INSTITUTE OF ACTUARIES OF INDIA

EXAMINATIONS

23rd May 2011

Subject ST6 — Finance and Investment B

Time allowed: Three hours (9.45* – 13.00 Hrs)

Total Marks: 100

INSTRUCTIONS TO THE CANDIDATES

1. *Please read the instructions on the front page of answer booklet and instructions to examinees sent along with hall ticket carefully and follow without exception*

2. ** You have 15 minutes at the start of the examination in which to read the questions. You are strongly encouraged to use this time for reading only, but notes may be made. You then have three hours to complete the paper.*

3. *You must not start writing your answers in the answer sheet until instructed to do so by the supervisor*

4. *The answers are not expected to be any country or jurisdiction specific. However, if Examples/illustrations are required for any answer, the country or jurisdiction from which they are drawn should be mentioned.*

5. *Attempt all questions, beginning your answer to each question on a separate sheet.*

6. *Mark allocations are shown in brackets.*

AT THE END OF THE EXAMINATION
Please return your answer book and this question paper to the supervisor separately.

Q. 1) A deposit instrument offered by a bank guarantees that investors will receive a return during a one-year period that is greater of zero and 30% of the return provided by NSE Nifty index over the one-year period. An investor is planning to put Rs. 5,000,000 in the instrument.

(a) Describe the payoff as an option on NSE Nifty index. (1)

The risk-free rate of interest is 6% per annum (with continuous compounding), dividend yield on NSE Nifty is 2% per annum (with continuous compounding) and the volatility of NSE Nifty is 20% annum (with continuous compounding). NSE uses Black-Scholes formula to price the options on NSE Nifty index.

(b) What is the value of the instrument? (5)

(c) Explain whether or not the instrument is a good deal for the investors when he/she compares the minimum return from the product with a risk-free investment? (2)

[8]

Q. 2) Consider the following two portfolios:

Portfolio A:

- One long European call on NSE Nifty with exercise price 100
- One short European call on NSE Nifty with exercise price 150
- One long European call on NSE Nifty with exercise price 200
- One long European put on NSE Nifty with exercise price 100
- One short European put on NSE Nifty with exercise price 150
- One long European put on NSE Nifty with exercise price 200

Portfolio B:

- One long European call on NSE Nifty with exercise price 150
- One long European put on NSE Nifty with exercise price 150
- a risk-free debt of face value 50 maturing at time T. The debt is redeemed at face value on maturity T.

All the options mature at time T.

(a) Make a table showing the payoff of each portfolio as a function of Nifty value at time T (S_T). (4)

(b) Make a table showing the payoff of each portfolio for the following values of NSE Nifty at time T: $S_T = 0, 100, 150, 200$. (2)

(c) Which portfolio requires greater initial outlay to establish? (2)

(d) For an arbitrage free market, prove that $c_1 - 2c_2 + c_3 > 25e^{-rT}$. Where c_1, c_2 and c_3 are the prices of the European call options on NSE Nifty at time 0 with exercise prices 100, 150 and 200 respectively and maturity time T. r is the risk-free rate of interest with continuous compounding. (3)

[11]

Q. 3) Consider a risk-free asset B and two stocks, X and Y, where

$$dB_t = rB_t dt\,,\ B_0 = 1$$
$$dS_t^X = \mu^X S_t^X dt + \sigma^X S_t^X dW_t\,,\ S_0^X = s_X$$
$$dS_t^Y = \mu^Y S_t^Y dt + \sigma^Y S_t^Y dW_t\,,\ S_0^Y = s_Y$$

Where W_t is a P-Wiener process.

Assume that the two stocks pay continuous dividend where the dividend processes are given by:

$$dD_t^X = \delta^X S_t^X dt$$
$$dD_t^Y = \delta^Y S_t^Y dt$$

(a) Using Girsanov's theorem, derive the stochastic differential equations of S_t^X and S_t^Y under Q-Wiener process (risk-neutral measure). (5)

(b) Consider a contract with maturity time T where an investor at time T receives the amount $D_T^X - \gamma D_T^Y$, where $\gamma > 0$. If $D_T^X - \gamma D_T^Y$ is positive, the investor receives $D_T^X - \gamma D_T^Y$ and if $D_T^X - \gamma D_T^Y$ is negative, the investor pays $\gamma D_T^Y - D_T^X$. Determine γ such that the price of the contract is 0 at time 0. (5)

[10]

Q. 4) Consider a standard Heath-Jarrow-Morton (HJM) forward rate model under the risk neutral martingale measure Q of the form:

$$dF(t,T) = m(t,T)dt + s(t,T)dz(t)$$

$z(t)$ is assumed to be a Q-Wiener process.

(a) Explain the HJM drift condition. What does it say? Why is it needed? How is it derived (only qualitative answer is needed)? (3)

(b) Given the above for ward rate dynamics, derive the dynamics of short rate under Q. Does the HJM drift conditions imply restrictions on the parameters of the short rate under Q? (8)

[11]

Q. 5) Consider a standard Black-Scholes market, that is, a market consisting of a risk-free asset, B, with P-dynamics given by

$$\begin{cases} dB(t) = rB(t)dt \\ B(0) = 1 \end{cases}$$

and a stock S, with P-dynamics given by

$$\begin{cases} dS(t) = \mu S(t)dt + \sigma S(t)dW(t) \\ S(0) = s(0) \end{cases}$$

Here W denotes a P-Wiener process and r, μ and σ are assumed to be constants

(a) Verify whether the portfolio with holding $h(t)^B$ units in risk free asset B and $h(t)^S$ units in stock S at time t defined by

$$h(t) = \left(h(t)^B, h(t)^S\right) = \left(\frac{S(t)}{B(t)}, \frac{B(t)}{S(t)}\right)$$ is self financing or not. (4)

(b) Determine whether the following process Y represents a tradable asset or not

$$Y(t) = S(t)^{-\alpha} \text{ where } \alpha = \frac{2r}{\sigma^2}$$ (4)

[8]

Q. 6) A stock price is currently Rs.1000. Over each of the next two 1-year periods it is expected to go up by 10% or down by 10%. The probability of an up move is 0.55 under the original P-measure. The risk-free interest rate is zero.

(a) Compute the price of an up-and-out put option (barrier option) with barrier H = 1050 and exercise price K = 1050. (2)

(b) Find the replicating portfolio for the option in (a) and verify that the portfolio is self-financing. (4)

[6]

Q. 7) Suppose that the prices of zero-coupon bonds with various maturities are given below in the table. The face value of each bond is Rs. 1000.

Maturity (Years)	**Price (Rs.)**
1	934.5794
2	869.3715
3	804.9606
4	741.8753
5	680.5831

(a) Suppose that an investor buys today (at time t = 0) three-year maturity zero coupon bonds with face value of Rs. 10 million. How many five year maturity zeros would you have to sell to make your initial cash flow equal to zero at time t = 0? (2)

(b) What are the cash flows on this strategy in each year? (2)

(c) What is the effective two-year interest rate on the effective three-year-ahead forward loan of principal value of Rs. 10 million? (2)

(d) Confirm that the effective two-year interest rate equals $(1+f_4)(1+f_5)-1$. Where f_4 is the forward rate per annum (with annual compounding) between the years 3 and 4 and f_5 is the forward rate per annum (with annual compounding) between the years 4 and 5. (3)

[9]

Q. 8) **(a)** "Prices of long-term bonds are more volatile than prices of short-term bonds". "Yield to maturity of short-term fluctuates more than yields of long-term bonds". How do you reconcile these two observations? (2)

(b) A fixed income portfolio manager is unwilling to realise a rate of return of less than 5% per annum (with annual compounding) over a six-year investment period currently valued at Rs. 10 million. Four years later, two-year zero rate is 7% per annum (with annual compounding). What must be the value of the portfolio at this time such that the portfolio manager is assured of achieving the minimum possible return? (2)

(c) In what way is owning a corporate bond similar to writing a put option? A call option? (2)

(d) You will receive bonus next month that you hope to invest in long-term corporate bonds. You believe that bonds today are selling at quite attractive yields, and you are concerned that bond prices will rise over the next few weeks. How might you use financial futures to hedge your risk? (2)

(e) Suppose that the spot price of one US dollar is Rs. 45. The one year futures price is Rs. 46. Is the interest rate higher in India or in United States? Explain. (1)

(f) Two bonds have identical times to maturity and coupon rates. One is callable at 104, the other at 108 on the same call date. Given all the other factors identical, which should have the higher yield to maturity? Why? (2)

[11]

Q. 9) Alpha has just purchased NSE Nifty fund currently selling at Rs. 5900 per share. To protect against losses, Alpha also purchased an at-the-money European put option on the fund for Rs. 110, with three-month time to expiration. Gamma, Alpha's financial adviser points out that Alpha is spending a lot of money on the put. He notes that 3-month puts with exercise price of 5800 cost only Rs. 70, and suggest that Alpha use the cheaper put.

(a) Analyse Alpha's and Gamma's strategies by making tables showing the profit for stock-plus-put positions for various value of stock funds in three months. (3)

(b) When does Gamma strategy do better? When does it do worse? (2)

(c) Which strategy entails greater systematic risk? (1)

[6]

Q. 10) **(a)** Should researchers use real-world or risk-neutral default probabilities for (i) calculating credit value at risk and (ii) adjusting the price of derivative for defaults? (2)

(b) "A long forward contract subject to credit risk is a combination of a short position in a no-default put and long position in a call subject to credit risk". Explain this statement. Assume that defaults happen only at the end of the life of the forward contract. (4)

(c) "The position of a buyer of a credit default swap is similar to the position of some one who is long a risk-free bond and short a corporate bond? Explain this statement. (3)

(d) Explain the structure of a CDO. (2)

[11]

Q. 11) **(a)** Suppose that European call option on Infosys stock with time to maturity three-months and strike price Rs. 3000 are selling at an implied volatility of 28% per annum. Infosys stock currently is Rs. 3000 per share, and the risk-free rate per annum (with continuous compounding) is 6%. If you believe the true volatility of the stock is 30% per annum, how can you trade on your belief without taking on exposure to the performance of Infosys. How many shares of stock you will hold for each option contract bought or sold such that it will give you profit when the option prices come into alignment? (3)

(b) Using the data in problem 11(a), suppose that three-month put options with a strike price of Rs. 3000 are selling at an implied volatility of 32%. Calculate a delta neutral portfolio comprising in calls and puts that will profit when the option prices come back into alignment. (3)

(c) The current risk-free rate of interest is 6% per annum (with continuous compounding). The 3-month futures price for Infosys stock is Rs. 3045, where as the 6-month futures price is Rs. 3100. Is there an arbitrage opportunity here? If so, how would you exploit it? (3)

[9]

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