

DIPLOMA IN NAUTICAL SCIENCE

Term-End Examination

December, 2007

BNA-011 : APPLIED MATHEMATICS

Time : 2 hours

Maximum Marks : 70

Note : Attempt **all** questions. All questions carry equal marks. Non-programmable scientific calculator is allowed.

1. (a) Prove

$$(1 - \omega^2 + \omega^4)(1 + \omega^2 - \omega^4) = 4 \quad 5$$

(b) Prove

$$\begin{vmatrix} x & x^2 & yz \\ y & y^2 & zx \\ z & z^2 & xy \end{vmatrix} = (x - y)(y - z)(z - x)(xy + yz + zx) \quad 5$$

2. (a) Solve the differential equation :

$$\frac{dy}{dx} - \frac{y}{x} = 2x^2 \quad 5$$

(b) If $\cos^{-1}\left(\frac{y}{b}\right) = n(\log x - \log n)$

prove that $x^2 \frac{d^2y}{dx^2} + x \frac{dy}{dx} + n^2y = 0.$ 5

3. (a) Evaluate

$$\int \frac{x}{\sqrt{x+2}} dx \quad 5$$

- (b) Show that

$$\int_0^{2a} \sqrt{2ax - x^2} dx = \frac{\pi a^2}{2} \quad 5$$

4. (a) Find λ so that the two vectors

$\hat{i} + (2\lambda - 1)\hat{j} + 3\hat{k}$ and $-3\hat{i} + 2\hat{j} - \lambda\hat{k}$ are perpendicular. Find also the magnitude of the unit vector. 5

- (b) Find the equation of the ellipse given foci $(\pm 3, 0)$ and passing through $(4, 1)$. 5

5. (a) In a spherical triangle ABC, angle $A = 124^\circ 21'$, side $AB = 41^\circ 30'$ and side $AC = 51^\circ 30'$. Calculate side BC using Haversine formula. 5

- (b) In a spherical right angled triangle, angle $B = 90^\circ$, angle $A = 43^\circ 30'$ and side $a = 41^\circ 45'$. Calculate sides b and c. 5

6. (a) Find the vector equation of the line whose Cartesian equations are :

$$\frac{x-1}{2} = \frac{y+1}{-3} = \frac{z+2}{4} \quad 5$$

- (b) Find the 13th term in the expansion of

$$\left(9x - \frac{1}{3\sqrt{x}}\right)^{18}, \quad x \neq 0. \quad 5$$

7. (a) From a pack of 52 cards, two are drawn one by one without replacement. Find the probability that both of them are kings. 5
- (b) Find the line of regression of y on x for the following data : 5

x	10	9	8	7	6	4	3
y	8	12	7	10	8	9	6