

Option : II (Mathematics)

1. Let $\alpha(x) = \sin^{-1} 2x$ and $\beta(x) = \log(1+x)$ where $x \rightarrow 0$, which of following is true ?

- (a) The infinitesimal $\alpha(x)$ and $\beta(x)$ are equivalent
- (b) The infinitesimal $\alpha(x)$ is of lower order than that of the infinitesimal $\beta(x)$
- (c) The infinite $\beta(x)$ is of lower order than that of the infinitesimal $\alpha(x)$
- (d) The infinitesimals $\alpha(x)$ and $\beta(x)$ are of same order

2. The value of

$$\lim_{x \rightarrow 0} \left(\frac{\tan x - \sin x}{x^3} \right) \text{ is :}$$

- | | |
|-------------------|-------------------|
| (a) 0 | (b) $\frac{1}{2}$ |
| (c) $\frac{1}{4}$ | (d) 1 |

3. For the function $y = x \log x$, then nth derivative y_n , $n \geq 1$, equals :

- | | |
|-------------------------------|-----------------------------------|
| (a) $(-1)^{n-2} (n-2)! / x^n$ | (b) $(-1)^n (n-2)! / x^{n-1}$ |
| (c) $(-1)^{n-1} (n-1)! / x^n$ | (d) $(-1)^{n-1} (n-1)! / x^{n-1}$ |

4. If

$$V = \cos^{-1} \left(\frac{x+y}{\sqrt{x+y}} \right),$$

then $x \frac{\partial V}{\partial x} + y \frac{\partial V}{\partial y} = \dots$

(a) $-\frac{1}{2}$	(b) $\tan v$
(c) $-\cot v$	(d) $-\frac{1}{2} \cos v \cosec v$

5. The length of the polar subtangent for the curve

$$r = b(1+\cos \theta)$$

at the point $\theta = \tan^{-1} \left(\frac{3}{4} \right)$ is :

- | | |
|----------------------|----------------------------|
| (a) $-\frac{3b}{5}$ | (b) $\sqrt{\frac{7}{2}} b$ |
| (c) $-\frac{27b}{5}$ | (d) $\frac{\sqrt{21}a}{5}$ |

6. Which of the following is the evolute of the parabola $y^2 = 4x$?
 (a) $27y^2 = 4(x-2)^3$ (b) $27x^2 = 4(y-z)^3$
 (c) $8y^2 = 27(x-2)^3$ (d) $8x^2 = 27(y-z)^3$
7. The general value of θ satisfying the equation
 $\text{cis } \theta \text{ cis } 2\theta \text{ cis } 3\theta \dots \text{cis } n\theta = 1$ is :
 (a) $\frac{m\pi}{n(n+1)}$ (b) $\frac{4m\pi}{n(n+1)}$
 (c) $\frac{2m\pi}{n(n+1)}$, m is an integer (d) None of the above
8. For an integer k , $\sinh \{k\pi i + (-1)^k \mu\}$ equals :
 (a) $\sinh \mu$ (b) $\sin \mu$
 (c) $i \sinh \mu$ (d) $i \sin \mu$
9. The sum to infinity of the series :
 $1+r \cos \theta + r \cos 2\theta + \dots, |r| < 1$, is :
 (a) θ (b) $(1-r \cos \theta)/(1-2r \cos \theta + r^2)$
 (c) $r \sin \theta / (1-2r \cos \theta + r^2)$ (d) $1-r \cos \theta$
10. The co-ordinates of the focus of the parabola :
 $y^2 - 6y - 8y + 17 = 0$ are :
 (a) $(1, 3)$ (b) $(3, 1)$
 (c) $(3, 3)$ (d) $(1, 1)$
11. What is the equation of chord of the parabola $y^2 = 8x$ which is bisected at the point $(2, -3)$?
 (a) $3x + 4y + 6 = 0$ (b) $3x - 5y - 21 = 0$
 (c) $2x + y - 1 = 0$ (d) $4x + 3y + 1 = 0$
12. For what value of k , the line $y = x + k$ is the tangent to the ellipse $2x^2 + 3y^2 = 1$?
 (a) $\sqrt{5}$ (b) $-\sqrt{5}/6$
 (c) $\sqrt{5}/\sqrt{6}$ (d) $\pm\sqrt{6}$

13. Let P and D be the extremities of two conjugate diameters of the hyperbola

$$3x^2 - 4y^2 = 12$$

If C is the centre of the hyperbola, then $CP^2 - CD^2$ equals :

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|--------|-------|
| (a) 0 | (b) 1 |
| (c) 12 | (d) 7 |

14. The conic $34x^2 + 24xy + 41y^2 + 48x + 14y - 108 = 0$ represents :

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|---------------|-----------------------|
| (a) Parabola | (b) Ellipse |
| (c) Hyperbola | (d) None of the above |

15. A line makes angles of 60° and 45° with the positive directions of axes of x and y respectively, what angle does it make with positive direction of z-axis ?

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|-------------------------------|-------------------------------|
| (a) 60° or 120° | (b) 45° or 135° |
| (c) 90° | (d) None of the above |

16. Which of the following are the direction cosines of the normal to the plane

$$2x + 3y - z + 1 = 0$$

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|---|--|
| (a) $\left(\frac{2}{\sqrt{14}}, \frac{3}{\sqrt{14}}, -\frac{1}{\sqrt{14}} \right)$ | (b) $\left(\frac{2}{14}, \frac{3}{14}, -\frac{1}{14} \right)$ |
| (c) $\left(\frac{-2}{\sqrt{14}}, \frac{-3}{\sqrt{14}}, \frac{-1}{\sqrt{14}} \right)$ | (d) None of the above |

17. For what value of λ , the spheres

$$\lambda(x^2 + y^2 + z^2) + x + 3y + 4 = 0$$

and $x^2 + y^2 + z^2 + 2x - 2y + 4z - 3 = 0$ cut orthogonally ?

- | | |
|-------|--------------------|
| (a) 1 | (b) $\frac{1}{2}$ |
| (c) 2 | (d) $-\frac{2}{3}$ |

18. The equation of the cone whose vertex is at origin and which passes through the curve

$$x^2 + y^2 = 4, z = 2$$

- | | |
|-----------------------|---------------------------|
| (a) $x^2 + y^2 = z^2$ | (b) $x^2 + y^2 + z^2 = 0$ |
| (c) $z^2 + y^2 = y^2$ | (d) $x^2 - y^2 = z^2$ |

19. The equation of the cylinder whose generator are parallel to the line $x = y = z$ and touches the sphere $x^2 + y^2 + z^2 = 1$ is :

- (a) $(x + y + z)^2 = 3(x^2 + y^2 + z^2 - 2)$
- (b) $3(x + y + z)^2 = (x^2 + y^2 + z^2 - 1)$
- (c) $(x + y + z)^2 = (x^2 + y^2 + z^2 + 1)$
- (d) None of the above

20. If $\int \frac{x^2 dx}{(2x^2 + 1)(x^2 + 1)} = \tan^{-1} x + f(x)$,

then $f(x)$ equals :

- (a) $-\frac{1}{\sqrt{2}} \cot^{-1}\left(\frac{1}{x\sqrt{2}}\right)$
- (b) $\frac{1}{\sqrt{2}} \tan^{-1}\left(\frac{x}{\sqrt{2}}\right)$
- (c) $\sqrt{2} \tan^{-1}\left(\frac{x}{2}\right)$
- (d) $\frac{1}{\sqrt{2}} \cot^{-1}(x\sqrt{2})$

21. If m is an odd positive integer such that

$$K \int \frac{x^m}{\sqrt{1-x^2}} dx = \frac{\pi}{2}, \text{ then } k \text{ equals :}$$

- (a) $\frac{m!!}{(m-1)!!}$
- (b) $\frac{(m-1)!!}{m!!} \frac{\pi}{2}$
- (c) $\frac{(m-1)!!}{m!!} \pi$
- (d) None of the above

22. The value of

$$\lim_{n \rightarrow \infty} \left\{ \frac{1}{n^3+1} + \frac{1}{n^3+8} + \frac{9}{n^3+27} + \dots + \frac{1}{2n} \right\} \text{ equals :}$$

- (a) $\log 2$
- (b) $\log \sqrt[3]{2}$
- (c) $\log \sqrt{3}$
- (d) None of the above

23. Given that

$$\int_0^{\pi/2} \log \cos x dx = -\frac{\pi}{2} \log 2,$$

What is the value of $\int_0^{\pi/2} x \cot x dx$ is :

- (a) $-\pi \log 2$ (b) $\frac{\pi}{2} \log 2$
(c) $\frac{\pi}{2} \log \frac{1}{2}$ (d) $-\pi \log \frac{1}{2}$

24. The area bounded by the curve

$$y = \cos x, 0 \leq x \leq 2\pi,$$

the x-axis and the ordinate $x = 2\pi$ is :

- (a) 1 (b) 2
(c) 4 (d) π

25. Let f and g be two point function such

$$\operatorname{div}(f \times g) = h - f \cdot \operatorname{curl} g, \text{ then } h \text{ equals :}$$

- (a) $g \cdot \operatorname{curl} f$ (b) $f \cdot \operatorname{div} g$
(c) $g \cdot \operatorname{div} f$ (d) None of the above

26. Which of the following differential can be reduces to variable separable form by the transformation $x + y = z$?

- (a) $dy = (x^2 + y^2 - 2xy) dx$ (b) $(x-y) dy = (3x + 3y + 19) dx$
(c) $(x+y)^2 dy = (x^2 + y^2 + xy) dx$ (d) $(x^2 + y^2) dy = 5dx - 2xy dy$

27. If $xM(x, y) + yN(x, y) \neq 0$ and the differential equation $M(x, y) dx + N(x, y) dy = 0$ is homogeneous, then the integrating factor of the differential equation is :

- (a) $xM(x, y) + yN(x, y)$ (b) $1/(xM(x, y) + yN(x, y))$
(c) $\int (xM(x, y) + yN(x, y)) dx$ (d) None of the above

28. Which of the following is the particular integral of the differential equation

$$(D - a)^k y = e^{ax}, D \equiv \frac{d}{dx} ?$$

(a) $\frac{x^k}{k!} e^{ax}$

(b) $x^k e^{ax}$

(c) $(1 + x + x^2 + \dots + x^k) e^{ax}$

(d) None of the above

29. The solution of the differential equation

$$y = 2x \frac{dy}{dx} + y^2 \left(\frac{dy}{dx} \right)^3 \text{ is :}$$

(a) $y = 2cx + c^3$

(b) $y = 2cx - c^3$

(c) $y^2 = 2c^3 x + c$, c constant

(d) None of the above

30. For the Legendre polynomial $P_n(x)$ of degree n , $P_n^{-1}(-1)/P_n^{-1}(1)$ equals :

(a) $-\frac{1}{2}$

(b) $\frac{1}{2} (-1)^n$

(c) $(-1)^{n-1}$

(d) $\frac{1}{2} (n^2 + n)$

31. For any square matrix A , which of the following is not always true ?

(a) $A (\text{adj } A) = |A| I$

(b) $(\text{adj } A^t) = (\text{adj } A)^{-1}$

(c) $A^{-1} = (\text{adj } A)/|A|$

(d) $AA^t, A^t A$ are both symmetric matrices

32. For any non-singular matrix A of order n , $\text{adj}(\text{adj } A)$ equals :

(a) $|A|^{n-1}$

(b) $|A|^{n-2} A$

(c) $|A|^{n-1} I_n$

(d) $n |A|$

33. The characteristic roots of the matrix

$$\begin{bmatrix} 1 & 2 & 7 \\ 0 & -4 & 19 \\ 0 & 0 & 5 \end{bmatrix} \text{ are :}$$

(a) 1, 2, 7

(b) 2, -4, 0

(c) 1, -4, 5

(d) 7, 19, 5

34. The rank of matrix

$$\begin{bmatrix} 3 & 2 & 1 & 4 \\ 1 & 4 & 3 & 2 \\ 4 & 6 & 4 & 6 \\ 7 & 8 & 5 & 10 \end{bmatrix} \text{ is :}$$

35. The number of linearly independent solutions of the system of equations

$x + y + z = 0, 3x + 4y + 5z = 0, 2x + 3y + 4z = 0$ is :

36. For what value of λ and μ , the simultaneous equations,

$x + y + z = 6$, $x + 2y + 3z = 10$, $x + 2y + \lambda z = \mu$ have no solution?

- (a) $\lambda \neq 3, \mu = 10$ (b) $\lambda = 3, \mu \neq 10$
 (c) $\lambda = 3, \mu = 10$ (d) All real values of λ and μ .

37. If $1, \alpha, \beta, \gamma, \dots, \delta$ are the roots of equation $x^n - 1 = 0$, then

$(1-\alpha)(1-\beta)(1-\gamma)\dots(1-\delta)$ equals:

- (a) $n!$ (b) 1

$$(c) \quad \frac{n(n+1)}{2}$$

38. The equation of squared difference of the roots of the equation

$$x^3 - 6x^2 + 11x - 6 = 0$$
 is:

- (a) $x^3 - 2x^2 - 4 = 0$ (b) $x^3 - 6x^2 + 9x - 4 = 0$
 (c) $x^3 - 2x^2 + x + 4 = 0$ (d) None of the above

39. The sum of the cubes of the roots of the equation

$$x^5 - x^2 - x - 1 = 0$$

40. If $b^2 + a^3$ is positive, then the roots of the equation

$$x^2 + 3ax + 2b = 0, a, b \in \mathbb{R},$$

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- (a) All real and distinct (b) All real and rational
 (c) All real and equal (d) One real and two imaginary

41. The infimum of the set

$\left\{-2, -\frac{3}{2}, -\frac{4}{3}, -\frac{5}{4}, \dots\right\}$ is :

42. Which of the following is true? The sequence

$$\left\{ (-1)^n \left(1 = \frac{1}{n} \right) \right\}$$

- (a) Converges to the limit 1 (b) Converges to the limit -1
(c) Diverges to $+\infty$ (d) Oscillates finitely

43. For what value of x , the series

$$\sum_{n=0}^{\infty} \frac{2 \cdot 4 \cdot 6 \cdots (2n+2)}{3 \cdot 5 \cdot 7 \cdots (2n+3)} x^{n+1}, \quad x > 0 \text{ converges:}$$

- (a) $1 < x \leq 2$ (b) $0 < x < 1$
 (c) $x \geq 1$ (d) $x \geq 0$.

44. Let

$f(x) = |x| + |x-1|$, $x \in \mathbb{R}$, then at the point $x=0$, the function $f(x)$ is:

- (a) Continuous but not derivable (b) Continuous as well as derivable
(c) Derivable but not continuous (d) Neither continuous nor derivable

45. If Q is a refinement of a partition P of an interval $[a, b]$, then for a bounded function $f(x)$ on $[a, b]$, which of the following is always true?

- (a) $\angle(Q, f) < \angle(P, f)$ (b) $U(Q, f) > U(P, f)$
 (c) $\angle(Q, f) > U(Q, f)$ (d) $U(Q, f) \leq U(P, f)$

46. Let

$$f(x) = x, \text{ when } x \text{ is rational}$$

$= -x$, when x is an irrational, then which of the following is true?

- (a) $f(x)$ and $|f(x)|$ are both integrable on $[0,1]$
 - (b) $f(x)$ is not integrable on $[0,1]$ but $|f(x)|$ is integrable on $[0,1]$
 - (c) $f(x)$ is integrable on $[0,1]$ but $|f(x)|$ is not integrable on $[0,1]$
 - (d) Neither $f(x)$ nor $|f(x)|$ is integrable on $[0,1]$

47. Let

$$f(x, y) = \frac{xy(x^2 - y^2)}{(x^2 + y^2)}, (x, y) \neq (0, 0)$$

$$= 0, \quad x = y = 0,$$

then what is the value of $f_{xy}(0,0)$?

48. Given that $xyz = abc$, what is the minimum value of $bcx + cay + abz$?

49. The value of

$$\int_C (ydx - xdy)$$

Where c is the ellipse $x = a \cos t$, $y = b \sin t$, taken in the clockwise direction, is :

- (a) πab (b) $2\pi ab$
 (c) $\frac{\pi}{2}ab$ (d) None of the above

50. The volume of solid bounded by the sphere $x^2 + y^2 + z^2 = 4$ and the surface of the paraboloid $x^2 + y^2 = 3z$ is :

- (a) $\frac{5\pi}{6}$ (b) $\frac{19\pi}{3}$
 (c) 19π (d) None of the above

51. If Q^+ is the set of all positive rational numbers and \ast a binary operation on Q^+ defined

by $a * b = \frac{ab}{2}$, then the inverse of an element $a \in Q^+$ is:

- (a) $\frac{1}{a}$ (b) 2
 (c) $\frac{4}{a}$ (d) None of the above

52. Assume that R is a relation on a set A , which of the following is not true ?

- (a) If R is symmetric, then R^{-1} is symmetric
- (b) If R is anti-symmetric, then R^{-1} is anti-symmetric
- (c) If R is reflexive, then $R \cap R^{-1} \neq \emptyset$
- (d) If R is symmetric, then $R \cap R^{-1} \neq \emptyset$

53. Which of the following is a proper subgroup of the multiplicative group

$$G = \{1, -1, i, -i\} \text{ where } i^2 = -1 ?$$

- (a) $\{1, i\}$
- (b) $\{1, -1\}$
- (c) $\{i, -i\}$
- (d) None of the above

54. Let G be a group (finite) and let $a \in G$. If $o(a) = n$ where $o(a)$ denotes the order of an element a and if a positive integer k divides n , then which of the following is equal to $o(a^k)$?

- (a) $\frac{n}{k}$
- (b) $n + k$
- (c) nk
- (d) None of the above

55. Let f be a homomorphism of a group G into a group \bar{G} with Kernel K_f . If f is an isomorphism of G into \bar{G} , then which of the following is true ?

- (a) $K_f = \bar{G}$
- (b) $K_f \neq \{e\}$
- (c) $K_f = \{e\}$
- (d) None of the above

56. The number of distinct cycles of length r , $r \leq n$ in S_n , the symmetric group of order n , is :

- (a) $\frac{n!}{(n-r)!}$
- (b) $\frac{n!}{r(n-r)!}$
- (c) $\frac{n!}{r!(n-r)!}$
- (d) $\frac{n!}{r!(n-r)!}$

57. Consider the Ring

$$R = \{0, 1, 2, 3, 4, 5, 6, 7, 8\}$$

under the addition and multiplication modulo 8. Then the units of the ring are :

- (a) 0, 2, 4, 6
- (b) 1, 3, 5, 7
- (c) 0, 1, 2, 3
- (d) None of the above

58. An ideal m of a Commutative Ring R with unity is a maximal ideal if and only if $\frac{R}{M}$ is:

59. If U and W are the subspaces of a finite dimensional vector space V and if $\dim U = \alpha$, $\dim W = \beta$ and $\dim (U \cap W) = \delta$, then $\dim (U+W)$ equals :

- (a) $\frac{\alpha\beta}{\delta}$ (b) $\frac{\alpha+\beta}{\delta}$
 (c) $\alpha\beta - \delta$ (d) $\alpha + \beta - \delta$

60. Under which of the following condition, the vectors

$(1, 1, 1), (1, x, x^2)$ and $(1, -x, x^2)$ form a basis of C^3 ?

- (a) $\chi \in \{0, 1, -1\}$ (b) $\chi \in \{0, 1, -1\}$
 (c) $\chi \in \{1, 2, 4\}$ (d) None of the above