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2008-GP

Test Paper Code: GP

Time: 3 Hours **Maximum Marks: 300**

INSTRUCTIONS

1. The question-cum-answer booklet has **52** pages and has **66** questions. Please ensure that the copy of the question-cum-answer booklet you have received contains all the questions.
2. Write your **Roll Number, Name and the name of the Test Centre** in the appropriate space provided on the right side.
3. Answer questions from **any two** Sections selected from (i) Geology Section; (ii) Physics Section; and (iii) Mathematics Section.
4. Write the answers to the objective questions against each Question No. in the **Answer Table for Objective Questions**, provided on Page No. **11**. Do not write anything else on this page.
5. Each objective question has **4 choices** for its answer: (A), (B), (C) and (D). Only **ONE** of them is the correct answer. There will be **negative marking** for wrong answers to objective questions. The following marking scheme for objective questions shall be used:
   (a) For each correct answer, you will be awarded **3 (Three)** marks.
   (b) For each wrong answer, you will be awarded **–1 (Negative one)** mark.
   (c) Multiple answers to a question will be treated as a wrong answer.
   (d) For each un-attempted question, you will be awarded **0 (Zero)** mark.
   (e) Negative marks for objective part(s) will be carried over to total marks.
6. Answer subjective questions only in the space provided after each question.
7. Do not write more than one answer for the same question. In case you attempt a subjective question more than once, please cancel the answer(s) you consider wrong. Otherwise, the answer appearing last only will be evaluated.
8. All answers must be written in blue/black/blue-black ink only. Sketch pen, pencil or ink of any other colour should not be used.
9. All rough work should be done in the space provided and scored out finally.
10. No supplementary sheets will be provided to the candidates.
11. **Clip board, log tables, slide rule, calculator, cellular phone, pager and electronic gadgets in any form are NOT allowed.**
12. The question-cum-answer booklet must be returned in its entirety to the Invigilator before leaving the examination hall. Do not remove any page from this booklet.
13. Refer to special instructions/useful data on the reverse.

A

2008-GP

READ INSTRUCTIONS ON THE LEFT SIDE OF THIS PAGE CAREFULLY

ROLL NUMBER						

Name:

Test Centre:

Do not write your Roll Number or Name anywhere else in this question-cum-answer booklet.

I have read all the instructions and shall abide by them.

.....

Signature of the Candidate

I have verified the information filled by the Candidate above.

.....

Signature of the Invigilator

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Special Instructions / Useful Data

Planck's constant $h = 6.6 \times 10^{-34}$ Js; $\hbar = 1.06 \times 10^{-34}$ Js
Charge of electron $|e| = 1.6 \times 10^{-19}$ C
Mass of electron $m_e = 9.1 \times 10^{-31}$ kg
$\frac{1}{4\pi\varepsilon_0} = 9 \times 10^9$ N m^2 C^{-2}

$\mathbb{R}$ denotes the set of real numbers
$\mathbb{C}$ denotes the set of complex numbers.

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IMPORTANT NOTE FOR CANDIDATES

- **Geology Section : Q. Nos. 1-15 (Objective Questions) and Q. Nos. 46-52 (Subjective Questions).**
- **Physics Section : Q. Nos. 16-30 (Objective Questions) and Q. Nos. 53-59 (Subjective Questions).**
- **Mathematics Section: Q. Nos. 31-45 (Objective Questions) and Q. Nos. 60-66 (Subjective Questions).**
- **Select any *TWO* Sections.**
- **Attempt objective and subjective questions of the selected *TWO* sections.**
- **Questions 1-45 (objective questions) carry *three* marks each and questions 46-66 (subjective questions) carry *fifteen* marks each.**
- **Write the answers to the objective questions in the *Answer Table for Objective Questions* provided on page 11 only.**

GEOLOGY SECTION

Q.1 Which of the following ways of measuring the size of an earthquake does not need an instrumental record?

(A) Richter Magnitude (B) Mw (C) Mb (D) Intensity

Q.2 Which of the following indicates fault movement associated with conservative plate margin?

(A) Thrust Faulting (B) Strike Slip Faulting (C) Normal Faulting (D) Reverse Faulting

Q.3 Which of the following tectonic settings is NOT associated with active volcanoes?

(A) Divergent Plate Boundaries
(B) Convergent Plate Boundaries
(C) Transform Fault
(D) Hot Spots

Q.4 Which of the following has the deepest Moho?

(A) The Himalayas
(B) Hawaii
(C) The Mid-Atlantic Ridge
(D) The East African Rift Valley

Q.5 Compared to the earth, the mass of a planet is approximately 1/80 and its radius is 1/4. How does its surface gravity compare to that of the earth?

(A) 1/2 (B) 1/3 (C) 1/4 (D) 1/5

Q.6 Match the land forms in **Group 1** with the geologie agents in **Group 2**.

	Group 1		**Group 2**
P	Bad Lands	1	Glaciers
Q	Pedestal Rocks	2	Oceans
R	Arete	3	Wind
S	Trenches	4	River

(A) P-1, Q-3, R-4, S-2
(B) P-4, Q-3, R-1, S-2
(C) P-3, Q-2, R-1, S-4
(D) P-4, Q-3, R-2, S-1

Q.7 In a reclined fold

(A) the axis plunges parallel to the axial plane
(B) the dip of axis and axial plane are same
(C) the axis plunges directly down the dip of the axial plane
(D) both the limbs dip towards each other

Q.8 If the dip of a fault plane is 35°, how much is its hade?

(A) 55° (B) 65° (C) 45° (D) 35°

Q.9 Birefringence represents

(A) the difference between maximum and minimum refractive indices in a particular mineral
(B) uniform refractive index in a mineral
(C) the difference between refractive index in a mineral as compared to calcite
(D) the difference between refractive index in a mineral as compared to quartz

Q.10 Match the silicate structure in **Group 1** with the associated mineral in **Group 2**.

Group 1	**Group 2**
P Nesosilicate	1 Beryl
Q Sorosilicate	2 Forsterite
R Cyclosilicate	3 Hemimorphite
S Inosilicate	4 Albite
T Phyllosilicate	5 Hypersthene
U Tektosilicate	6 Tremolite

(A) P-2, Q-1, R-3, S-4, T-5, U-6
(B) P-2, Q-3, R-4, S-1, T-6, U-5
(C) P-2, Q-4, R-1, S-3, T-5, U-6
(D) P-2, Q-3, R-1, S-5, T-6, U-4

Q.11 Paleozoic Era is divided into which of the following successive Periods?

(A) Cambrian, Silurian, Ordovician, Carboniferous, Devonian, Permian
(B) Cambrian, Silurian, Jurassic, Carboniferous, Ordovician, Permian
(C) Cambrian, Ordovician, Silurian, Devonian, Carboniferous, Permian
(D) Cambrian, Devonian, Ordovician, carboniferous, Silurian, Permian

Q.12 Vesicular and amygdaloidal structures are typically observed in

(A) Syenites (B) Basalts (C) Granites (D) Sandstones

Q.13 Greywacke

(A) is light in color, represents oxidizing environment, shows current bedding and forms under shallow marine conditions
(B) is dark in color, represents reducing environment, shows graded bedding and forms under deep freshwater conditions
(C) is light in color, represents oxidizing environment, shows current bedding and forms under shallow marine conditions
(D) is dark in color, represents reducing environment, shows graded bedding and forms under deep water conditions

Q.14 Match the localities in **Group 1** with the associated mineral/metal deposits in **Group 2**.

Group 1	**Group 2**
P Agnigundla	1 Manganese
Q Kiriburu	2 Copper
R Mansar	3 Iron
S Sukinda	4 Lead
T Zawar	5 Chromite
U Koderma	6 Mica

(A) P-2, Q-3, R-1, S-5, T-4, U-6
(B) P-3, Q-2, R-1, S-5, T-4, U-6
(C) P-2, Q-3, R-5, S-1, T-6, U-4
(D) P-1, Q-3, R-4, S-2, T-6, U-5

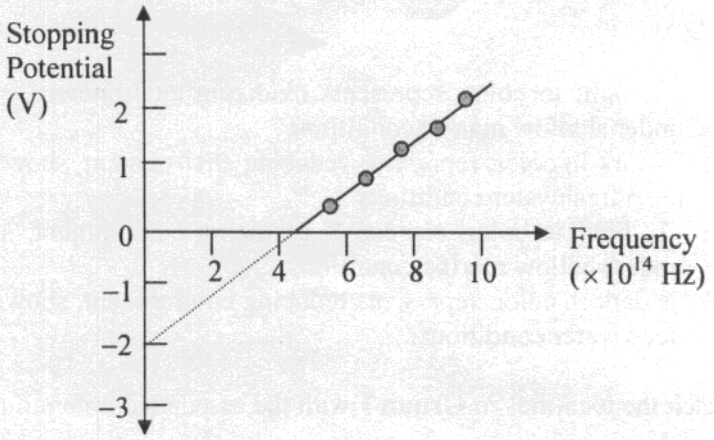
Q.15 Which of the following is correct?

(A) The metal content of an ore is called ore mineral.
(B) Pegmatite deposits are formed towards the end of consolidation of magma in which the residual fraction is highly enriched in volatile constituents.
(C) Chromite deposits of Sukinda are an example of immiscible liquid injection process.
(D) Natural separation of heavy and light minerals by means of moving water is called immiscible liquid segregation.

Space for rough work

PHYSICS SECTION

Q.16 The figure shows the results obtained during a photoelectric effect experiment where the stopping potential is plotted against the frequency of the incident radiation. The work function of the metal is close to



(A) −2.0 eV (B) 0.5 eV (C) 2.0 eV (D) 4.5 eV

Q.17 A metal of atomic weight W and density ρ has fcc structure. The side a of its conventional cubic cell is given by

(A) $\left(\frac{W}{\rho}\right)^{1/3}$ (B) $\left(\frac{2W}{\rho}\right)^{1/3}$ (C) $\left(\frac{4W}{\rho}\right)^{1/3}$ (D) $\left(\frac{6W}{\rho}\right)^{1/3}$

Q.18 When a point object is placed at a distance u = 36 cm from a biconvex lens, the image forms at a distance v = 18 cm on the other side of the lens. If the distance of the object is reduced to 12 cm, the image will be formed at

(A) 12 cm (B) ∞ (C) 6 cm (D) $-\infty$

Q.19 An electron is moving inside a uniformly charged sphere with total charge $+e$. The uncertainty principle $\Delta p_x \, \Delta x \geq \hbar$ is used for finding the smallest radius r_{min} of the charged sphere such that the kinetic energy of the electron is equal to its potential energy on the surface of the sphere. Then r_{min} is approximately in the range

(A) $10^{-5} - 10^{-4}$ Å (B) $10^{-2} - 10^{-1}$ Å (C) 1 – 10 Å (D) $10^{4} - 10^{5}$ Å

Q.20 For a black body at temperature T, the average energy per mode is E_1 for frequencies close to zero, and is E_2 for very high frequencies. As a function of temperature T and frequency ν, E_1 and E_2 are given as

(A) $E_1 = E_2 = k_B T$
(B) $E_1 = E_2 = h\nu$
(C) $E_1 = k_B T$ and $E_2 = h\nu \exp(-h\nu 7 k_B T)$
(D) $E_1 = h\nu \exp(-h\nu / k_B T)$ and $E_2 = k_B T$

Q.21 In kinetic theory of gases, the pressure of a gas in a container can be written as $p = \frac{2}{3}U$. For a monatomic gas, the quantity U is

(A) kinetic energy per molecule
(B) total kinetic energy of all the molecules in the container
(C) total average kinetic energy of molecules in one mole
(D) total average kinetic energy of molecules in unit volume

Q.22 A mass of 0.5 kg moving with a speed of 2 m/s hits another mass 1 kg moving in the same direction with a speed of 1 m/s. Kinetic energy of the centre of mass is

(A) 4/3 J (B) 3/4 J (C) 0 J (D) 8/3 J

Q.23 If a function of distance x and time t is given by $f(x - \mathrm{v}t)$, then it represents

(A) a wave traveling along $+x$ direction and satisfies the wave equation $\frac{\partial f}{\partial x} + \frac{1}{\mathrm{v}}\frac{\partial f}{\partial t} = 0$
(B) a wave traveling along $+x$ direction and satisfies the wave equation $\frac{\partial f}{\partial x} - \frac{1}{\mathrm{v}}\frac{\partial f}{\partial t} = 0$
(C) a wave traveling along $-x$ direction and satisfies the wave equation $\frac{\partial f}{\partial x} + \frac{1}{\mathrm{v}}\frac{\partial f}{\partial t} = 0$
(D) a wave traveling along $-x$ direction and satisfies the wave equation $\frac{\partial f}{\partial x} - \frac{1}{\mathrm{v}}\frac{\partial f}{\partial t} = 0$

Q.24 Consider the following conditions satisfied by an electric field $\vec{E}$

$$\vec{\nabla}.\vec{E} = X \text{ and } \vec{\nabla} \times \vec{E} = \vec{Y}$$

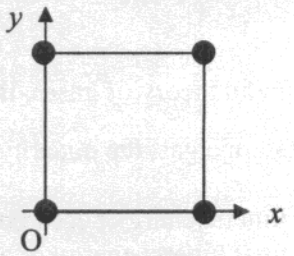
Which of the following conditions is true if the field can be derived from a potential

(A) Both X and $\vec{Y}$ must be zero
(B) X must be zero but $\vec{Y}$ can be non zero
(C) X can be non zero but $\vec{Y}$ must be zero
(D) both X and $\vec{Y}$ can be non zero

Q.25 The dispersion relation for a wave is given as $\omega(k) = C|\sin ka|$, where C is a constant. The group velocity of the wave vanishes at

(A) $k = 0$ (B) $k = \pm \pi/2a$ (C) $k = \pm \pi/a$ (D) $k = \pm 2\pi/a$

Q.26 Four masses, each of mass m, are at the corners of a square of side a as shown in the figure. The product of inertia I_{xy} of the system about O is



(A) ma^2 (B) $4ma^2$ (C) $-ma^2$ (D) $-4ma^2$

Q.27 A point charge $+q$ is kept at a distance d from the centre of a conducting sphere of radius R. If $d > R$, the potential on the sphere is

(A) $\frac{q}{4\pi\varepsilon_0 d}$ (B) $\frac{q}{4\pi\varepsilon_0 R}$ (C) $\frac{q}{4\pi\varepsilon_0 (d-R)}$ (D) $\frac{q}{4\pi\varepsilon_0}\left(\frac{1}{d}-\frac{1}{R}\right)$

Q.28 A region in space contains uniform electric and magnetic fields, $E = E_0 \hat{k}$ and $B = B_0 \hat{k}$. If a charged particle enters the region with an initial velocity $\vec{v} = v_0 \hat{i}$, then the resultant trajectory of the particle will be a helix with

(A) constant radius, but with increasing pitch.
(B) constant pitch, but with increasing radius.
(C) constant radius and constant pitch.
(D) increasing radius and increasing pitch.

Q.29 After being hit, a golf ball reaches a maximum height of 60 m with a speed of 20 m/s. Right after being hit, the speed of the ball is (take g as 10 m/s^2)

(A) 100 m/s (B) 80 m/s (C) 60 m/s (D) 40 m/s

Q.30 An ideal gas with a C_p/C_v ratio of 1.5 is compressed to half its original volume. Then the

(A) pressure increases by a factor of 2
(B) pressure increases by a factor of $\sqrt{2}$
(C) temperature increases by a factor of 2
(D) temperature increases by a factor of $\sqrt{2}$

MATHEMATICS SECTION

Q.31 Let $f(x)=\begin{cases} x^2\sin\left(\frac{1}{x}\right), & \text{if } x\neq 0 \\ 0, & \text{if } x=0 \end{cases}$ *for* $x\in\mathbb{R}$.

Then at $x=0$, the function $f(x)$ is

(A) discontinuous
(B) continuous but not differentiable
(C) continuous with a continuous derivative
(D) continuous with a discontinuous derivative

Q.32 The value of the integral $\int_{-\pi}^{\pi/4}\sqrt{\frac{1-\cos 2x}{2}}\,dx$ is

(A) $-1-\frac{1}{\sqrt{2}}$ (B) $1-\frac{1}{\sqrt{2}}$ (C) $3-\frac{1}{\sqrt{2}}$ (D) $2-\frac{1}{\sqrt{2}}$

Q.33 Consider the differential equation $\frac{dy}{dx}-2x=\phi(x)$, $x\in\mathbb{R}$, satisfying $y(0)=0$, where $\phi(x)=\begin{cases}0, & x\le 0\\ 1, & x>0\end{cases}$. This initial value problem

(A) has a continuous solution which is not differentiable at $x=0$
(B) has a continuous solution which is differentiable at $x=0$
(C) has a continuous solution which is differentiable on $\mathbb{R}$
(D) does not have a continuous solution on $\mathbb{R}$

Q.34 Suppose the power series $f(z)=\sum_{n=0}^{\infty}a_n(z-i)^n$, $a_n\in\mathbb{C}$, converges for $z=1$ and diverges for $z=-1+2i$. Consider the following statements about $f(z)$:

I. the radius of convergence of $f(z)$ is 2
II. $f(z)$ can be developed as a power series about $z=1$
III. $f(z)$ is infinitely differentiable whenever $|2z-1-i|<\frac{1}{2}$

Then,

(A) only **II** is false
(B) only **III** is true
(C) only **I** is false
(D) None among **I**, **II** and **III** is true

Q.35 Let $T:\mathbb{R}^n \longrightarrow \mathbb{R}^m$, $m > n$, be a linear transformation. Consider the following statements about T:

I. T can be one-to-one
II. T can be onto
III. $\dim(T(\mathbb{R}^n)) \leq n$

Then,

(A) only **I** is true (B) only **II** is true (C) only **II** is false (D) only **III** is true

Q.36 Let $f(x,y) = \begin{cases} \dfrac{xy}{x^2+y^2}, & \text{if } (x,y) \neq (0,0) \\ 0, & \text{if } (x,y) = (0,0). \end{cases}$

Then at (0,0), the function $f(x,y)$ is

(A) continuous, but its first partial derivatives do not exist
(B) discontinuous, but its first partial derivatives exist
(C) continuous and the first partial derivatives exist
(D) neither continuous nor do the first partial derivatives exist

Q.37 Let $f(x)$ be such that $|f(x_1)-f(x_2)| \leq |x_1-x_2|^2$, $\forall\, x_1, x_2 \in \mathbb{R}$. Then $f(x)$

(A) need not be continuous
(B) is continuous but not differentiable
(C) is strictly monotone
(D) is a constant function

Q.38 The value of the integral $\int_1^4 \int_{\sqrt{y}}^2 \frac{e^{x^2}}{y}\, dx\, dy$ is

(A) $e^4 - e$ (B) $\sqrt{\pi}$ (C) $e^3 - e$ (D) $5\pi - e$

Q.39 If $y(x)$ satisfies $\frac{dy}{dx} + 2y = 2 + e^{-x^2}$ with $y(0) = 0$, then $\lim_{x\to\infty} y(x)$ equals

(A) 0 (B) 1 (C) 2 (D) -1

Q.40 Three distinct positive integers are chosen from the first fifty natural numbers at random. The probability that each integer chosen is divisible by both 2 and 3 is:

(A) $1/350$ (B) $3/250$ (C) $1/150$ (D) $3/50$

Q.41 The fourth divided difference of the function $f(x) = 3x^3 + 2x^2 - 1$, relative to the points $x_0 = 0$, $x_1 = 1$, $x_2 = 2$, $x_3 = 3$, $x_4 = 4$ in the interval $0 \leq x \leq 4$ is

(A) -1 (B) 2 (C) 0 (D) 3

Q.42 The equation $x3^x = 1,\ x \in \mathbb{R}$, has

(A) infinitely many roots
(B) only two roots
(C) no root
(D) only one root

Q.43 The differential equation $\frac{d^2y}{dx^2} + y = 0$ satisfying $y(0) = 1, y(\pi) = 0$ has

(A) a unique solution
(B) a singly infinite family of solutions
(C) no solution
(D) a doubly infinite family of solutions

Q.44 Let $C_1 : x^2 + y^2 = 1$ and $C_2 : x^2 + \frac{y^2}{2} = 1$ be the curves from $(1,0,1)$ to $(-1,0,1)$, drawn counterclockwise in the upper half $(y \geq 0)$ of the plane $z = 1$, and $\vec{F} = yz\,\hat{i} + zx\,\hat{j} + xy\,\hat{k}$. Let the line integrals be $I_1 = \int_{C_1} \vec{F} \cdot \vec{dr}$ and $I_2 = \int_{C_2} \vec{F} \cdot \vec{dr}$. Then

(A) $I_2 > \frac{1}{2}$
(B) $I_1 > I_2$
(C) $I_2 > I_1$
(D) $I_1 = I_2$

Q.45 The radius of convergence of the series $\sum_{n=0}^{\infty} 2^{-n} x^{3n}$ is

(A) ∞
(B) $\sqrt[3]{2}$
(C) 0
(D) 2

Space for rough work

Space for rough work

A

Answer Table for Objective Questions

Write the Code of your chosen answer only in the 'Answer' column against each Question No. Do not write anything else on this page.

Question No.	Answer	Do not write in this column	Question No	Answer	Do not write in this column
01			31		
02			32		
03			33		
04			34		
05			35		
06			36		
07			37		
08			38		
09			39		
10			40		
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FOR EVALUTAION ONLY

Correct		**Marks**	**(+)**
Incorrect		**Marks**	**(−)**
Total Marks in Q. Nos. 1-15			**( )**

Correct		**Marks**	**(+)**
Incorrect		**Marks**	**(−)**
Total Marks in Q. Nos. 16-30			**( )**

Correct		**Marks**	**(+)**
Incorrect		**Marks**	**(−)**
Total Marks in Q. Nos. 31-45			**( )**

GEOLOGY SECTION

Q.46 (a) Differentiate
(i) between convergent margin-subduction zone and convergent margin-collision zone;
(ii) between earthquake hypocenter and earthquake epicenter. (6)

(b) Briefly describe:
(i) Wadati-Benioff zone
(ii) Magnetic stripes
(iii) Island Arc. (9)

A

Q.47 Identify and explain the differences between the two kinds of seismic body waves. (15)

A

Q.48 (a) A large area of continent consisting 30 km of crust with an average density of 2.8 Mg/m^3 is over 90 km thick material with density 3.1 Mg/m^3. It is covered with a 1.6 km thick layer of ice (density 0.9 Mg/m^3), and is in isostatic equilibrium. Then the ice melts. After equilibrium has been regained, by how much has the rock surface of the continent changed ? (density of the asthenosphere is 3.2 Mg/m^3). (9)

(b) Draw a velocity-depth model of P- and S-waves for the interior of the earth. (6)

A

Q.49 (a) Show the stratigraphic succession of the Cuddapah Super Group. (9)

(b) An aquifer has a total volume of 1200 m^3 and the volume of open spaces is 300 m^3. Find the porosity (in %) of the aquifer. (6)

A

Q.50 Drawing figures wherever necessary, write briefly on

(i) Mafic Rocks,

(ii) Drag Folds,

(iii) Alluvial Fans and Bajadas. (15)

A

Q.51 (a) What is "Bowen's Reaction Series" ? What is congruent- and incongruent melting ? (9)

(b) Complete the following reactions giving also the names of the minerals formed:

(i) Mg_2SiO_4 + SiO_2 =
(Forsterite) (Quartz)

(ii) $NaAlSi_2O_6$ + SiO_2 =
(Jadeite) (Quartz)

(6)

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Q.52 Drawing figures wherever necessary, write briefly on

(i) Recumbent Fold.

(ii) Angular Unconformity.

(iii) Ripple Marks. (15)

PHYSICS SECTION

Q.53 In a hydrogen-like atom, an electron in the ground state requires 476 eV to reach an excited level with quantum number $2n$. If it makes a transition from this level to a lower level with quantum number n, it emits a photon of energy 40.8 eV. (The ground-state energy of hydrogen atom is −13.6 eV).

Find (i) n, (ii) the atomic number Z of the element and (iii) the ionization energy of the atom? (15)

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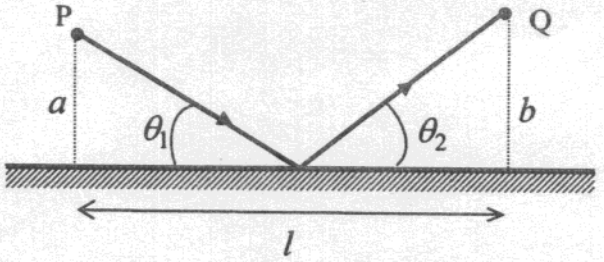
Q.54 Consider a ray of light starting from point P in front of a mirror and going to another point Q after getting reflected from the mirror, as shown in the figure.

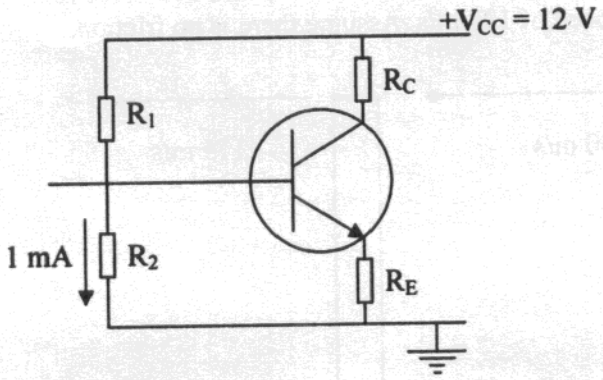


Show that the incident and reflected light make the same angle from the mirror using Fermat's principle (light travels from P to Q by taking a path such that the time of travel is the least). (15)

A

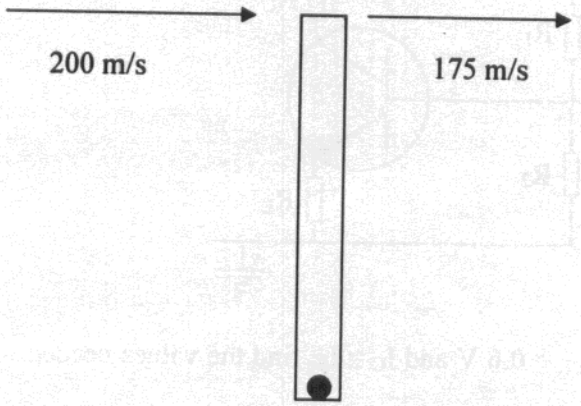
Q.55 A capacitor of capacitance 2 μF is charged fully by a battery of 5 V. It is now disconnected from the battery and connected across an inductor of inductance 200 μH. Find the charge on the capacitor and the current through the inductor as a function of time after the capacitor and the inductor are connected. (15)

Q.56 The figure shows a transistor circuit with a voltage divider bias. The operating point of the transistor is to be obtained as $(I_{CQ}, V_{CQ}) = (2\text{ mA}, 6\text{ V})$.



Given that $V_E = 2$ V, $V_{BE} = 0.6$ V and $I_C \cong I_E$, find the values needed for the resistances R_1, R_2, R_E and R_C. (15)

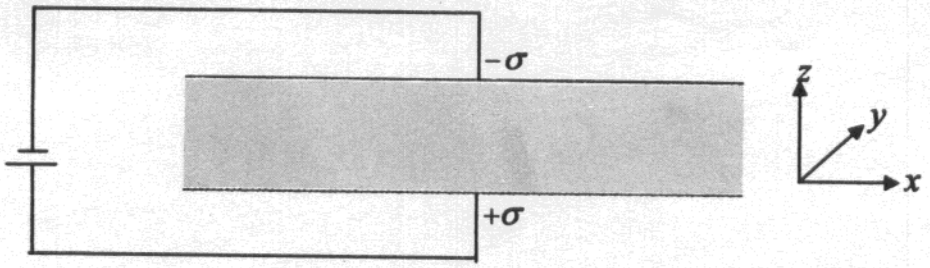
Q.57 A bullet of mass 0.1 kg travelling at a speed of 200 m/s hits one end of a stick of length 1.0 m and mass 0.5 kg lying on a horizontal table and pivoted at the other end, as shown in the figure. After hitting the stick, the bullet continues to move along its original path but with a reduced speed of 175 m/s. Assume there is no friction.



(a) Find the angular speed of the stick after it is hit. (9)

(b) Find the magnitude of the impulse on the pivot because of the hit. (6)

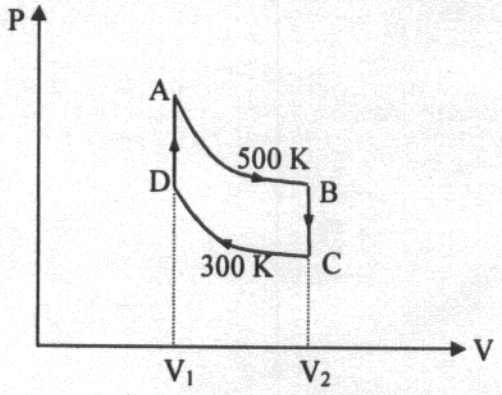
Q.58 The figure shows a parallel plate capacitor assembly where the space between the two conducting plates is filled with a dielectric material of dielectric constant $\kappa = 2$. The plates have a surface charge density σ.



(a) Find the polarization vector in the dielectric and the bound surface charge density at the lower surface of the dielectric. (6)

(b) The battery is now disconnected and the dielectric is pulled out by a distance x along the x-direction. Find the force experienced by the dielectric in that position. Assume the plates to be squares of sides 1.0 unit each and separated by a distance d. (9)

Q.59 The figure shows P-V diagram for one mole of a monatomic ideal gas. AB and CD are isothermal processes at 500 K and 300 K, respectively, and $V_2 = 2V_1$.



Find the heat absorbed or rejected in each section, AB, BC, CD, and DA. What is the total work done during the complete cycle ABCDA ? (15)

A

MATHEMATICS SECTION

Q.60 Suppose $f(z)$ is analytic in $\{z \in \mathbb{C} : |z-i| \leq 2\}$ and $f(z) = ze^z$ for all z such that $|z-i| = 2$. Find $f(z)$ when $|z-i| < 2$. (15)

A

Q.61

Consider the series $\sum_{k=1}^{\infty} \frac{(-1)^k x^k}{k}$, for $x \in \mathbb{R}$. Find the values of x for which the series

(i) converges,
(ii) converges absolutely, and
(iii) converges uniformly.

Justify your claims. (15)

A

Q.62 A real 3×3 matrix M has eigenvalues ±1 and 2. Show that

(i) M is invertible,

(ii) M^3-2M^2 is singular and

(iii) M is diagonalizable. (15)

Q.63 Let $f:\mathbb{R}\longrightarrow\mathbb{R}$ be a function satisfying $f(0)\neq 0$ and $f(x+y)=f(x)f(y)\ \forall\ x,y\in\mathbb{R}$.
Show that $f(0)=1$, and that f is continuous on $\mathbb{R}$ if it is continuous at 0. (15)

Q.64 Let S be the boundary of the region consisting of the parabolic cylinder $z = 1 - x^2$ and the planes $y = 0$, $y = 2$ and $z = 0$. Evaluate the integral

$\oiint_S \vec{F} \cdot \hat{n}\, dS$, where $\vec{F} = xy\,\hat{i} + (y^2 + e^{xz^2})\,\hat{j} + \sin(xy)\,\hat{k}$ and $\hat{n}$ is the outwardly drawn unit normal to S. (15)

A

A

Q.65 Suppose a function $g(x)$ is defined and its first derivative $g'(x)$ exists and is continuous on an interval $I = [x_* - \delta,\ x_* + \delta]$ around the fixed point x_* of $g(x)$. If $g'(x) \leq \alpha < 1$ and $x_{n+1} = g(x_n),\ n = 0, 1, 2, \ldots,$ is the iterative sequence starting from any $x_0 \in I$, show that

$$|x_* - x_0| \leq \frac{\alpha^n}{1-\alpha} |x_0 - x_1|.$$

(15)

A

Q.66 Let X be a continuous random variable with probability density function

$$f(x)=\begin{cases} \alpha x, & 0\le x<1 \\ \alpha, & 1\le x<2 \\ -\alpha(x-3), & 2\le x\le 3 \\ 0, & \text{elsewhere.} \end{cases}$$

Determine

(i) the constant α and

(ii) $P(X\le \frac{3}{2})$.

(15)

A

Space for rough work

Space for rough work

Space for rough work