AC-3370-A-B

Seat No.__________

M. Phil. Examination

April / May – 2003

Statistics : Paper-III

(Optional Paper)

1 - Demography

2 - Statistical Methods for Reliability and Life testing

Time : **3** Hours] [Total Marks : **100**

Instructions : (i) **Each** question carries **20** marks.
(ii) Use of statistical **tables** and scientific **calculator** is permitted.

1 - Demography

1 Define population projection. Discuss different methods in detail used for population projection.

OR

1 (a) Explain the following terms with suitable examples :
(i) Crude Death Rate
(ii) Specific Death Rate
(iii) Age Specific Death Rate
(iv) Standardised Death Rate.

(b) From the following data, calculate
(i) the unemployment rate for the country as a whole
(ii) the crude unemployment rate for districts A and B separately.
(iii) the standardised unemployment rate for districts A and B separately.

Age distribution per 1000 of the population				***Percentage unemployed***		
Age group	**District A**	**District B**	**Whole Country**	**District A**	**District B**	**Whole Country**
15-29	300	200	240	3.5	3.0	4.0
30-44	320	350	360	9.0	8.0	7.5
45-59	325	300	290	11.5	15.0	12.5
60 and over	55	150	110	22.0	25.0	18.0

2 (a) Define the terms :
(i) $p(x, n($

(ii) $l(x)$

(iii) $L(x)$

(iv) $T(x)$

(v) $\mathring{e}(x)$.

(b) In usual notations show that

(i) $\sum_{i=1}^{\infty} l(x+i) = T(x) - \frac{1}{2} l(x)$

(ii) $e(x) = \mathring{e}(x) - \frac{1}{2}$

(iii) $e(x) = \sum_{i=1}^{\infty} \frac{l(x+i)}{l(x)}$

(iv) $l\left[x + \frac{1}{2}\right] = L(x)$.

(c) The number of persons dying at age 42 is 492 and the complete expectation of life at age 42 and 43 years are 5.36 and 5.02 years respectively. Find the number of persons living at ages 42 and 43.

OR

2 (a) Explain the terms :

(i) Stable population

(ii) Stationary population

(iii) Expectation of life

(iv) Force of mortality.

(b) In usual notations show that

(i) $T(x) = \mathring{e}(x) \cdot l(x)$

(ii) $q(x) = \frac{1 - [e(x) - e(x+1)]}{1 + e(x+1)}$

(iii) $\frac{dL(x)}{d(x)} = -d(x)$

(iv) $\frac{dT(x)}{d(x)} = -l(x)$.

(c) A book seller takes in a stock of 100 books in statistics subject of a new publishing house. In the first week he sells 20% of them, in the second week 45% of the remaining, in the third week 50% of the remaining and in the fourth week 65% of the remaining. In the fifth week he sells all he as left.

(i) Draw up a weekly 'life table' for 100 books of statistics.

(ii) What is the chance that one of the original 100 books
 (i) is sold in the first week ?
 (ii) is sold in the second week
 (iii) is unsold in the fourth week ?

(iii) Assuming that sales may take place at any time of the day and on any day of the week; how long on the average would a book of statistics lie on the book case before being sold ?

3 (a) Define Central Mortality Rate $m(x)$.

Show that

(i) $q(x) = \dfrac{2m(x)}{2+m(x)}$

(ii) $\mu\left(x+\dfrac{1}{2}\right) = m(x)$.

(b) (i) If $p(x,n) = \dfrac{x}{x+n}$, find $\mu(x)$

(ii) If $\mu(x) = ax^2+bx+c$, find $l(x)$.

(c) Show that the approximate value of the force of mortality $\mu(x)$ is :

$$\frac{1}{12l(x)}\left[8\{l(x-1)-l(x+1)\}-\{l(x-2)-l(x+2)\}\right]$$

OR

3 (a) Explain the structure of Abridged life table. Describe King's method of constructing an abridged life table.

(b) In usual notations show that

$$q(x,n) = \frac{2n\cdot m(x,n)}{2+m(x,n)-\frac{1}{6}\left\{m(x,n)^2-\frac{d}{dx}\left(m(x,n)^2\right)\right\}}.$$

(c) Using Grevilles method of construction of an Abridged life table, calculate the expectation of life from the following data :

Age $(x - x+10)$	Central Rate of Mortality
35 – 44	.0711
45 – 54	.1028
55 – 64	.1571
65 – 74	.2400
75 – 84	.3061
85 – 94	.4221

4 (a) Explain the following terms with suitable examples :

(i) Crude Birth Rate

(ii) General Fertility Rate

(iii) Age Specific Fertility Rate

(iv) Total Fertility Rate

(v) Gross Reproduction Rate

(vi) Net Reproduction Rate.

(b) The following table relates to the female population in one State of India in year 1991. The number of female live births are classified according to the age of mothers and the survival rate for females. Calculate the various measures of population Growth and comment on your findings.

Age	***Female population (100)***	***Total number of female live births (100)***	***Sarvival rate per 1,00,000***
15 – 19	**15,7670**	**4,632**	**58,065**
20 – 24	**14,7624**	**14,443**	**55,870**
25 – 29	**1,24,200**	**14,058**	**52,981**
30 – 34	**1,05,865**	**8,329**	**48,963**
35 – 39	**89,265**	**4,036**	**44,146**
40 – 44	**77,887**	**2,158**	**39,154**
45 – 49	**61,161**	**689**	**34,198**

OR

4 (a) Discuss Method of Pearl and Reed for fitting a Logistic Curve for Population Growth.

(b) For the following values of $l(x)$ of a life table, fit a Makeham Curve. Also fit a Gompertz Curve to the following data and find the fitted values.

x	$l(x)$	x	$l(x)$
43	79,737	49	73,896
44	78,842	50	72,795
45	77,918	51	71,651
46	76,964	52	70,458
47	75,878	53	69,215
48	74,957	54	67,919

5 Write short notes on any **three** of the following :

(i) Census

(ii) Uses of Vital Statistics

(iii) Migration

(iv) Demographic process and Basic Measures

(v) Construction of Life tables

(vi) Nuptiality and Natality.

2 - Statistical Methods for Reliability and Life testing

1 (a) Define the terms :

(i) Reliability

(ii) Hazard function.

Derive the relationship between hazard function and probability function in case of continuous lifetime random variable.

(b) Derive and identify the lifetime model whose hazard function is $\frac{\beta}{\eta}\left(\frac{x-\vartheta}{\eta}\right)^{\beta-1}$; $x \geq \vartheta$ and discuss the properties of the hazard function.

(c) Find MTTF of a system with reliability function $R(t) = \frac{1}{2}e^{-t/2} + \frac{1}{2}e^{-t/3}$. Show that the failure rate function of this system is $h(t) = \left[\frac{1}{4} + \frac{1}{3} e^{-\frac{1}{6}t}\right] \Big/ \left[1 + e^{-t/6}\right]$. Also show that $h(t)$ is a decreasing failure rate function. What is the probability that the unit will fail between $t = 2$ and $t = 3$ given that it survived 2 units of time ?

OR

1 (a) Explain the terms :
MTBF, MTTF.
Discuss the nonparametric method of estimating reliability and hazard rate in case of grouped and ungrouped failure data.

(b) The following ordered failure times were observed for a heat exchanger used in the alkylation unit of a gasoline refinery. The failure mode was "leaking in one or more sections of a four-in-a-bank unit". The observed failure times were : 0.41, 0.58, 0.75, 0.83, 1.00, 1.08, 1.17, 1.25 and 1.35 years. Calculate hazard rate and reliability at time $t = 1.00, 1.08, 1.17$.

(c) Explain the method of calculating the failure rate using the χ^2–distribution with $(1-\alpha)$ 100% confidence level. If 100 resistors were placed on test for 1000 hrs and during this period 10 resistors failed, calculate the failure rate to provide a confidence of 95%.

2 (a) If $T_1,\ T_2, ...T_n$ are independent and identically distributed exponential with failure rate $\lambda\ (\lambda > 0)$ and let $G_k = T_{(k)} - T_{(k-1)}$ for $k = 1, 2, ..., n$; $T_{(0)} = 0$, where T_i is the i^{th} ordered observation, then show that $G_1,\ G_2, ..., G_n$ are independent and $P(G_k \geq t) = e^{-(n-k+1)\lambda t}, t \geq 0$.
Hence or otherwise find $E\left[T_{(r)}\right]$ and $V\left[T_{(r)}\right]$.

(b) If n items are put up to test and test is terminated after all the items have failed. If failure time distribution is exponential with mean $\theta,\ \theta > 0$ then obtain MLE and UMVUE of $R(t)$. What will be expected termination time of the test ?

OR

2 (a) Define Type I and Type II censoring. Suppose n items are under test with replacement and failure time distribution is exponential with mean $\frac{1}{\lambda}, \lambda > 0; w_1, w_2, \ldots\ldots$ are the interevent times for a point process then show that the number of events in the interval $[0, t)$ has the Poisson distribution with parameter $n\lambda t$.

(b) Let n items are put up to test for t hours under exponential failure without replacement. Derive Mle of $\theta, \theta > 0$, where θ is the mean of the lifetime model. Also obtain its bias and MSE.

3 (a) Explain progressive censoring in connection of failure censored scheme.

(b) For k-stage Type II progressive censoring without replacement derive ML estimators of the parameters for exponential life-time model having failure rate $\frac{1}{\theta}, \theta > 0$. Also discuss some properties of the estimators. Derive ML estimator of reliability of a component at time t.

(c) Derive UMPU test for testing $\boldsymbol{H}: \theta = \theta_0$ versus $\boldsymbol{K}: \theta \neq \theta_0$ on the basis of the data obtained under Type II censoring without replacement, where θ is the hazard rate of a continuous lifetime model.

OR

3 (a) Explain Type I progressive censoring with k-stages. Derive the composite pdf in case of Type I progressive censoring without replacement scheme. Obtain ML estimators of the parameters for exponential lifetime model under the above censoring scheme when observations are available in group.

(b) Derive expressions for bias and MSE of the maximum likelihood estimator derived in the above part (a).

4 (a) Explain the concept of reliability of series and parallel systems. In case of a series system with k-components each component c_i has exponential lifetime distribution with mean $\theta_i, i = 1, 2, \ldots\ldots, k$; derive failure time distribution and reliability function of the system. Obtain an expression for mean time between failures for the system.

(b) For a series system of n identical independent components, such that each component has an exponential failure time distributions with mean $\theta, \theta > 0$. If n is large derive the failure time distribution of the system.

(c) Explain the concept of k out of n system. Give some practical illustrations for the system. What will be the case when $k = 1$ and $k = n$. Give an expression for the system reliability.

OR

4 (a) Define different types of standby redundancy. For cold standby redundancy derive the general expression of the system relaibility assuming that the switching and sensing over device is not 100% reliable. What will be the change when switching and sensing is 100% reliable ?

(b) Show that the reliability of standby system with n-components having constant and equal failure rate λ, $\lambda > 0$, is given by

$$R_{sb}(t) = e^{-\lambda t} \sum_{r=0}^{n-1} \frac{(\lambda t)^r}{r!}.$$

5 (a) Explain non parametric estimation of the survival function. Give an expression for the estimate of variance of an estimator of the survival function. List out the properties of the estimator.

(b) How can you derive interval estimates of the survival function in case of uncensored data ? Explain fully.

OR

5 (a) What is Bridge structure of a system ? Explain the method based on Bayes' theorem to obtain reliability of a bridge structure with suitable illustration.

(b) In connection of system reliability define minimal cuts and minimal paths. Explain the method with illustration to obtain system reliability using minimal cuts and minimal paths.