Punjab Technical University
Master of Computer Application Examination

MCA 5th Semester ADVANCEDCOURSE – II

Time: Three hours Maximum: 100 marks

Answer any FIVE questions. All questions carry equal marks.

1. (a) State and prove fundamental theorem for homomorphism of rings.
(b) Show that an ideal M of a ring R is maximal if and only if ~ is a field.

2. (a) Show that any two elements a and b in a Euclidean ring R have a greatest common divisor.
(b) Let F be a field. Prove that the ring of polynomials F over F is an Euclidean ring.

3. (a) State and prove Eisenstein irreducibility criterion.
(b) If R is a unique factorization domain, then show that R is also a unique factorization domain.

4. (a) Define a generating set in a vector space. Show that {VI'V2'". vn} is a minimal generating set of a vector space V if and only ifit is a basis of V .
(b) If VI and V2 are subspaces of a vector space V , then prove that dim {VI+V2) =dim VI + dim V2 - dim (VI n V2).

5. (a) If V and Ware vector spaces of dimensions m and n respectively over F, then show that Horn (V, W) is of dimension mn over F .
(b) If V is a finite dimensional vector space and v =I:0 is in V , prove that there is an element rEV (V is the dual of V), such that r(v) =I:O.

6. (a) Show that m an M =MI E9M2 E9... E9Mn if and only if R -module,
(i) M =MI +M2 +... +Mn
(ii) Mi n(MI +M2 +... Mi-I +Mi+1 +
.., +M n) =(0) for allIi, 1 S;is; n.
(b) Let M be a finitely generated module over a principal ideal domain R. Show that M can be expressed as M = F EBt(M), where F is free.

7. (a) Let Fe K c L be field extensions slow the K/F and L/K are finite. Then prove that L/F is finit, and [L: F] =[L: K] [K: F].
(b) Let K be an extension of a field Fane a EK . Then show that a is algebraic over F if anc only if F(a) is a finite extension of F .

8. (a) Let f(x) be any polynomial of degree n ~1 over a field F . Prove that there is an extension K of F of degree at most n! in which f(x) has n roots. .
(b) Obtain a splitting field of X4 - 2 over Q. ,I

9. State and prove the fundamental theorem of Galois theory. ~

10. (a) Prove that the field of complex numbers' algebraically dosed.
(b) Show that for every prime number p ani! n ~ 1, there exists a field with p' element.. - I