DipIETE – ET / CS (OLD SCHEME)

Code: DE23 / DC23 **Subject: MATHEMATICS - II**
Time: 3 Hours **UNE 2009** **Max. Marks: 100**

NOTE: There are 9 Questions in all.

- **Question 1 is compulsory and carries 20 marks. Answer to Q. 1. must be written in the space provided for it in the answer book supplied and nowhere else.**
- **Out of the remaining EIGHT Questions answer any FIVE Questions. Each question carries 16 marks.**
- **Any required data not explicitly given, may be suitably assumed and stated.**

Q.1 Choose the correct or the best alternative in the following: (2x10)

a. Argument of $\sqrt{\frac{1+i}{1-i}}$ is

(A) 0 **(B)** $\frac{\pi}{4}$

(C) $\frac{\pi}{2}$ **(D)** π

b. $\frac{(\cos 3\theta + i \sin 3\theta)^4}{(\cos 4\theta + i \sin 4\theta)^3} \times \frac{(\cos 4\theta - i \sin 4\theta)^5}{(\cos 5\theta + i \sin 5\theta)^{-4}}$ is equal to

(A) 1 **(B)** –1
(C) 0 **(D)** None of these

c. $(i \times j) \cdot k + (j \times k) \cdot i + (k \times i) \cdot j$ is equal to

(A) 0 **(B)** 1
(C) 2 **(D)** 3

d. If $2i + j - mk$ is perpendicular to the sum of the vectors $i - j + 2k$ and $3i + 2j + k$, then m is equal to

(A) 1 **(B)** 2
(C) 3 **(D)** 4

e. $\begin{vmatrix} 1 & bc & a(b+c) \\ 1 & ca & b(c+a) \\ 1 & ab & c(a+b) \end{vmatrix}$ is equal to

(A) a **(B)** b
(C) c **(D)** 0

f. The sum and product of eigen values of $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 3 & -1 \\ 0 & -1 & 3 \end{bmatrix}$ are

(A) 7, 8 **(B)** 7, 3
(C) 3, 0 **(D)** 3, 1

g. If $A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ p & q & r \end{bmatrix}$ and I is the unit matrix of order 3, then A^3 is equal to

(A) $pA^2 + qA + rI$ **(B)** $rA^2 + qA + pI$
(C) $qA^2 + rA + pI$ **(D)** $rA^2 + pA + qI$

h. The inverse Laplace transform of $\frac{1}{s(s^2+1)}$ is

(A) 1 + sin t **(B)** 1 – sin t
(C) 1 + cos t **(D)** 1 – cos t

i. The period of the function $\text{Sin}\,\frac{2\pi x}{n}$ is

(A) 2 **(B)** π
(C) n **(D)** $\frac{1}{n}$

j. The solution of the differential equation $\frac{d^2y}{dx^2} - \frac{4dy}{dx} + 4y = \text{Sin } 2x$ is

(A) $y = (c_1 + c_2x)e^{2x} + \frac{1}{8}\text{Cos}2x$ **(B)** $y = (c_1 + c_2x)e^{2x} + \frac{1}{8}\text{Sin } 2x$
(C) $y = c_1e^{2x} + c_2e^{2x} + \text{Cos}2x$ **(D)** $y = (c_1 + c_2x)e^{2x} + \text{Sin } 2x$

Answer any FIVE Questions out of EIGHT Questions.
Each question carries 16 marks.

Q.2 a. Find the real and imaginary parts of tan (x + iy). **(8)**

b. Use De-Moivre's Theorem to solve the equation $x^5 + 1 = 0$. **(8)**

Q.3 a. If Z_1 and Z_2 are two complex numbers, show that
$|Z_1+Z_2|^2+|Z_1-Z_2|^2=2|Z_1|^2+2|Z_2|^2$. **(8)**

b. ABCDEF is a regular hexagon whose centroid is 0. Show that
$\overrightarrow{AB}+\overrightarrow{AC}+\overrightarrow{AD}+\overrightarrow{AE}+\overrightarrow{AF}=6\overrightarrow{AO}$ **(8)**

Q.4 a. Show that $(\vec{A}\times\vec{B})^2=(\vec{A}\cdot\vec{A})(\vec{B}\cdot\vec{B})-(\vec{A}\cdot\vec{B})^2$. **(8)**

b. A particle acted on by constant forces $4\hat{i}+\hat{j}-3\hat{k}$ and $3\hat{i}+\hat{j}-\hat{k}$ is displaced from the point $\hat{i}+2\hat{j}+3\hat{k}$ to the point $5\hat{i}+4\hat{j}+\hat{k}$. Find the total work done. **(8)**

Q.5 a. Evaluate $\begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix}$. **(8)**

b. Use Cramer's rule to solve the equations
$$3x+y+2z=3$$
$$2x-3y-z=-3$$
$$x+2y+z=4$$ **(8)**

Q.6 a. For what values of k the equations
$$x+y+z=1$$
$$2x+y+4z=k$$
$$4x+y+10z=k^2$$
has a solution and solve them completely in each case. **(8)**

b. Use Cayley-Hamilton theorem to find inverse of
$$\begin{bmatrix} 3 & 2 & 4 \\ 4 & 3 & 2 \\ 2 & 4 & 3 \end{bmatrix}$$ **(8)**

Q.7 a. Find the Laplace transform of $te^{-at}\sin 3t$. **(8)**

b. Find the inverse Laplace transform of $\log\frac{s+a}{s+b}$. **(8)**

Q.8 a. Solve the differential equation

$$\frac{d^2y}{dx^2} + 4y = e^x + \text{Sin } 2x \quad .$$ **(8)**

b. Use Laplace transform to solve the initial value problem

$$\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = e^{2x}$$

Given that $y = 0, \frac{dy}{dx} = 0$ at x = 0. **(8)**

Q.9 a. Find all values of $\left[\frac{1}{2} + i\frac{\sqrt{3}}{2}\right]^{3/4}$. **(6)**

b. Find a Fourier series expansion of the function

$$f(x) = x^2, \quad -\pi < x < \pi$$ **(10)**