JUNE 2007

Code: AE-07

Subject: NUMERICAL ANALYSIS & COMPUTER PROGRAMMING
Time: 3 Hours
Max. Marks: 100

NOTE: There are 9 Questions in all.

- **Question 1 is compulsory and carries 20 marks. Answer to Q. 1. must be written in the space provided for it in the answer book supplied and nowhere else.**
- **Out of the remaining EIGHT Questions answer any FIVE Questions. Each question carries 16 marks.**
- **Any required data not explicitly given, may be suitably assumed and stated.**

Q.1 Choose the correct or best alternative in the following: (2x10)

a. The initial value problem $y' = y, y(0) = 1$ is given. An approximation to $y(0.2)$ by Taylor series method of second order with $h = 0.2$, is given by

(A) 1.22. **(B)** 1.2.
(C) 0.98. **(D)** 1.42.

b. The integration formula $\int_{-1}^{1} f(x)dx = af(-1) + bf(1/3)$ is exact for $f(x) = 1, x, x^2$.
The values of a, b respectively are

(A) $1/8, 7/8$. **(B)** $1/4, 5/4$.
(C) $1/4, 3/4$. **(D)** $1/2, 3/2$.

c. In the numerical differentiation formula
$f'(x_0) = [af(x_0) + 4f(x_0 + h) - f(x_0 + 2h)]/(2h)$, the value of a is

(A) 3. **(B)** –1.
(C) –3. **(D)** 1.

d. The linear least squares approximation to the following data is

x	-2	-1	1	2
$f(x)$	-7	0	2	9

(A) $1+3.4x$. **(B)** $3+7x$.

(C) $3.4+x$. (D) $5-3.x$.

e. The polynomial $p(x)$ of degree 2 satisfying the data $p(1)=1, p(3)=27, p(4)=64$ is

(A) $8x^2-7$. (B) $8x^2-19x+12$.

(C) $3x^2-3x+1$. (D) $5x^2-6x+2$.

f. The first divided difference $f[x_0, x_1]$ is approximately equal to

(A) f_0'. (B) $-f_0'$.

(C) f_0''. (D) f_0'/h.

g. What is the output of the following C program

```
#include<stdio.h>
main()
{
        int a, b = 1, c = 3 ;
        a = 2*b + c;
        b = 2*a + b/3;
        c /= 2;
        a = b + c + a ;
        printf ("\n b = % d \ c = % d \ a = % d \ n", b, c, a );
}
```

(A) $b=8\ c=1\ a=16$. (B) $b=10\ c=1\ a=14$.

(C) $b=10\ c=6\ a=16$. (D) $b=10\ c=1\ a=16$.

h. The body of a C program is as follows

```
main()
{
        int sum = 0, digit;
        long number, a;
        clrscr();
        printf ( "\nEnter positive integer number\n " );
        scanf (" %ld", &number );
        a = number;
        while (a>0)
        {
                digit = a%10;
                a/= 10;
                sum+= digit;
        }
printf ( "\n Result of %ld = %d \n", number, sum );
}
```

If the input is 5276, then output is : Result of 5276 = ? . The value of ? is

(A) 16. **(B)** 20.
(C) 26. **(D)** 52.

i. If the *if* statement does not have associated *else*, then what happens if the condition is *false*.

(A) gives an error message.
(B) Control is passed to the beginning of the program.
(C) Control is passed to the next statement after the *if* statement.
(D) Control is passed to *return*.

j. A root of $x^4 - x - 10 = 0$ is being determined by the Regula-Falsi method. It was found that it lies in the interval (1.8385, 2). In the next iteration, the root lies in

(A) (1.8385, 1.8536). **(B)** (1.8465, 2).
(C) (1.8536, 2). **(D)** (1.8465, 1.8536).

Answer any FIVE Questions out of EIGHT Questions.
Each question carries 16 marks.

Q.2 a. A positive root of the equation $e^x - \left[1 + x + \frac{x^2}{2} + \frac{x^3}{6} e^{0.3x}\right] = 0$ lies in the interval (2, 3).

(i) Starting with this interval, apply bisection method two times.

(ii) Taking the mid-point of the last interval obtained in (i) as initial approximation, refine the root using two iterations of Newton-Raphson method. **(9)**

b. Find the rate of convergence of the secant method for finding a simple root of the equation $f(x) = 0$. **(7)**

Q.3 a. Find the inverse of the matrix $A = \begin{bmatrix} 1 & 1 & 1 \\ 4 & 3 & -1 \\ 3 & 5 & 3 \end{bmatrix}$ using the LU decomposition method. **(8)**

b. The system of equations $x\cos(x\,y) + 0.5 = 0,\ \sin(x\,y) + 2x - y - 0.95 = 0$ has one solution near $x = 1, y = 2$. Obtain the approximation to the solution using two iterations of Newton's method. **(8)**

Q.4 a. Perform three iterations of Gauss-Jacobi method for the solution of the system of equations

$$\begin{bmatrix} 6 & 0 & 2 \\ 0 & 8 & 3 \\ 5 & 3 & 12 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -8 \\ 13 \\ -11 \end{bmatrix}$$

taking the initial approximations as $x_1^{(0)} = -0.9,\ x_2^{(0)} = 2.1,\ x_3^{(0)} = -1.1$. **(6)**

b. Obtain the iteration matrix of the Gauss-Jacobi method for the problem in (a). Hence, find the rate of convergence of the method. **(10)**

Q.5 a. Use divided differences to fit a polynomial for the data

x	−2	0	1	3	4	5	6
f(x)	−19	1	−1	91	233	471	829

(6)

b. If $\sum_{k=0}^{n-1} \Delta^2 f_k = a\Delta f_n + b\Delta f_0$, then find the values of *a* and *b*. **(3)**

c. Write a C program for finding a simple root of $f(x) = 0$ using Regula-Falsi method. Input the end points of the interval (*a, b*) in which the root lies, maximum number of iterations allowed (*n*), and the error tolerance tol. Output the value of the root and number of iterations taken. If number of iterations *n* is not sufficient, the program should output the same. Assume that $f(x) = \cos(x) - x * \exp(x)$. **(7)**

Q.6 a. Construct the forward difference table for the data

x	0	0.2	0.4	0.6	0.8	1.0
f(x)	−0.5	−0.476	−0.308	0.148	1.036	2.5

and hence find the value of f (0.3) **(7)**

b. A given data is to be approximated by the quadratic polynomial $f(x) = a + bx + cx^2$. Derive the normal equations using the least squares approximation. Hence, find the approximation to the data

x	−2	−1	0	1	3
f(x)	8.0	5.2	2.6	4.2	24.2

(9)

Q.7 a. Define a differentiation rule as $D(h) = [f(x+h) - 2f(x) + f(x-h)]/h^2$. Using Taylor series expansions, show that $f''(x) - D(h) = c_1h^2 + c_2h^4 + \ldots$ and write the expressions for c_1 and c_2. Using the above formula, compute approximations to $f''(0.5)$ with step lengths *h* = 0.2 and 0.1 from the following table of values.

x	0.3	0.4	0.5	0.6	0.7
f(x)	1.027	1.064	1.125	1.216	1.343

(9)

b. The error in the composite trapezoidal rule for evaluating $\int_a^b f(x)dx$ is bounded by $|\text{Error}| \le \frac{(b-a)h^2}{12} M_2$, $M_2 = \max |f''(x)|$, $x \in [a, b]$. Composite trapezoidal rule is being used to compute $\int_0^1 \frac{dx}{3+2x}$. Using the above error estimate, find *h* such that $|\text{Error}| \le 10^{-6}$. **(7)**

Q.8 a. Write a C program to evaluate $\int_a^b f(x)dx)$ by Simpson's rule of integration based on $2n+1$ points. Input the values of the limits *a, b* and *n*. Write $f(x) = x^2/(x^3+x+1)$ as a function program. Output all the data and the computed value. **(7)**

b. Evaluate the integral $\int_0^2 \frac{dx}{x^3+1}$ using Simpson's rule with 2, 4 and 8 subintervals. Hence, obtain a better estimate using Romberg integration (extrapolation). **(9)**

Q.9 a. Evaluate the integral $\int_0^2 \frac{dx}{5+2x}$ using the Gauss-Legendre two point formula. **(8)**

b. Given the initial value problem $y' = t^2 + y^2$, $y(1) = 2$, estimate *y*(1.4) with *h* = 0.2, using the Heun's method. **(8)**