University of Hyderabad

Entrance Examination, 2007 8

M.Sc. (Mathematics/Applied Mathematics)

Hall Ticket No.	

Time: **2** hours	Part A: **25**
Max. Marks: **75**	Part B: **50**

Instructions

1. The OMR sheet contains space for answers to 100 questions. Answer Part A in 1 to 25 and Part B in 26 to 50. Ignore the remaining spaces.
2. Fill in your hall ticket number in the space provided in both the OMR sheet and on this page.
3. Read the instructions provided in the OMR sheet carefully.
4. Calculators are not allowed.
5. Each correct answer in Part A carries **1 mark** and each wrong answer carries 0.33 $\frac{1}{4}$ mark.
6. Each correct answer in Part B carries **2 marks** and each wrong answer carries 0.66 $\frac{1}{2}$ mark.
7. Do not gamble as there is negative marking. There will be no penalty if a question is unanswered.
8. Answers are to be given on the OMR sheet provided.
9. The appropriate answer should be coloured in by either a blue or black ball point/sketch pen ~~or a black sketch pen.~~ DO NOT USE A PENCIL.
10. The set of real numbers is denoted by $\mathbb{R}$, the set of complex numbers by $\mathbb{C}$, the set of rational numbers by $\mathbb{Q}$ and the set of integers by $\mathbb{Z}$.

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PART A

Each question carries 1 mark. $\frac{1}{4}$ mark will be deducted for each wrong answer. There will be no penalty if the question is left unanswered. The set of real numbers is denoted by $\mathbb{R}$, the set of complex numbers by $\mathbb{C}$, the set of rational numbers by $\mathbb{Q}$ and the set of integers by $\mathbb{Z}$.

1. The function $f(x) = |\sin x|$ is

   (A) continuous everywhere but not differentiable anywhere.

   (B) not continuous at $n\pi$; n is an integer.

   (C) continuous everywhere but not differentiable at $n\pi$; n is an integer.

   (D) differentiable everywhere.

2. The function $f(x) = (x-1)^2$ for $x \in [0,3]$ has

   (A) a maximum but no minimum on $[0,3]$.

   (B) both maximum and minimum on $[0,3]$.

   (C) a minimum at $x = 1$ but no maximum on $[0,3]$.

   (D) a minimum at $x = 0$ but no maximum on $[0,3]$.

3. The function $f(x) = \sin x \cos x$, $x \in \mathbb{R}$ is

   (A) an odd function which is periodic with period 2π.

   (B) an even function which is periodic with period 2π.

   (C) an odd function which is periodic with period π.

   (D) an even function which is periodic with period π.

4. Let $f(x) = \cos^{-1} x$. Then f is one-one and onto if the domain and range are specified respectively as

   (A) $[-1,1]$ and $(-\infty,\infty)$. (B) $[-1,1]$ and $[0,2\pi]$.

   (C) $[-1,1]$ and $[0,\frac{\pi}{2}]$. (D) $[-1,1]$ and $[0,\pi]$.

5. The only point at which the function $2|x+1|-1$ fails to be differentiable is

   (A) -1. (B) 1. (C) 0. (D) 2.

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6. The set $\{x \in \mathbb{R} : x^2 > x\}$ is same as

(A) the interval $(0, 1)$.

(B) the complement of the interval $(0, 1)$.

(C) the complement of the interval $[0, 1]$.

(D) the interval $[0, 1]$.

7. A bijection is a map that is both one-one and onto. It is given that $f : \mathbb{R} \to \mathbb{R}$ is not a bijection. Which of the following must be true?

(A) If f is not one-one, then f is not onto.

(B) If f is not onto, then f is not one-one.

(C) f is neither one-one nor onto.

(D) If f is one-one, then f is not onto.

8. Let A be a 3×3 matrix with eigenvalue 1. Then

(A) A is invertible.

(B) $det A = 0$.

(C) $det(A - I) = 0$ where I the identity 3×3 matrix.

(D) $A - I$ is invertible.

9. The rank of the matrix $\begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 6 & 8 \\ 3 & 6 & 9 & 12 \\ 4 & 8 & 12 & 16 \end{pmatrix}$ is

(A) 0. (B) 1. (C) 2. (D) 3.

10. Let $S = \{v_1, \ldots, v_4\}$ be linearly independent vectors in $\mathbb{R}^5$. Consider the following statements :
(i) There exists a basis of $\mathbb{R}^5$ containing S.
(ii) Any non-empty subset of S will be linearly independent.
(iii) $\{v_1 + v_4, v_2, v_3, v_4\}$ are linearly independent.

(A) all three statements are true.

(B) none of the statements is true.

(C) only (i) and (ii) are true.

(D) only (ii) and (iii) are true.

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11. If A is a 3×3 matrix with eigenvalues $1, 1$ and -2, then the determinant of A is

(A) 2. (B)1. (C) 0. (D) -2.

12. In a cyclic group of order 24 the number of elements of order 12 is

(A) 1. (B) 2. (C) 3. (D) 4.

13. In the group S_5 of permutations on six symbols, an element conjugate to (1 3)(2 4 6) is

(A) (1 3) (2 4) (5 6). (B) (1 2 3 4) (5 6).

(C) (1 2 3) (4 5 6). (D) (1 2 3) (4 5).

14. Let C_n denote a cyclic group of order n and let $\phi : C_{52} \to C_{52}$ be the homomorphism $\phi(x) = x^7$. Then the order of the kernel of ϕ is

(A) 13. (B) 7. (C) 1. (D) 4.

15. How many solutions of the equation $\sin x = \cos x$ are there in the interval $[0, 10]$?

(A) 3.
(B) 4.
(C) Infinitely many.
(D) Nil.

16. The maximum value of $f(x) = 5\cos x + 12\sin x + 13$ is

(A) 17. (B) 18. (C) 25. (D) 26.

17. Consider the following statements :
(i) There exist integers x and y such that $107x + 49y = 511$.
(ii) There exists an integer x such that 59 divides $x^2 + 1$.

Then

(A) both statements are true.
(B) the first statement is true but the second is false.
(C) the second statement is true but the first is false.
(D) both statements are false.

18. The angle between the planes $2x - 4y + 5z = 0$ and $3x - y - 2z = 0$ is

(A) 0. (B) $\frac{\pi}{2}$. (C) $\frac{\pi}{3}$. (D) π.

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19. The equation of the sphere through the circle $x^2 + y^2 + z^2 = 4$; $2x + 3y + 4z = 6$ and the point $(1, 2, 2)$ is

(A) $2(x^2 + y^2 + z^2) - 2x - 3y - 4z - 2 = 0$.

(B) $2(x^2 + y^2 + z^2) + 2x + 3y + 4z - 34 = 0$.

(C) $(x^2 + y^2 + z^2) - 2x - 3y - 4z - 7 = 0$.

(D) $(x^2 + y^2 + z^2) + 2x + 3y + 4z - 25 = 0$.

20. The particular integral of

$$\frac{d^3y}{dx^3} + \frac{d^2y}{dx^2} - \frac{dy}{dx} - y = e^x$$

is given by

(A) $4xe^x$. (B) $\frac{1}{4}xe^x$. (C) $\frac{1}{4}xe^{-x}$. (D) $\frac{1}{2}xe^x$.

21. The general solution of the equation $e^y dx + (xe^y + 2y)dy = 0$ is given by

(A) $xe^y + y^2 = c$. (B) $ye^x + y^2 = c$.
(C) $xe^y + x^2 = c$. (D) $ye^x + x^2 = c$.

22. Suppose $f : [a, b] \to \mathbb{R}$ is a polynomial function such that $f(a)f(b) < 0$ and $f'(x) \neq 0$ for any $x \in (a, b)$. Then f has

(A) no root in (a, b).

(B) exactly one root in (a, b).

(C) two roots in (a, b).

(D) more than two roots in (a, b).

23. Let $f(x) = \frac{1}{2} + \int_0^x \sin t \, dt$. Then

(A) f is differentiable with $f'(x) = \sin x$.

(B) f is continuous but not differentiable at $x = 0$.

(C) $f'(\frac{\pi}{2})$ does not exist.

(D) $f'(x) = \cos x$.

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24. Consider the sequence $\left\{\frac{(-1)^n}{n}\right\}$ and the series $\sum_{n=1}^{\infty}\frac{(-1)^n}{n}$. Then

(A) the sequence converges but not the series.

(B) the series converges but not the sequence.

(C) neither the series nor the sequence converges.

(D) both the series and the sequence converge.

25. If $P(A) = 0.5$ and $P(A \cup B) = 0.9$, then the value of $P(B \cap A^c)$

(A) is 0.1. (B) is 0.5.

(C) is 0.4. (D) cannot be evaluated from the information given.

PART B

Each question carries 2 marks. $\frac{1}{2}$ mark will be deducted for a wrong answer. There will be no penalty if a question is unanswered.

26. Let G be a cyclic group of order 36 and H a cyclic group of order 17. Then the number of homomorphisms from G to H is

(A) 0. (B) 1. (C) 2. (D) 3.

27. An example of a continuous function $f(x)$ on $(-1, 1)$ which is not bounded is

(A) $\tan x$. (B) e^x. (C) $\tan\left(\frac{\pi x}{2}\right)$. (D) $\frac{1}{1+x^2}$.

28. Consider the following statements :

(i) a differentiable function on $(-1, 1)$ must be bounded.

(ii) a differentiable function on $[-1, 1]$ must be bounded.

(iii) a continuous bounded function on $\mathbb{R}$ must be differentiable.

(A) all three statements are true.

(B) only (i) is true.

(C) only (ii) is true.

(D) only (iii) is true.

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29. For which of the following statements is the converse also true:

(i) a finite subset of $\mathbb{R}$ is bounded.

(ii) an absolutely convergent series of real numbers is convergent.

(iii) a group of prime order must be cyclic.

(A) (i) and (iii) only.

(B) none of the statements.

(C) (ii) only

(D) (iii) only.

30. The number of real roots of the polynomial

$$p(x) = x(x-2)(x-4)(x-6) + 2$$

is

(A) 1. (B) 2. (C) 3. (D) 4.

31. The largest natural number n such that $m^3 - m$ is divisible by n for all natural numbers $m \geq 10$ is

(A) 2. (B) 3. (C) 5. (D) 6.

32. Suppose $f : [a, b] \to \mathbb{R}$ is a strictly monotone continuous function, differentiable on (a, b) and $e^b f(a) = e^a f(b)$. Then there is a $c \in (a, b)$ such that

(A) $f'(c) = 0$.

(B) $f(c)f'(c) = -1$.

(C) $f(c) = f'(c)$.

(D) none of the above holds.

33. Consider the differential equation $\left(\frac{dy}{dx}\right)^2 + y^2 = 0$. It has

(A) no solution.

(B) a unique solution.

(C) two linearly independent solutions.

(D) many solutions.

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34. Define $f : \mathbb{R} \to \mathbb{R}$ as follows :

$$f(x) = \begin{cases} 1, & \text{if } x \text{ is rational} \\ \dfrac{\sin x}{x}, & \text{if } x \text{ is irrational} \end{cases}$$

Then

(A) f is continuous everywhere.

(B) f is continuous only at $x = 0$.

(C) f is continuous all rational points.

(D) f is continuous at all irrational points.

35. Let A be a 3×3 real matrix and suppose $\alpha \in \mathbb{C}$ is such that there exists a non-zero vector $\vec{x}$ such that $A\vec{x} = \alpha\vec{x}$. Then

(A) the number of such α's is at least 3.

(B) the number of such α's is exactly 3.

(C) there are infinitely many such α's.

(D) there are at most 3 such α's.

36. Let A be a 3×3 real matrix and let $\alpha \in \mathbb{R}$. Suppose there exists a non-zero vector $\vec{x}$ such that $A\vec{x} = \alpha\vec{x}$. Then

(A) there are an uncountable number of such $\vec{x}$.

(B) the number of such $\vec{x}$ is exactly 3.

(C) there are only countable number of such $\vec{x}$.

(D) there is exactly one such $\vec{x}$.

37. Let A be a 4×4 singular matrix with real entries. Then it is always true that

(A) all eigenvalues are purely imaginary.

(B) there are at least two real eigenvalues.

(C) there is at most one eigenvalue.

(D) all eigenvalues are real.

38. Which of the following is not always true?

(A) Every convergent sequence is bounded.

(B) Every 3×3 real matrix has at least one eigenvector.

(C) Every differentiable function is continuous.

(D) Any three vectors will always span a two dimensional vector space.

39. Let

$$f(x) = \begin{cases} xe^{-1/x^2} & \text{if } x \neq 0 \\ 0, & \text{if } x = 0. \end{cases}$$

Then

(A) f is differentiable everywhere.

(B) f is continuous everywhere but not differentiable at $x = 0$.

(C) f is neither continuous nor differentiable at $x = 0$.

(D) f is not differentiable anywhere.

40. Let $\vec{a} = (1, -1, 1)$ and $\vec{b} = (3, 1, -1)$. Then

(A) $\vec{a}, \vec{b}, (1, 0, 0), (1, 1, -1)$ will span $\mathbb{R}^3$.

(B) $\vec{a}, \vec{b}, (2, -1, 1), (2, 1, -1)$ will span $\mathbb{R}^3$

(C) $\vec{a}, \vec{b}, (0, 0, 1)$ will span $\mathbb{R}^3$

(D) $\vec{a}, \vec{b}, (1, 1, -1)$ will span $\mathbb{R}^3$.

41. 9 students are to be seated in 3 rows with 3 in each row. The probability that student no.1 is seated in the second row is

(A) $\frac{1}{9}$. (B) $\frac{1}{6}$. (C) $\frac{1}{3}$. (D) $\frac{2}{3}$.

42. A non-empty subset is to be drawn from a set of 10 objects. The probability that the subset will contain and even number of elements is

(A) $\frac{513}{1023}$. (B) $\frac{512}{1023}$. (C) $\frac{511}{1024}$. (D) $\frac{511}{1023}$.

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43. The value of the integral

$$\int_0^a \int_0^{\sqrt{a^2-y^2}} (x^2+y^2)dxdy$$

is equal to

(A) $\frac{\pi a^4}{2}$. (B) $\frac{\pi a^4}{4}$. (C) $\frac{\pi a^4}{8}$. (D) $\frac{\pi a^4}{16}$.

44. The solution of the initial value problem

$$\frac{dy}{dx} = x^2 y - 3x^2,\ y(0) = 1$$

is

(A) $y = 3 - 2e^{x^3/3}$. (B) $y = 3 - 2e^{-x^3/3}$.
(C) $y = 3 - 2e^{x^2/3}$. (D) $y = 3 - 2e^{-x^2/3}$.

45. Let $f(x, y)$ be a homogeneous function of degree 4. Suppose that $g(x, y) = f(x^3 - y^3, 3x^2y + 3xy^2)$, then $x\frac{\partial g}{\partial x} + y\frac{\partial g}{\partial y}$ is a homogeneous function of degree

(A) 3. (B) 4. (C) 7. (D) 12.

46. Consider the following statements :

(I) The order of a subgroup of a finite group must divide the order of the group.
(II) If G is a group of order n and m divides n, then G must have a subgroup of order m.
(III) If G is a cyclic group of order n and m divides n, then G must have a subgroup of order m. Then

(A) all statements are true.
(B) (I) and (II) are true but (III) is false.
(C) (II) and (III) are true but (I) is false.
(D) (I) and (III) are true but (II) is false.

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47. The system of equations

$$\begin{aligned} x + 2y + 3z &= 1 \\ (a-5)y + 4z &= 1 \\ (b-6)z &= (c-5)(d-6) \end{aligned}$$

has no solution when

(A) $a = 5, b = 6, c = 5, d = 6$.
(B) $a \neq 5, b \neq 6, c \neq 5, d \neq 6$.
(C) $a = 5, b \neq 6, c = 5, d \neq 6$.
(D) $a \neq 5, b = 6, c = 5, d \neq 6$.

48. Which of the following is not a periodic function?

(A) $[x] - x$. (B) $x + \cos x$. (C) $\tan x$. (D) $\frac{1}{2}\sin^2(2x)$.

49. $\left(\frac{1+i\sqrt{3}}{2}\right)^{20}$ is equal to

(A) $\left(\frac{1+i\sqrt{3}}{2}\right)$. (B) $\left(\frac{1+i\sqrt{3}}{2}\right)^2$. (C) $\left(\frac{1+i\sqrt{3}}{2}\right)^3$. (D) $\left(\frac{1+i\sqrt{3}}{2}\right)^4$.

50. $\int_0^{\pi/2} \left(\frac{1-\tan x}{1+\tan x}\right) dx$ is equal to

(A) 0. (B) 1. (C) 2. (D) 3.

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