INSTITUTE OF ACTUARIES OF INDIA

EXAMINATIONS

15th November 2010

Subject CT1 – Financial Mathematics

Time allowed: Three Hours (15.00 – 18.00 Hrs)

Total Marks: 100

INSTRUCTIONS TO THE CANDIDATES

1. *Please read the instructions on the front page of answer booklet and instructions to examinees sent along with hall ticket carefully and follow without exception*

2. *Mark allocations are shown in brackets.*

3. *Attempt all questions, beginning your answer to each question on a separate sheet. However, answers to objective type questions could be written on the same sheet.*

4. *In addition to this paper you will be provided with graph paper, if required.*

AT THE END OF THE EXAMINATION

Please return your answer book and this question paper to the supervisor separately.

Q. 1)   **a)**   Calculate the accumulated value after 5 years of an investment of ₹1000 made now for the following interest rate scenarios :

       i)   At simple rate of interest of 6% p.a. for first 3 years, reinvested at a simple rate of interest of 7% p.a. for next 2 years.   (2)

       ii)   At interest rate of 5% p.a. convertible monthly for first 2 ½ years; 2% per quarter for next 1 year; and at a force of interest of 8% p.a. thereafter.   (2)

   b)   The rate of discount per annum convertible quarterly is 8%.
Calculate up to 4 decimal places:

       (i)   The equivalent rate of interest per annum convertible half-yearly.   (2)

       (ii)   The equivalent rate of discount per annum convertible monthly.   (1)

       (iii) The equivalent rate of interest per annum convertible once in two years.   (1)

       [8]

Q. 2)   **a)**   Define the following and express algebraically explaining the symbols used :

       (i)   Real rate of return   (2)

       (ii)   Time weighted rate of return   (2)

   b)   You wish to purchase a car in 4 years' time. The current cost of the car is ₹450,000 which is expected to increase by 5% p.a. over the next 4 years.

In order to match your expected expenditure of car purchase, you invest ₹400,000 now in a 4-year index linked security which pays nominal coupon of 5% annually in arrear and is redeemable at par. The coupon and capital payments are inflated by reference to an inflation index value with 2 months time lag. The inflation index value 2 months before issue was 115.

The table below shows the index values at other times.

Time	0	$^{10}/_{12}$	1	$1^{10}/_{12}$	2	$2^{10}/_{12}$	3	$3^{10}/_{12}$	4
Index	117	123	124	128	130	133	134	141	142

The coupons received are invested till the end of 4 years in a bank account which provides an annual return of 4% p.a. Determine whether your investment proceeds at the end of 4 years from now will be sufficient to buy the car or not?   (5)

       [9]

Q. 3)   **a)**   An investor buys a 20-year deferred annuity plan from a finance company. Under the plan the investor will pay a monthly installment of premium every month in advance for 20 years. At the end of 20 years, he will receive a monthly annuity in arrears payable for 20 years. The amount of the monthly annuity payment will be ₹ 10000 in the first year. From second year and onwards, monthly annuity for each year will increase by 3% p.a. Determine the monthly installment that the investor would pay to achieve an effective rate of return of 4% per half year.   (6)

b) A 3-year annuity has the following payment schedule:

Year 1 ₹ 6,500 per annum paid continuously.
Year 2 ₹ 6,500 per annum paid monthly in advance.
Year 3 ₹ 6,500 per annum paid half yearly in advance.

Calculate the total present value of these payments at the beginning of the first year at a rate of interest of 9% per annum convertible quarterly. (8)

[14]

Q. 4) Jia takes a loan which is to be repaid in installments annually in arrears. The first installment is ₹ 160, the second ₹ 155 and so on with the payments reducing by ₹ 5 p.a. until the end of the 15th year after which there are no further payments.

The rate of interest charged by the lender is 8% p.a. effective.

a) Calculate the amount of the loan. (3)

b) Calculate the interest and capital components of the third payment. (3)

c) Calculate the amount of capital repaid in the instalment at the end of the thirteenth year. (3)

[9]

Q. 5) a) Define the following investments and describe the associated cash-flows :

   i) Index linked security (2)

   ii) Call Deposits (2)

   iii) Equity (2)

b) The shares of a company pay dividends every 6 months, with the next dividend being due in exactly 2 months' time. The next dividend is expected to be ₹ 32.00. Each half yearly dividend payment is expected to grow at a compound rate of 4% per annum from the level of ₹ 32.00. Dividends are expected to be paid in perpetuity.

Determine the price of the share for an investor who is subject to Income Tax of 20% on the dividend and who wishes to realize a net annual effective yield of 9%. (4)

c) Eva purchased 100 shares of the above company at a price of ₹ 989 per share. The next dividend of ₹ 32 is due in exactly 6 months' time. As above, it is expected to grow at a compound rate of 4% p.a. from the level of ₹ 32.00. She sold 50 shares at the end of 2 years, at a price of ₹ 1114 per share and the remaining at the end of 7 years at a price of ₹ 2340 per share. The sale made at any point is after receipt of the dividend then due. All capital gains are taxed at 10%. The income tax rates are 20% of the dividend. Calculate the net effective annual return earned by Eva on the whole investment over the 7 year period. (8)

[18]

Q. 6) An investor is considering investing in a project with cash flows as below :

Investments :

₹ 150 crores – at the start of the project
₹ 20 crores – at the end of each month for the following six months
₹3 crores – per annum payable monthly at the end of each month with first payment at the end of 7th month till 20 years from the start of the project

Income :

₹ 3 crores – per month for 19 ½ years at the start of each month beginning from 7th month
₹ 500 crores – at the end of 20 years

a) Calculate the Net Present Value of the project at 8% p.a. effective.   (7)

b) Calculate the DPP for the project assuming an effective rate of interest of 8% p.a.   (5)

[12]

Q. 7) A finance company has the following liabilities:

Annuity payments of ₹ 85,000 per annum to be paid annually in arrears for the next 14 years and a lump sum of ₹ 300,000 to be paid in 11 years. It wishes to invest in two fixed-interest bonds in order to immunize its liabilities. Bond P has a coupon rate of 9% per annum and a term to redemption of nine years. Bond Q has a coupon rate of 5.5% per annum and a term to redemption of 24 years. Both securities are redeemable at par and pay coupons annually in arrears.

a) Calculate the present value of the liabilities at a rate of interest of 7% per annum effective.   (2)

b) Calculate the discounted mean term of the liabilities at a rate of interest of 7% per annum effective.   (4)

c) Calculate the nominal amount of each security that should be purchased so that both the present value and discounted mean terms of assets and liabilities are equal.   (7)

d) Without further calculation, comment on whether, if the conditions in (c) are fulfilled, the finance company is likely to be immunized against small, uniform changes in the rate of interest.   (2)

[15]

Q. 8)   An investor wishes to invest ₹ 1000 in a fixed deposit scheme. There are two possible investment scenarios.

Scenario 1 :
The amount will be invested in a fund where expected return will be 6.75% p.a. effective and the standard deviation of annual returns is 5%. The annual effective rates of return are independent and $(1+i_t)$ is log-normally distributed where i_t is the return in year t.

Scenario 2:
The amount would earn 7% p.a. effective for the first four years. After 4 years, the accumulated sum will be invested for a further 6 years at the then prevailing market rate. This market rate is expected to be 4% p.a. with probability 0.1, 5% p.a. with probability 0.3, 6.5% p.a. with probability 0.3, 8% p.a. with probability 0.3.

The investor will receive the accumulated amount at the end of 10 years. There are no intermediate payments made to him during the 10-year period.

For the accumulated amount received at the end of 10 years, Calculate

a)   The expected value and Standard deviation if invested in Scenario 1.   (8)

b)   The expected value and Standard deviation if invested in Scenario 2.   (7)

[15]

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