating class but {1, 4} is not.

22. A large district is subdivided into 125 non overlapping blocks. Five blocks are selected at random and completely enumerated. This procedure of sampling is known as

(a) Proportional to size sampling.

(b) Stratified sampling.

(c) Systematic sampling.

(d) Cluster sampling.

(e) none of the above.

23. In a connected Block design with v treatments and b blocks, the rank of the C matrix is

(a) v-1.

(b) $< v$.

(c) 1.

(d) b-1.

(e) none of the above.

24. In a 2^4 factorial design with two blocks of 8 plots each in a replication, it was decided to confound effect ABCD. If two blocks in a replication were incompletely constructed with following treatment combinations:

Block I: ab, cd, ac, ad, bc, bd

Block II: a, b, abc, acd, abd, bcd

The remaining treatments in Block I and Block II respectively are

(a) (1, c) and (d, abcd).

(b) (1,d) and ((c, abcd).

(c) (c, abcd) and (d, abcd).

(d) (1, abcd) and (c, d).

(e) none of the above.

25. When the availabilities and demands are integers for a balanced Transportation problem then any basic feasible solution is

(a) mixed type.

(b) real type.

(c) integer type.

(d) may not exist.

(e) none of the above.

Part B

There are **10** questions in this part. Answer any **5** questions. Each question carries **10 marks**. The maximum you can score is **50 marks**. The answers should be written in the separate answer script supplied to you.

1. Let X be a random variable with cumulative distribution function

$$F(x) = \begin{cases} 0, & x < 0 \\ 0.6, & 0 \le x < 2 \\ 0.8 + \frac{1}{10}(x-2), & 2 \le x < 4 \\ 1, & x \ge 4. \end{cases}$$

Find (a) $P[0 \le X \le 2]$. (b) $E(X)$. (c) Median of X.

2. Consider the joint distribution of (X, Y) given below, where a,b,c and d are nonnegative, adding up to unity.

	X	
Y	0	1
0	a	b
1	c	d

   (a) If X and Y are uncorrelated, then show that X and Y are independent.

   (b) Give an example to show that this result may not hold in general if at least one of the random variable takes three values.

3. Suppose $X_1, \cdots, X_n$ are i.i.d.r.v.s with common probability distribution given by

$$P[X = x] = \frac{c(\theta)}{2^{x/\theta}}, \quad x = \theta,\ \theta+1, \cdots, \quad 1 < \theta < \infty.$$

   (a) Find $c(\theta)$.

   (b) Find a minimal sufficient statistic for θ.

4. $X_1, \cdots, X_n$ is a random sample with common probability mass function $P[X = x]$ defined on $\{1, 2, \cdots, N\}$. Based on the sample we wish to test $H_0: P[X = x] = \dfrac{1}{N}$ against $H_1: P[X = x] = \dfrac{2x}{N(N+1)}$.

   (a) Determine the MP level α test.

   (b) For $N = 10$ the sample observed is 5, 2. Will you reject H_0 at 0.01 level of significance? Justify.

5. T_1, T_2 and T_3 are unbiased estimators of $\theta_1 + \theta_3$, $\theta_1 - \theta_2$ and $2\theta_1 - \theta_2 + \theta_3$ respectively. The dispersion matrix of $(T_1, T_2, T_3)'$ is given to be $10\begin{pmatrix} 1 & 1/2 & 1/3 \\ 1/2 & 1 & 1/3 \\ 1/3 & 1/2 & 1 \end{pmatrix}$.

   (a) Verify whether i) $\theta_2 + \theta_3$ ii) $\theta_1 - 2\theta_3$ is estimable.

   (b) Find the BLUE and evaluate the variance of the BLUE of the estimable function(s) in (a) above.

6. Suppose

$$\underline{X} \sim N_3\left(\underline{0}, \begin{pmatrix} 1 & 1/2 & 1/2 \\ 1/2 & 1 & 1/2 \\ 1/2 & 1/2 & 1 \end{pmatrix}\right)$$

   Show that $X_1^2 - 2X_1X_2 + X_2^2 + X_3^2 \sim \chi_2^2$.

7. Suppose X_n, $n = 1, 2, \cdots$ are independent variables with $X_n \sim U\left(-\frac{1}{\sqrt{n}}, \frac{1}{\sqrt{n}}\right)$. and $S_n = X_1 + \cdots + X_n$.

   (a) Verify whether $\frac{S_n}{n} \to 0$ almost surely. State the results used.

   (b) What can you say about the limiting distribution of $\sum_{i=1}^{n} \frac{X_n}{\sqrt{n}}$? Justify your answer.

8. For a connected, quadruplicate and variance balanced block design D with v treatments and b blocks, prove or disprove: b $\geq$ v.

9. Solve the following LPP for $\theta_1 = 0$ and $\theta_2 = 0$ by graphical method and carry out the sensitivity for different values of θ_1.

$$\begin{aligned} \text{Maximize} \quad & Z = 2x_1 + 3x_2 \\ \text{subject to} \quad & \\ & x_1 + 4x_2 \leq 4 + \theta_1 \\ & x_1 + x_2 \leq 2 + \theta_2 \\ & x_1 \text{ and } x_2 \geq 0. \end{aligned}$$

10. On a particular day, a truck company has four trucks for sending material to six terminals. The costs of sending material by different trucks to different terminals are given in the table below.

C-48

	Trucks				
Terminals	A	B	C	D	E
1	3	6	2	6	5
2	7	1	4	4	7
3	3	8	5	8	3
4	6	4	3	7	4
5	5	2	4	3	2
6	5	7	6	2	5

(a) Find the optimal assignment solution to the company.

(b) Find the optimal assignment solution to the company under the restriction that truck D is assigned to terminal 1.

(c) Compute the percentage of loss due to the restriction in (b) above.