# DIPLOMA IN NAUTICAL SCIENCE

## Term-End Examination

## December, 2006

## BNA-011 : APPLIED MATHEMATICS

*Time : 2 hours*            *Maximum Marks : 70*

**Note :**   All questions are **compulsory**. Use of calculator is allowed.

1. Solve any **two** of the following :          2×4=8

   (a) Find the square roots of $(5 - 12i)$.

   (b) Show that

   $$\begin{vmatrix} b+c & a & a \\ b & c+a & b \\ c & c & a+b \end{vmatrix} = 4abc$$

   (c) Prove that

   $$\log 2 + 2\left[\frac{1}{5} + \frac{1}{3}\left(\frac{1}{5}\right)^3 + \frac{1}{5}\left(\frac{1}{5}\right)^5 + \ldots \infty\right] = \log 3$$

**2.** Solve any *one* of the following :　　　　　　　1×4=4

　(a)　Find the cosine of the angle between $3\hat{i} + \hat{j} + 2\hat{k}$
　　　and $2\hat{i} - 2\hat{j} + \hat{k}$.

　(b)　Find the moment of the force $3\hat{i} + 4\hat{j} - 5\hat{k}$ about
　　　the point $(1, 2, 3)$ if the force acts at the point
　　　$(2, -1, 4)$.

**3.** Solve any *one* of the following :　　　　　　　1×7=7

　(a)　Find the coefficient of correlation $\rho(x, y)$ when
　　　Cov.$(x, y) = -8\cdot25$, Var.$(x) = 8\cdot25$, Var.$(y) = 8\cdot25$.

　(b)　A bag contains 6 black and 3 white balls. Another
　　　bag contains 5 black and 4 white balls. If one ball is
　　　drawn from each bag, find the probability that these
　　　two balls are of the same colour.

**4.** Solve any *one* of the following :　　　　　　　1×7=7

　(a)　Find the centre and radius of the circle
　　　$3x^2 + 3y^2 - 18x + 6y + 7 = 0$.

　(b)　Find the eccentricity and latus rectum of the ellipse
　　　$3x^2 + 2y^2 = 6$.

**5.** Solve any *one* of the following :　　　　　　　1×4=4

　(a)　Find the equation of the plane passing through three
　　　points $(2, 3, 4)$, $(-3, 5, 1)$ and $(4, -1, 2)$.

　(b)　Find the centre and radius of the sphere
　　　$3x^2 + 3y^2 + 3z^2 - 2x + 4y - 6z = 1$.

**6.** Solve any **two** of the following : 2×4=8

(a) Differentiate w.r.t. x

$$\tan^{-1}\left(\frac{5x}{1-6x^2}\right)$$

(b) If $x^y = e^{x-y}$, prove that

$$\frac{dy}{dx} = \frac{\log x}{(1+\log x)^2}$$

(c) If $y = \frac{\sin^{-1} x}{\sqrt{(1-x^2)}}$, show that

$$(1-x^2)\frac{d^2y}{dx^2} - 3x\frac{dy}{dx} - y = 0$$

**7.** Solve any **two** of the following : 2×4=8

(a) $\displaystyle\int \frac{3x+2}{(x-1)(2x+3)}\ dx$

(b) $\displaystyle\int x^2 \tan^{-1} x\ dx$

(c) $\displaystyle\int \frac{\sec x + \tan x}{\sec x - \tan x}\ dx$

**8.** Solve any **one** of the following :  $1\times4=4$

(a) $\dfrac{dy}{dx} = \dfrac{xy + y}{xy + x}$

(b) $(x + y)^2 \dfrac{dy}{dx} = a^2$

**9.** Solve any **one** of the following :  $1\times10=10$

(a) In a right angled spherical triangle PXY, angle $X = 92°\ 5'$, angle $Y = 90°$ and side $p = 53°\ 20'$. Calculate angle P.

(b) In a spherical triangle PZX, sides $p = 87°\ 10'$, $z = 62°\ 37'$ and $x = 100°\ 10'$. Calculate angle P.

**10.** Solve any **one** of the following :  $1\times10=10$

(a) In a quadrantal spherical triangle XYZ, angle $X = 73°\ 01'$, side $y = 47°\ 47'$, side $x = 90°$. Calculate angle Y.

(b) In a spherical triangle ABC, angles $A = 88°\ 36'$, $B = 121°\ 36'$, $C = 69°\ 35'$. Calculate side a.