This question paper contains 8 printed pages.]

Your Roll No ....

5165

B.Sc. Prog./B.Sc. (Hons.)/I J

M.A. 107-B – MATHEMATICS

(For Life Sciences)

(NC – Admission of 2008 onwards)

Time : 3 Hours **Maximum Marks : 75**

(Write your Roll No on the top immediately on receipt of this question paper.)

There are three Sections in this question paper.

Attempt any **two** questions from each Section.

Students are allowed to use calculators.

Section – I

1. (a) Consider a spherical cell of volume V and surface S. Express V as a function of S Is it a linear function ? 4

   (b) A culture of bacteria initially weighs 1 gm and is doubling in size every hour How long will it take to reach a weight of 3 gms 4

(c) The weight of a certain stock of fish is given by W = nw, where n is the size of the stock and w the average weight of each fish. If n and w change with time t according to the formulas $n = (2t^2 + 3)$ and $w = (t^2 - t + 2)$, find the rate of change of W w.r.t. time t. 4½

2\. (a) Assume that a population of size 25000 (at time t = 0) grows according to the formula $N = 25000 + 45t^2$ where the time t is measured in days. Find the average growth rate in the time intervals from t = 0 to t = 2. 4½

(b) Find :

(i) $\lim_{h \to 0} \frac{4 - h}{2 + 7h}$

(ii) $\lim_{h \to 0} \frac{4 - (2 + h)^2}{1 - (1 - h)^2}$ 4

(c) Show that for Fibonacci numbers

$a_1 + a_2 + \quad + a_n = a_{n+2} - 1$ 4

3\. (a) Integrate

(i) $\int (3x - 7)^5 dx$

(ii) $\int \sin(5 - 3x)\,dx$

(iii) $\int \frac{\log x}{x}\,dx$. 7½

(b) An individual suffering from a certain disease is administered an amount x of a suitable drug. His probability of being cured is $\frac{\sqrt{x}}{3(1+x)}$.

Find the value of x that gives him the maximum probability of being cured. 5

Section – II

4 (a) If $A = \begin{pmatrix} 1 & 1 & 1 & 1 \\ -1 & 1 & -1 & 1 \end{pmatrix}$ and $B^T = (a\ b\ c\ d)$

when T stands for Transpose

Calculate

(i) $A(B^T)^T$ (ii) $B^T A^T$, and show that

$(AB)^T = B^T A^T$. 4

(b) A signal operated by a laboratory mouse has only two faces : R = red, Y = yellow. At each trial the mouse may or may not change the signal Suppose that the following transition probabilities are given :

$$R \longrightarrow R \cdot p_{11} = 0.8$$

$$R \longrightarrow Y \cdot p_{12} = 0.2$$

$$Y \longrightarrow R \,.\, p_{21} = 0.6$$

$$Y \longrightarrow Y \quad p_{22} = 0.4$$

Assume further that each trial is independent of past experience. Then the outcomes of each trial form a Markov chain with two states (R and Y). Establish the transition matrix with the above probabilities. Also, calculate the probabilities for two-step transitions keeping into mind the fact that under the assumption of Markov chains the multiplication rule holds. 4½

(c) If $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}, B = \begin{pmatrix} 2 & 5 \\ 0 & 4 \end{pmatrix}, C = \begin{pmatrix} -1 & -2 \\ 3 & 0 \end{pmatrix}$

Find out A(B + C) in two ways according to the distributive law 4

5. (a) If $Q = (n^2 + y^2)^{1/2}$, verify that

$$\frac{\partial^2 Q}{\partial x^2} + \frac{\partial^2 Q}{\partial y^2} = \frac{1}{Q}$$ 4

(b) Some biological rhythms are described by the second order differential equation

$$\frac{d^2 x}{dt^2} + kn = 0 \ (k > 0)$$

Show that n = A cos wt + B sin wt is the solution of the differential equation where $w^2 = k$ 4½

(c) If $z = ax^2 + 2hxy + by^2$, verify that

$$\frac{\partial^2 z}{\partial y \partial x} = \frac{\partial^2 z}{\partial x \partial y}$$ 4

6 (a) Show that $y = \frac{c}{x} + d$ is a solution of the differential equation $\frac{dy}{dx} + \frac{c}{x^2} = 0$

Further, plot this solution for c = 1, d = 0 and c = − 1, d = 0, take $x > 0$ 6½

(b) Assume that a population grows in such a way that the specific growth rate $\frac{1}{N}\frac{dN}{dt}$ remains constant Let N_1 be the number of individuals at the time instant t_1. Find $N = N(t)$ **6**

Section – III

7. (a) The following are the weights (kg) of the 6 subjects in the sample studied by a scientist

83 9, 99.0, 63.8, 71 3, 65.3, 79 6

Compute the mean and standard deviation. **6½**

(b) Suppose that over a period of several years the average number of deaths from a certain non-contagious disease has been 10 If the number of deaths from this disease follows the Poisson distribution, what is the probability that during the current year

(i) exactly seven people will die from the disease

(ii) ten or more people will die from the disease (Given $e^{-10} = 0\ 000045$) **6**

8 (a) The heights of a certain population of individuals are normally distributed with a mean of 70 inches and a standard deviation of 3 inches What is the probability that a person picked at random from the group will be between 65 and 74 inches tall ?

(Area under the standard normal curve from 0 to 1 33 = 0 4082

Area under the standard normal curve from 0 to 1 67 = 0 4525) **6½**

(b) Find the equations of regression lines for the following values of x and y

x	1	2	3	4	5
y	2	5	3	8	7

Also estimate y for $x = 10$ **6**

9. (a) In a health survey of school children, the mean haemoglobin level of 55 boys was found to be 10.2 g per 100 ml with a standard deviation 2.1 g Can it be considered that this group of boys is identified from a population with a mean of 11.0 g / 100 ml **6**

(b) Hearing levels in two groups of school children with normal hearing in frequency of 500 cycles per second was found as follows :

	No of Children	Hearing ($\bar{x}$) threshold	S D. (σ)
Group I	62	15.5 dB	6.5 dB
Group II	76	20 dB	7.1 dB

Test at 5% level of significance if there is any difference between hearing levels recorded in two groups. 6½