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Your Roll No . . . . .

**5164**

**B.Sc. Prog./I**

**J**

**MA-107 (a) – MATHEMATICS – I**

**(For Physical Sciences)**

**(NC – Admission of 2008 onwards)**

**Time : 3 Hours**

**Maximum Marks : 75**

*(Write your Roll No on the top immediately on receipt of this question paper )*

Attempt any **two** questions from each Section

**Section – I**

- 1 (a) A boat is travelling towards east across a river at the rate of 4 miles per hour while the river's current is flowing at the rate of 3 miles per hour towards south. Find the resultant velocity and sketch its approximate direction **6**
- (b) Show that the set  $\left\{ \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \end{bmatrix} \right\}$  is a basis for  $\mathbb{R}^2$  **6**

- 2 (a) Let  $T : \mathbb{R}^2 \rightarrow \mathbb{R}^2$  be defined by

$$T \left( \begin{bmatrix} x \\ y \end{bmatrix} \right) = \begin{bmatrix} -y \\ -x \end{bmatrix}$$

Show that  $T$  is a linear transform. 6

- (b) Let  $T : \mathbb{R}^2 \rightarrow \mathbb{R}^2$  be the linear transformation denoting reflection about  $y$ -axis. Find the eigen values and eigen vectors of  $T$  6

3. (a) Solve by elementary row operations, the following system of equations

$$2x + 3y + z = 9$$

$$x + 2y + 3z = 6$$

$$3x + y + 2z = 8 \quad \text{6}$$

- (b) Find the rank of the following matrix

$$\begin{bmatrix} 1 & 1 & 2 & 3 \\ 1 & 3 & 0 & 3 \\ 1 & -2 & -3 & 0 \\ 1 & 1 & 2 & 3 \end{bmatrix} \quad \text{6}$$

### Section - II

4. (a) Examine the convergence of the sequence

$$\left\{ \frac{\cos n}{n} \right\} \quad \text{6}$$

- (b) Find the  $n^{\text{th}}$  derivative of  $y = \frac{x}{1 + 3x + 2x^2}$  6

- (c) If  $y = e^{m \sin^{-1} x}$ , show that 6

$$(1 - x^2) y_{n+2} - (2n + 1) x y_{n+1} - (n^2 + m^2) y_n = 0$$

- 5 (a) Sketch the graph of  $y = |x - 2| + 1$  6

- (b) If the population of a country doubles itself in 50 years, in how many years will it be three times under the assumption that the rate of increase is proportional to the number of inhabitants ? 6
- (c) Find the Maclaurin's series for  $x^{n+1}$ , assuming that  $\lim_{n \rightarrow \infty} R_n(x) = 0$  6
6. (a) Draw the level curves at heights  $R = 0, 1, 2$  of the function  $Z = \sqrt{9 - x^2 - y^2}$ . 6
- (b) Find the slope of the surface  $Z = \sqrt{x^2 + 2y^2}$  along  $x$ -axis at  $(1, 2, 3)$  6
- (c) Verify that the function  $u(x, t) = \sin(x + ct) + \cos(3x + 3ct)$  is a solution of the wave equation  $\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$  6

### Section – III

7. (a) The probability that a student of Mathematics will not pass M.B.A. examination is  $\frac{3}{5}$  and that a student of History will not pass it is  $\frac{4}{5}$ . Calculate the probability that at least one of them passes the examination 4½

- (b) In a throw of dice, a random variable  $X$  is defined in the following way .  
 $X$  takes the value equal to 2 times  
 The outcome if it is odd and  $-1$  times  
 The outcome if it is even, write down  $X$   
 Also write down the probability distribution of  $X$  3

8. (a) Find mean and variance of Binomial distribution 3½

- (b) A random variable  $X$  has the following probability distribution .

$X$	0	1	2	3	4	5	6	7
$P(X)$	0	$k$	$2k$	$2k$	$3k$	$k^2$	$2k^2$	$7k^2+k$

Find  $k$ . Also evaluate  $P(0 < X < 5)$  4

9. (a) The following statistics were collected during a chemistry experiment to study a possible relationship between output( $X$ ) in milligrams and temperature setting ( $Y$ ) in degrees Celsius. 3½

$$N = 4, \bar{X} = 639, \bar{Y} = 188, \sigma_x^2 = 210,$$

$$\sigma_y^2 = 407, \Sigma XY = 240700$$

Calculate Karl Pearson's coefficient of correlation between  $X$  and  $Y$ .

- (b) A sample of 900 members is found to have a mean of 34 cm. Can it be reasonably regarded as a simple sample from a large population with mean 32 cm and s d 23 cm ? 4