

SATHYABAMA UNIVERSITY

(Established under section 3 of UGC Act, 1956)

Course & Branch: B.E/B.Tech – Common to ALL Branches
(Except to Bio Groups)

Title of the paper: Engineering Mathematics - IV

Semester: IV

Max.Marks: 80

Sub.Code: 401(2003-2004-2005)- 6C0054(2006-2007)Time: 3 Hours

Date: 24-04-2009

Session: FN

PART – A (10 x 2 = 20)
Answer All the Questions

1. Write the Dirichlet's conditions for expansion of Fourier series.
2. Write the complex form of the Fourier series for $f(x) = e^{-x}$ in the interval $(-1, 1)$.
3. Form the p.d.e by eliminating the arbitrary function from $z = f(x^2 + y^2)$.
4. Find the particular integral of $(D^2 - DD')z = \sin x \cos 2y$.
5. A string is fixed at the end points $x = 0$ and $x = l$, is initially at rest in its equilibrium position. It is set vibrating by giving a velocity $\lambda x (l - x)$. Write down the boundary conditions for the problem.
6. State any two of the empirical laws used to derive the one dimensional heat equation.
7. Write down the differential equation for two dimensional heat flow for unsteady case.

8. Write the most general solution of the steady – state temperature at an arbitrary point (r, θ) in the annulus.
9. If $F(s) = F(f(x))$ then show that $F(f'(x)) = -is F(s)$.
10. Find the finite fourier cosine transform of $f(x) = x$ in $(0, \pi)$.

PART – B

(5 x 12 = 60)

Answer All the Questions

11. (a) Obtain the Fourier series for the function $f(x) = |x|$ in the interval $(-\pi, \pi)$ and using this series find the RMS value of $f(x)$ in the interval.

(b) Obtain the fourier series for $f(x)$ upto the first harmonic from the following data:

x	0	$\frac{\pi}{3}$	$\frac{2\pi}{3}$	π	$\frac{4\pi}{3}$	$\frac{5\pi}{3}$	2π
f(x)	3	4	3	1	0	1	3

(or)

12. (a) Expand $f(x) = x - x^2$ as a Fourier series in $-1 < x < 1$.
- (b) Express $f(x) = x$ in half range sine series of periodicity $2l$ in the range $0 < x < l$ and deduce the value of

$$\left(\frac{1}{1^4}\right) + \left(\frac{1}{3^4}\right) + \left(\frac{1}{5^4}\right) + \dots \text{to } \infty$$

13. (a) Solve $(D^2 - 6DD' + 5D'^2)z = e^x \sinh y$.
- (b) Solve $(mz - ny) p + (nx - lz) q = ly - mx$.

(or)

14. (a) Solve $z = px + qy + p^2 + q^2$.

(b) Solve $(D^2 + 4DD' - 5D'^2)z = x + y^2 + \pi$.

15. A tightly stretched string with fixed end points $x = 0$ and $x = l$ is initially in a position given by $y(x, 0) = y_0 \sin^3 \left(\frac{\pi x}{l} \right)$. If it is released from rest from this position, find the displacement y at any time and at any distance from the end $x = 0$.

(or)

16. Find the temperature $u(x, t)$ in a silver bar which is perfectly insulated laterally, if the ends are kept at 0°C and it initially, the temperature is 5°C , at the center of the bar and falls uniformly to zero at its ends.
17. A long rectangular plate with insulated surface is one cm wide. If the temperature along one short edge $y = 0$ is $u(x, 0) = K(lx - x^2)$ degree, for $0 < x < l$. While the two long edges $x = 0$ and $x = l$ as well as the other short edge are kept at 0°C find the steady state temperature function $u(x, y)$.

(or)

18. Find the steady state temperature in a circular plate of radius 'a' which has one half of its circumference at 0°C and the other half at $K^\circ\text{C}$.
19. (a) State and prove convolution theorem on Fourier transform.

(b) Find the Fourier transform of $f(x)$ given by

$$f(x) = \begin{cases} 1 - |x| & \text{if } |x| < 1 \\ 0 & \text{for } |x| > 1 \end{cases}$$

and hence find the value of $\int_0^\infty \left(\frac{\sin^4 t}{t^4} \right) dt$.

(or)

20. (a) Find Fourier sine and cosine transform of x^{n-1} .

(b) Find the function if its sine transform is $\frac{e^{-as}}{s}$.