[This question paper contains 6 printed pages.]

5222 *Your Roll No.*

B.Sc./I **J**

MATHO-PHYSICS

MP-201 Mathematics – I

(NC – Admissions of 2008 & onwards)

Time : 3 hours *Maximum Marks : 112*

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt **six** *questions in all, taking* **two** *from Section* **I, three** *from Section* **II** *and* **one** *from Section* **III.**

SECTION – I

Attempt any **two** *questions from this section.*

1. (a) Sketch the following hyperbola and label the vertices, foci and asymptotes :

$$x^2 - 4y^2 + 2x + 8y - 7 = 0$$ (7)

(b) Find an equation of the ellipse for which length of major axis is 26 and foci at $(\pm 5, 0)$.

Also sketch the ellipse. (7)

(c) Determine whether $\vec{u}$ and $\vec{v}$ make an acute angle, an obtuse angle, or are orthogonal, where

$\vec{u} = \hat{i} - 2\hat{j} + 2\hat{k}, \ \vec{v} = 2\hat{i} + 7\hat{j} + 6\hat{k}$ (5)

2 (a) Find two unit vectors that are orthogonal to both

$$\vec{u} = -7\hat{i} + 3\hat{j} + \hat{k}$$

and $\vec{v} = 2\hat{i} + 4\hat{k}$ (6)

(b) If $\vec{A} = \left(2x^2y - x^4\right)\hat{i} + \left(e^{xy} - y\sin x\right)\hat{j} + \left(x^2\cos y\right)\hat{k}$,

find $\dfrac{\partial^2 \vec{A}}{\partial x^2}, \dfrac{\partial^2 \vec{A}}{\partial x \partial y}, \dfrac{\partial^2 \vec{A}}{\partial y^2}$ (6)

(c) Find $\nabla\phi$ if $\phi = \ell n \left\|\vec{r}\right\|$,

where $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ (7)

3 (a) Find, by vector method, the area of the triangle determined by the points

$P_1(2, 2, 0)$, $P_2(-1, 0, 2)$ and $P_3(0, 4, 3)$ (6)

(b) Prove that curlgrad $\phi = 0$, for any scalar function ϕ in x, y, z (6)

(c) (i) Determine the constant a so that the vector

$$\vec{V} = (x + 3y)\hat{i} + (y - 2z)\hat{j} + (x + az)\hat{k}$$

is solenoidal

(ii) If $\vec{A} = xz^3\hat{i} - 2x^3yz\hat{j} + 2yz^4\hat{k}$, find $\nabla \times \vec{A}$ at the point (1, −1, 1) (3,4)

SECTION – II

Attempt all the **three** *questions in this section*

4 (a) Find the asymptotes of the curve

$$x^3 + x^2y - xy^2 - y^3 + 2xy + 2y^2 - 3x + y = 0$$

(8)

(b) Trace the curve

$$9ay^2 = x(x-3a)^2$$

(10)

OR

(a) Find the position and nature of the double points of the curve

$$x^3 + 2x^2 + 2xy - y^2 + 5x - 2y = 0$$

(8)

(b) Trace the curve

$$y(x^2 + 4a^2) = 8a^3$$

(10)

5 (a) Evaluate

$$\int \frac{1}{(x+1)\sqrt{2x^2+3x+4}}\,dx$$

(8)

(b) Show that the length of the curve $y = \log \frac{e^x - 1}{e^x + 1}$ from $x = 1$ to $x = 2$ is $\log\left(e + \frac{1}{e}\right)$ (8)

OR

(a) Evaluate $\int \frac{1}{(2x^2 + 3)\sqrt{3x^2 - 4}}\, dx$ (8)

(b) Find the volume formed by the revolution of the loop of the curve $y^2(a + x) = x^2(a - x)$ about x-axis (8)

6 (a) Examine the continuity at $x = 0$, the function f defined by

$$f(x) = \frac{e^{1/x} - e^{-1/x}}{e^{1/x} + e^{-1/x}}, \text{ if } x \neq 0$$

$$f(0) = 1$$ (6)

(b) State and prove Rolle's Theorem (8)

(c) (i) Verify Lagrange's Mean Value Theorem for $f(x) = x(x - 1)(x - 2)$ in $[0, \frac{1}{2}]$

(ii) Show that the function

$f(\lambda) = 2 - 3x + 6x^2 - 4x^3$ is strictly decreasing in every interval (5+5)

OR

(a) Show that the function f defined by

$$f(x) = \frac{1}{x},\ 1 < x < 2,$$

is uniformly continuous (8)

(b) Show that $\frac{x}{1+x} < \log(1+x) < x\ \forall\ x > 0$ (8)

(c) Show that differentiability implies continuity Give an example to show that the converse is not true (8)

SECTION - III

Attempt **one** *question from this section*

7 (a) Solve the equation

$$x^4 - 8x^3 + 14x^2 + 8x - 15 = 0$$

the roots being in A P (5)

(b) If α, β, γ are the roots of the equation,

$$x^3 + px + q = 0,$$

then show that

$$\left(\frac{\alpha^7+\beta^7+\gamma^7}{7}\right) = \left(\frac{\alpha^2+\beta^2+\gamma^2}{2}\right)\left(\frac{\alpha^5+\beta^5+\gamma^5}{5}\right)$$

(6)

(c) If α, β, γ are the roots of the equation,

$$ax^3 + bx^2 + cx + d = 0,$$

form an equation whose roots are α^3, β^3, γ^3 (5)

8 (a) Prove that

$$32 \sin^4\theta \cos^2\theta = \cos 6\theta - 2\cos 4\theta - \cos 2\theta + 2$$ (5)

(b) Find sum of the series

$$\cos\theta \sin\theta + \cos^2\theta \sin 2\theta + \quad + \cos^n\theta \sin n\theta$$ (6)

(c) Show that a necessary condition that the points A, B, C representing Z_1, Z_2, Z_3 respectively on Argand plane to be vertices of an equilateral triangle is that

$$\frac{1}{Z_2 - Z_3} + \frac{1}{Z_3 - Z_1} + \frac{1}{Z_1 - Z_2} = 0$$ (5)