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## SATHYABAMA UNIVERSITY

(Established under section 3 of UGC Act,1956)
Course \& Branch :B.E/B.Tech - Common to ALL Branches Title of the Paper :Engineering Mathematics - IV Max. Marks :80 Sub. Code :401-6C0054
Date :12/11/2009

Time : 3 Hours
Session :AN

$$
\text { PART }-\mathrm{A} \quad(10 \times 2=20)
$$

Answer ALL the Questions

1. State Drichlet's conditions.
2. Find the RMS value of $f(x)=x^{2}, x \in[0, \pi]$
3. Find the partial differential equation of a family of spheres whose centers lie on X O Y plane.
4. Find the singular solution of $z=p x+q y+p^{2} q^{2}$
5. Write down any two assumptions in the problem of transverse vibration of finite elastic string.
6. Define steady state conditions.
7. Write down the three possible solution of Laplace equation in Cartesian coordinates and point out the equation to be considered for the discussion of heat flow in an infinite plate so long when compared parallel to Y axis.
8. Write down the solution of Laplace equation in polar coordinates that corresponds to heat flow in an annulus region.
9. Find the inverse sine transform of $F(s)=\frac{1}{s}$
10. Prove that $F s[x f(x)]=-\frac{d}{d s} F c[f(x)]$
PART - B

Answer All the Questions
11. (a) Find the Fourier series of $f(x)=x^{2}$ in the interval $(-\pi \leq X \leq \pi)$ and deduce $\sum_{1}^{\infty} \frac{1}{n^{2}}=\pi^{2} / 6$
(b) Find the complex form of Fourier series for $f(x)=e^{-a x}$ in the interval - $1 \leq \mathrm{x} \leq 1$.
(or)
12. (a) Derive Euler's formulae of determining Fourier coefficient when a function $f(x)$ is expressed in terms of Fourier series.
(b) Obtain the first three harmonics in the Fourier series expansion in $(0,12)$ for the function $y=f(x)$ defined by the table given below.

| X | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Y | 1.8 | 1.1 | 0.3 | 0.11 | 0.5 | 1.5 | 2.16 | 1.88 | 1.25 | 1.3 | 1.76 | 2.00 |

13. (a) Find the partial differential equation of a family of plane expressed in normal form:

$$
1 x+m y+n z=a, 1^{2}+m^{2}+n^{2}=1
$$

(b) Find the singular solution of the partial differential equation

$$
\begin{array}{r}
z=p x+q y+a \sqrt{\left(1+p^{2}+q^{2}\right)} \\
\text { (or) }
\end{array}
$$

14. (a) Solve $\left(z^{2}-2 y z-y^{2}\right) p+(x y+z x) q=(x y-z x)$
(b) Solve with usual notations:-

$$
r-4 s+4 t=e^{2 x-y}+\cos (2 x-3 y)
$$

15. A string AB of length ' $I$ ', tightly stretched between A and B is initially at rest in the equilibrium position. If each point of the string is suddenly given a velocity $f(x)=a\left[1 x-x^{2}\right]$, find the transverse displacement of the string at subsequent time $t$.
16. A $\operatorname{rod} \mathrm{AB}$ of length 1 is heated so that A is maintained at zero temperature while B at $100^{\circ}$ until the steady state conditions prevail. If at time $t=0$, both ends are insulated, find the temperature of the rod at any distance $x$ from one end $x=0$ and at any time t .
17. A square metal plate of side ' $a$ ' has the edges $x=0$ and $y=0$ insulated. The edge $\mathrm{y}=\mathrm{a}$ is kept at temperature $0^{\circ}$ and the edge $x=1$ is kept at temperature ky. Find the steady state temperature distribution in the plate.
18. A plate in the form of a circular sector is bounded by the lines $\theta=0, \theta=\alpha$ and $r=a$. Its surfaces are insulated and the temperatures along the straight line boundaries are kept at $0^{\circ} \mathrm{c}$. If the temperature along the circular boundary is $\mathrm{T}^{\circ} \mathrm{C}$, find the steady state temperature distribution in the plate.
19. (a) State the prove Parseval's identity.
(b) Find the Fourier transform of $f(x)$, if

$$
F(x)=\left\{\begin{array}{cc}
1-|x| & |x|<1 \\
0 & |x| \geq 1 \\
\text { (or) }
\end{array}\right.
$$

20. (a) State and prove convolution theorem.
(b) Find the finite Fourier sine and cosine transforms of

$$
f(x)=\frac{x}{\pi} \text { in }(0, \pi)
$$

