SECTION – I

Straight Objective Type

This section contains 6 multiple choice questions. Each question has 4 choices

(A), (B) , (C) , (D) out of which **ONLY ONE** is correct .

1. *If* $0 < x < 1$, *then* $\sqrt{1+x^2}\left[\{x\cos(\cot^{-1}x)+\sin(\cot^{-1}x)\}^2-1\right]^{\frac{1}{2}} =$

(A) $\frac{x}{\sqrt{1+x^2}}$ (B) x (C) $x\sqrt{1+x^2}$ (D) $\sqrt{1+x^2}$

Solution: (C)

Let $\cot^{-1}x = \theta \Rightarrow x = \cot\theta$

$$\Rightarrow \sin\theta = \frac{1}{\sqrt{1+x^2}}, \quad \cos\theta = \frac{x}{\sqrt{1+x^2}}$$

Given Expresion

$$= \sqrt{1+x^2}\left[(x\cos\theta+\sin\theta)^2-1\right]^{\frac{1}{2}}$$

$$= \sqrt{1+x^2}\left[\left(x\frac{x}{\sqrt{1+x^2}}+\frac{1}{\sqrt{1+x^2}}\right)^2-1\right]^{\frac{1}{2}}$$

$$= \sqrt{1+x^2}\left[\left(\frac{x^2+1}{\sqrt{1+x^2}}\right)^2-1\right]^{\frac{1}{2}}$$

$$= \sqrt{1+x^2}\,(x^2)^{\frac{1}{2}} = x\sqrt{1+x^2}$$

2. Consider the two curves

$C_1 : y^2 = 4x$

$C_2 : x^2 + y^2 - 6x + 1 = 0$

Then,

(A) C_1 and C_2 touch each other only at one point

(B) C_1 and C_2 touch each other exactly at two points

(C) C_1 and C_2 intersect (but do not touch) at exactly two points

(D) C_1 and C_2 neither intersect not touch each other

Solution: (B)

Replace $y^2 = 4x$ in C_2 to get

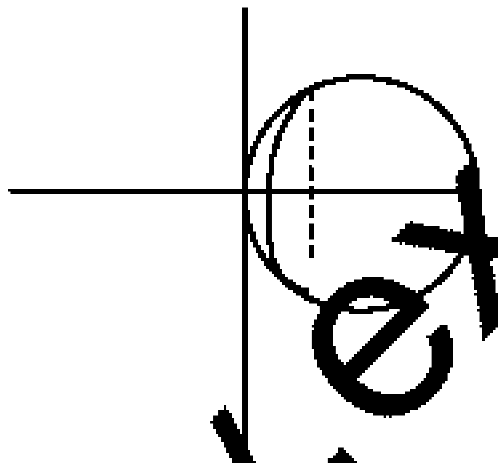
$x^2 + 4x - 6x + 1 = 0$

$x^2 - 2x + 1 = 0 \Rightarrow (x - 1)^2 = 0$

$\Rightarrow x = 1$ As $x = 1$ is double root,

Parabola & circle touch each other.

C_1 & C_2 touch each other at two parts.



3. The edges of a parallelopiped are of unit length and are parallel to non-coplanar unit vectors $\hat{a}, \hat{b}, \hat{c}$ such that $\hat{a} \cdot \hat{b} = \hat{b} \cdot \hat{c} = \hat{c} \cdot \hat{a} = \frac{1}{2}$.

Then the volume of the parallelopiped is

(A) $\frac{1}{\sqrt{2}}$ (B) $\frac{1}{2\sqrt{2}}$ (C) $\frac{\sqrt{3}}{2}$ (D) $\frac{1}{\sqrt{3}}$

Solution: (B)

$$[\hat{a}\ \hat{b}\ \hat{c}]^2 = \begin{vmatrix} \hat{a}\cdot\hat{a} & \hat{a}\cdot\hat{b} & \hat{a}\cdot\hat{c} \\ \hat{b}\cdot\hat{a} & \hat{b}\cdot\hat{b} & \hat{b}\cdot\hat{c} \\ \hat{c}\cdot\hat{a} & \hat{c}\cdot\hat{b} & \hat{c}\cdot\hat{c} \end{vmatrix}$$

$$= \begin{vmatrix} 1 & 1/2 & 1/2 \\ 1/2 & 1 & 1/2 \\ 1/2 & 1/2 & 1 \end{vmatrix} = \frac{1}{2}$$

$$\Rightarrow [\hat{a}\ \hat{b}\ \hat{c}]^2 = \frac{1}{2}$$

$$\Rightarrow [\hat{a}\ \hat{b}\ \hat{c}] = \pm\frac{1}{\sqrt{2}}$$

Volume of parallelopied defined by $\hat{a}, \hat{b}, \hat{c}$ as adjacent edges $= \left|[\hat{a}\ \hat{b}\ \hat{c}]\right| = \frac{1}{\sqrt{2}}$

4. Let a and b non-zero real numbers. Then the equation

$$(ax^2 + by^2 + c)(x^2 - 5xy + 6y^2) = 0$$

Represents

(A) four straight lines, when c = 0 and a, b are of the same sign

(B) two straight lines and a circle, when a = b, and c is of sign opposite to that of a

(C) two straight lines and a hyperbola, when a and b are of the same sign and c is of sign opposite to that of a

(D) a circle and an ellipse, when a and b are of the same sign and c is of sign opposite to that of a

Solution: (A)

$(ax^2 + by^2 + c)(x^2 - 5xy + 6y^2) = 0$

$(ax^2 + by^2 + c)(x - 2y)(x - 3y) = 0$

$x - 2y = 0$ & $x = 3y$ are straight lines passing through origin.

$ax^2 + by^2 + c = 0$ represents Circle if a =b. As sign of C is opposite to that of a & b, it is a real Circle with positive radius.

$ax^2 + by^2 - k = 0$ (Assuming k > 0)

=> $x^2 + y^2 = k/a$ & a > 0

As k/a > 0, it is real circle whose centre is (0, 0), $r = \sqrt{k/a}$

5. The total number of local maxima and local minima of the function

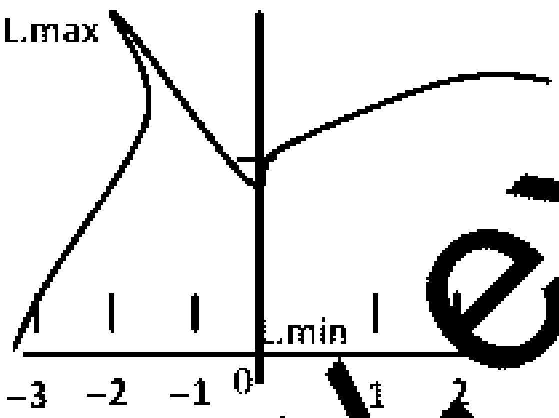
$$f(x) = \begin{cases} (2+x)^3, & -3 < x \le -1 \\ x^{2/3}, & -1 < x < 2 \end{cases}$$

is

(A) 0 (B) 1 (C) 2 (D) 3

Solution: (C)

Draw graph of f(x) as it is a easy catch.



From graph there is 1 local max. & 1 local min.

Total local max./local min. = 2

6. Let $g(x) = \frac{(x-1)^n}{\log\cos^m(x-1)}; 0 < x < 2,$ m and n are integers, $m \neq 0$ n>0, and let p be the left hand derivative of $|x-1|$ at x = 1.

If $\lim_{x\to 1^+} g(x) = p$, then

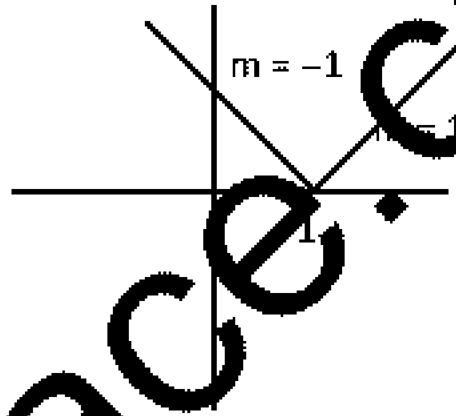
(A) n = 1, m = 1 (B) n = 1 , m = -1

(C) n = 2, m = 2 (D) n > 2 , m = n

Solution: (C)

LHD of |x − 1| = −1

=> p = −1



lim g(x) = lim (g1 + h) (RHL of g(x) at x = 1)

$$=\lim_{x\to 1} \frac{h^n}{\log[\cos^m h]} = \frac{h^n}{m\ \log(\cos h)}$$

Apply LH Rule

$$=\lim_{h\to 0} \frac{n\ h^{n-1}}{m(-\tan h)} = -\frac{n}{m} h^{n-2}\left[\frac{h}{\tan h}\right] = -\frac{n}{m} h^{n-2}$$

As p = −1, n can not be more than 2, if n > 2, then lim g(x) = 0 which is wrong

=> n = 2

Also $-\frac{n}{m} = -1 \Rightarrow m = 2$

SECTION - II

Multiple Correct Answers Type

This section contains 4 multiple correct answer(s) type questions. Each question has 4 choices (A) , (B) , (C) , (D) , out of which **ONE OR MORE** is/are correct.

7. Let P (x_1 , y_1) and Q(x_2, y_2) , $y_1 < 0$, $y_2 < 0$, be the end points of the latus rectum of the ellipse $x^2 + 4y^2 = 4$. The equations of parabolas with latus rectum PQ are

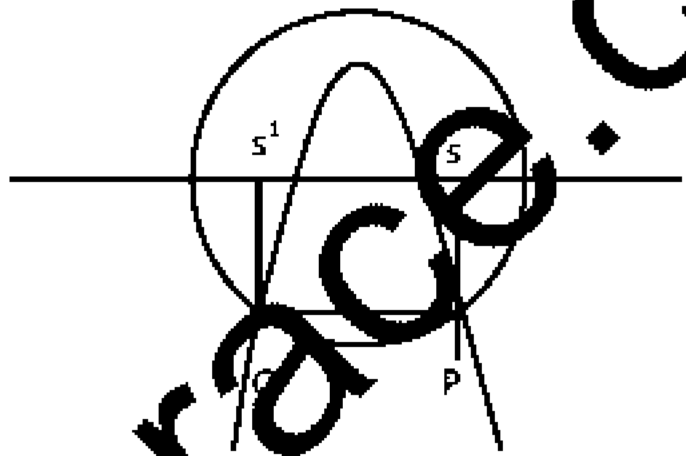
(A) $x^2 + 2\sqrt{3}y = 3 + \sqrt{3}$

(B) $x^2 - 2\sqrt{3}y = 3 + \sqrt{3}$

(C) $x^2 + 2\sqrt{3}y = 3 - \sqrt{3}$

(D) $x^2 - 2\sqrt{3}y = 3 - \sqrt{3}$

Solution:



$$\frac{x}{4} + \frac{y^2}{1} = 1$$

$$a = 2,\ b = 1$$

$$e = \frac{\sqrt{3}}{2}$$

$$s \equiv (ae, 0)\ \&\quad s^1 \equiv (-ae, 0)$$

$$\Rightarrow s \equiv (\sqrt{3} \qquad s^1 \equiv (-\sqrt{3},\ 0)$$

Solve ellipse & $x = \sqrt{3}$ *to get*

$$\frac{3}{4} + \frac{y^2}{1} = 1 \qquad \Rightarrow y = \pm\frac{1}{2}$$

$$\Rightarrow P \equiv \left(\sqrt{3},\ -\frac{1}{2}\right) \qquad Q \equiv \left(-\sqrt{3},\ \frac{1}{2}\right)$$

Replace P & Q in all choices & see parabolas given in (B) & (C) pass through P & Q

8. A straight line through the vertex P of a triangle PQR intersects the side QR at the points S and circumcircle of the triangle PQR at the point T. If S is not the centre of circumcircle, then

(A) $\frac{1}{PS} + \frac{1}{ST} < \frac{2}{\sqrt{QS \times SR}}$

(B) $\frac{1}{PS} + \frac{1}{ST} > \frac{2}{\sqrt{QS \times SR}}$

(C) $\frac{1}{PS} + \frac{1}{ST} < \frac{4}{QR}$

(D) $\frac{1}{PS}+\frac{1}{ST}>\frac{4}{QR}$

Solution: Using AM $\geq$ GM

$AM\ \ of\ \frac{1}{PS}\ \&\ \frac{1}{ST}$ should be greater than $am\ of\ \frac{1}{PS}\ \&\ \frac{1}{ST}$

$$\Rightarrow \frac{\frac{1}{PT}+\frac{1}{ST}}{2}\geq\sqrt{\frac{1}{PT}+\frac{1}{ST}}\ ---(1)$$

In circle, PS. ST = QS. SR ------(2)

Combining (1) & (2), we get

$$\frac{1}{PS}+\frac{1}{ST}\geq\frac{2}{\sqrt{QS\cdot SR}}\ ------(3)$$

$$\Rightarrow (B)\ is\ \ correct$$

As AM $\geq$ GM,

$$\sqrt{QS\cdot SR}<\frac{QS+SR}{2}\frac{QR}{2}\ ----(4)$$

Combining (3) & (4),

$\frac{1}{PS}+\frac{1}{ST}\geq\frac{4}{QR}$ => (D) is correct (We can replace lesser side by a smaller number)

9. Let f(x) be a non constant twice differentiable function defined on $(-\infty,\infty)$ such that

f(x) = f(1-x) and $f'\left(\frac{1}{4}\right)=0$. Then

(A) f''(x) vanishes at least twice on [0, 1]

(B) $f'\left(\frac{1}{2}\right)=0$

(C) $\int_{-1/2}^{1/2} f\left(x+\frac{1}{2}\right)\sin x dx = 0$

(D) $\int_{0}^{3/2} f(t)e^{\sin \pi t} dt = \int_{1/2}^{1} f(1-t)e^{\sin \pi t} dt$

Solution: (A, B, C, D)

f(x) = f(1− x) => $f'(x) = -f'(1-x)$

put x =1/2

$f'(1/2)$ $-f'(1/2)$ => $f'(1/2) = 0$ => B

f(x) = f(1−x)

$f'(x) = -f'(1-x)$

$f'(1/4) = -f(1-1/4) = -f'(3/4)$

$f'(1/4) = 0$ => $f'(3/4) = 0$

Apply Rolle's Them in (1/4, ½) & between [1/2, 1] of $f'(x)$

Hence $f''(x) = 0$ at two points => A is correct

f(x) = f(1−x)

Replace $x \to x + \frac{1}{2}$

F(1/2 + x) = f(1/2 − x) => f(x) is system about x= ½

$\int_{-\frac{1}{2}}^{\frac{1}{2}} f\left(x+\frac{1}{2}\right)\sin x dx = 0 \Rightarrow C$

even *odd*

$\int_{\frac{1}{2}}^{1} f(1-t)e^{\sin \pi t} dt = \int_{\frac{1}{2}}^{0} f(z)e^{\sin \pi(1-z)}(-dz)$

1 – t = z

$-dt = dz \qquad = \int_0^{\frac{1}{2}} f(z) e^{\sin \pi t} dz \qquad \Rightarrow D$

10.

Let

$$S_n = \sum_{k=1}^{n} \frac{n}{n^2 + kn + k^2} \quad \text{and} \quad T_n = \sum_{k=0}^{n-1} \frac{n}{n^2 + kn + k^2},$$

for $n = 1, 2, 3, \cdots$. Then,

(A) $S_n < \frac{\pi}{3\sqrt{3}}$

(B) $S_n > \frac{\pi}{3\sqrt{3}}$

(C) $T_n < \frac{\pi}{3\sqrt{3}}$

(D) $T_n > \frac{\pi}{3\sqrt{3}}$

Solution: (A, D)

$$S_n = \sum_{k=1}^{n} \frac{n}{n^2 + kn + k^2}$$

$$S_\infty = \lim_{n \to \infty} \sum_{k=1}^{n} \frac{n}{n^2 + kn + k^2}$$

$$= \lim_{n \to \infty} \frac{1}{n} \sum_{k=1}^{n} \frac{1}{1 + \frac{k}{n} + \frac{k^2}{n^2}}$$

$$= \int_0^1 \frac{dx}{1 + x + x^2} = \int_0^1 \frac{dx}{\left(x + \frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2}$$

$$= \frac{2}{\sqrt{3}} \tan^{-1}\left(\frac{2}{\sqrt{3}}\left(x + \frac{1}{2}\right)\right)_0^1$$

$$= \frac{2}{\sqrt{3}}\left(\tan^{-1}\sqrt{3} - \tan^{-1}\frac{1}{\sqrt{3}}\right)$$

$$= \frac{2}{\sqrt{3}}\left(\frac{\pi}{3} - \frac{\pi}{6}\right) = \frac{\pi}{3\sqrt{3}}$$

$$S_1 = \sum_{k=1}^{1} \frac{1}{1 + 1 + 1} = \frac{1}{3} < \frac{\pi}{3\sqrt{3}}$$

Also $n \uparrow \Rightarrow S_n \uparrow$

Hence $S_n < \frac{\pi}{3\sqrt{3}}$

$$T_n = \sum_{k=0}^{1} \frac{1}{n^2 + kn + k^2} \quad T_\infty = \lim_{n \to \infty} \frac{1}{n} \sum_{k=0}^{1} \frac{1}{1 + x + x^2}$$

$$= \int_0^1 \frac{dx}{1 + x + x^2} = \frac{\pi}{3\sqrt{3}}$$

$$= T_1 = \sum_{k=0}^{1} \frac{1}{1 + 0 + 0} = 1$$

As $n \uparrow$, T_n *decreases*

$$T_n > \frac{\pi}{3\sqrt{3}}$$

SECTION - III

Reasoning Type

This section contains 4 reasoning type questions. Each question has 4 choices

(A) , (B) , (C) , (D), out of which **ONLY ONE** is correct

11. Consider the system of equations

$$x - 2y + 3z = -1$$

$$-x + y - 2z = k$$

$$x - 3y + 4z = 1.$$

STATEMENT-1 : The system of equations has no solution for $k \neq 3$.

And

STATEMENT-2 : The determinant $\begin{vmatrix} 1 & 3 & 1 \\ -1 & -2 & k \\ 1 & 4 & 1 \end{vmatrix} \neq 0$ for $k \neq 3$.

(A) STATEMENT-1 is True, STATEMENT-2 is True; STATEMENT-2 is a correct explanation for STATEMENT-1

(B) STATEMENT-1 is True, STATEMENT-2 is True; STATEMENT-2 is NOT a correct explanation for STATEMENT-1

(C) STATEMENT-1 is True, STATEMENT-2 is False

(D) STATEMENT-1 is False, STATEMENT-2 is True

Solution: (A)

$$D = \begin{vmatrix} 1 & -2 & 3 \\ -1 & 1 & -2 \\ 1 & -3 & 4 \end{vmatrix} = 0$$

=> System of equations can have a solution if $D_1 = D_2 = D_3 = 0$

$$D_2 = \begin{vmatrix} 1 & -1 & 3 \\ -1 & k & -2 \\ 1 & 1 & 4 \end{vmatrix} = k - 3$$

$\forall\ k \neq 3,\ D_2 \neq 0 \qquad \Rightarrow \textit{No Solution.}$

=> Statement –1 is correct.

Statement –2 is also correct as shown above.

=> Statement –2 is correct.

It is obvious that statement –2 is correct explanation of statement –1.

12. Consider the system of equations

ax by = 0, cx + dy = 0, where a, b, c, d $\in \{0, 1\}$.

STATEMENT-1 : The probability that the system of equations has a unique solution is $\frac{3}{8}$.

and

STATEMENT-2 : The probability that the system of equations has a solution is 1.

(A) STATEMENT-1 is True, STATEMENT-2 is True; STATEMENT-2 is a correct explanation for STATEMENT-1

(B) STATEMENT-1 is True, STATEMENT-2 is True; STATEMENT-2 is NOT a correct explanation for STATEMENT-1

(C) STATEMENT-1 is True, STATEMENT-2 is False

(D) STATEMENT-1 is False, STATEMENT-2 is True

Solution: (B)

x = 0, y = 0 will definitely satisfy ax + by = 0 & cx + dy = 0

=> system of equations is consistent.

=> Prob system has a solution = 1.

=> Statement –2 is correct.

Unique solution would exist for the following cases:

a	b	c	d
0	1	0	1
1	0	1	0
1	1	1	0
1	1	0	1
1	0	1	1
0	1	1	1

a,b,c,d can take values in 2^4 ways (Each can be either 0 or 1)

P(Unique Solution) = $\frac{6}{16} = \frac{3}{8}$

=> Statement –1 is correct

As prob (solution exists) Cannot be used to find P(Unique Solution) Statement –2 is not correct explanation of statement –1 => B

13. Let f and g be real valued functions defined on interval (–1, 1) such that g" (x) is continuous, g (0) $\neq$ 0, g" (0) $\neq$ 0 , and f(x) = g(x) sin x.

STATEMENT-1 : $\lim_{x\to 0}\left[g(x)\cot x - g(0)\operatorname{cosec} x\right] = f''(0)$. and

SATEMENT-2 : f' (0) = g (0).

(A) STATEMENT-1 is True, STATEMENT-2 is True; STATEMENT-2 is a correct explanation for STATEMENT-1

(B) STATEMENT-1 is True, STATEMENT-2 is True; STATEMENT-2 is NOT a correct explanation for STATEMENT-1

(C) STATEMENT-1 is True, STATEMENT-2 is False

(D) STATEMENT-1 is False, STATEMENT-2 is True

Solution: (A)

f(x) = g(x)sinx

$f^{I}(x) = g^{I}(x)$ s9nx + g(x) cosx

$f^{'}(0) = 0 + g(0) \Rightarrow f^{I}(0) = g(0)$

=> Statement −2 is correct

$f''(x) = g''(x)\sin x + g'(x)\cos x + g'(x)\cos x + g(x)(-\sin x)$

$f''(0) = 0 + g'(0) + g'(0) + 0\ 2g'(0)$

As $g'(0) =)$, $f''(0) = 2g'(0) = 0$.

Statement-1

$$LHS = \lim_{x \to 0} \frac{g(x)\cos x - g(0)}{\sin x} \lim_{x \to 0} \frac{\cot x f(x) - g(0)}{\sin x}$$

Apply LH Rule

$$\lim_{x \to 0} - \frac{\operatorname{cosec}^2 x\, f(x) + \cot x\, f'(x)}{\cos x}$$

$$= \lim_{x \to 0} - \frac{\operatorname{cosec}^2 x\, g(x) + \cot x\, f'(x)}{\cos x}$$

$$= \lim_{x \to 0} - \frac{g(x) + \cot x\, f'(x)}{\frac{\sin 2x}{2}}$$

$N \to 0 \qquad as\ g(0) = f'(0) \qquad statement$ [illegible]

Apply LH Rule again

$$\lim_{x \to 0} - \frac{g'(x) + \cos x\, f''(x) - \sin x \text{[illegible]}}{\cos 2x(v \sin g \text{[illegible]}(0) = 0)} = f'(0)$$

=> statement −1 is correct

Ans is A as statement [illegible] used to prove statement-1

14. Consider three planes

P_1 : [illegible] y + z = 1

[illegible]: x + y − z = −1

P_3 : x − 3y + 3z = 2.

Let L_1, L_2, L_3 be the lines of intersection of the planes P_2 and P_3, P_3 and P_3. and P_3 and P_2, respectively.

STATEMENT-1 : At least two of the lines L_1, L_2 and L_3 are non-parallel.

and

STATEMENT-2 : The three planes do not have a common-point.

(A) STATEMENT-1 is True, STATEMENT-2 is True; STATEMENT-2 is a correct explanation for STATEMENT-1

(B) STATEMENT-1 is True, STATEMENT-2 is True; STATEMENT-2 is NOT a correct explanation for STATEMENT-1

(C) STATEMENT-1 is True, STATEMENT-2 is False

(D) STATEMENT-1 is False, STATEMENT-2 is True

Solution: (D)

let n_1, n_2 & n_3 be dR, of planes.

$$A \text{ vector } 11 \text{ to } L_1 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{vmatrix}$$

$$= 2(\hat{j} + \hat{k}) \text{--------} (1)$$

$$A \text{ vector } 11 \text{ to } L_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & -1 \\ 1 & 3 & \end{vmatrix}$$

$$= -4(\hat{j} + \hat{k}) \text{--------} (2)$$

$$A \text{ vector } 11 \text{ to } L_3 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 1 \\ & -3 & 3 \end{vmatrix}$$

$$= -2(\hat{j} + \hat{k}) \text{--------} (3)$$

Combining (1), (2) & (3) L_1, L_2 & L_3 are parallel.

=> P_1, P_2 & P_3 do not have a common point.

=> statement –2 is correct

Statement-1 is false as all lines are parallel is No two of there are Non-parallel

SECTION - IV

Linked Comprehension Type

This section contains 3 paragraphs. Based upon each paragraph , 3 multiple choice questions have to be answered. Each question has 4 choices (A) , (B) , (C) ,(D) ,out of which **ONLY ONE** is correct

Paragraph for 15-17

Let A, B, C be three sets of complex numbers as defined below

$A = \{z : \text{Im } z \geq 1\}$

$B = \{z : |z - 2 - i| = 3\}$

$C = \{z : \text{Re } ((1 - i) z) = \sqrt{2}\ \}$.

15. The number of elements in the set $A \cap B \cap C$ is

(A) 0 (B) 1 (C) 2 (D) ∞

16. Let z be any point in $A \cap B \cap C$ Then $|z + 1 - i|^2 + |z - 5 - i|^2$ lies between

(A) 25 and 29 (B) 30 and 34 (C) 35 and 39 (D) 40 and 44

17. Let z be any point in $A \cap B \cap C$ and let ω be any point satisfying $|w - 2 - i| < 3$. Then, $|z| - |w| + 3$ lies between

(A) −6 and 3 (B) −3 and 6 (C) −6 and 6 (D) −3 and 9

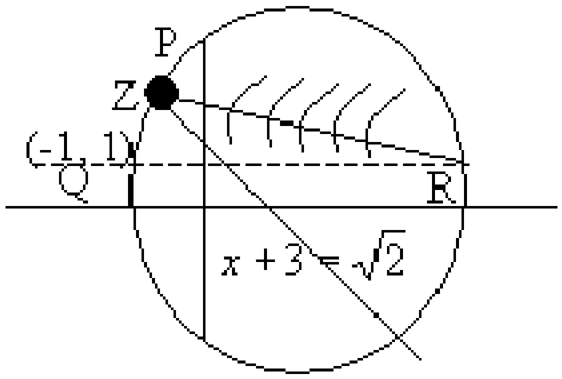
Solution: $I_m(z) \geq 1 \Rightarrow y \geq 1$ ------ (1)

$|z - 2 - i| = 3 \Rightarrow (x - 2)^2 + (y - 1)^2 = 9$

$\Rightarrow x^2 + y^2 - 4x - 2y - 4 = 0$ ------- (2)

$\text{Re}((1 - i)z) = \sqrt{2} \Rightarrow (1 - i)(x + iy) = \sqrt{2}$

$\Rightarrow x + y = \sqrt{2}$ --------- (3)



Solution 15: (B)

$n\{A \cap B \cap C\}$

no of elements in

$A \cap B \cap C = 1$

Solution 16: (C)

$|2 + 1 - i|^2 + |2 - 5 - i|^2 = PQ^2 + PR^2 = QR^2 = \text{diameter}^2 = 36$

Solution 17: (B)

Lows of w is interior of the circle

$0 < |z| < 3$ (z is p see figure)

$0 < |w| < 6 \quad -6 < |z| - |w| < 3$

On combining => $-3 < |z| - |w| + 3 < 6$

Paragraph for 18-20

A circle C of radius 1 is inscribed in an equilateral triangle PQR. The points of contact of C with the sides PQ, QR, RP are D, E, F, respectively. The line PQ is given by the equation $\sqrt{3}x + y - 6 = 0$ and the point D is $\left(\frac{2\sqrt{3}}{2}, \frac{3}{2}\right)$. Further, it is given that the origin and the centre of C are on the same side of the line PQ.

18. The equation of circle C is

(A) $(x - 2\sqrt{3})^2 + (y - 1)^2 = 1$ (B) $(x - 2\sqrt{3})^2 + (y + \frac{1}{2})^2 = 1$

(C) $(x - \sqrt{3})^2 + (y + 1)^2 = 1$ (D) $(x - \sqrt{3})^2 + (y - 1)^2 = 1$

19. Points E and F are given by

(A) $\left(\frac{\sqrt{3}}{2}, \frac{3}{2}\right), (\sqrt{3}, 0)$ (B) $\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right), (\sqrt{3}, 0)$

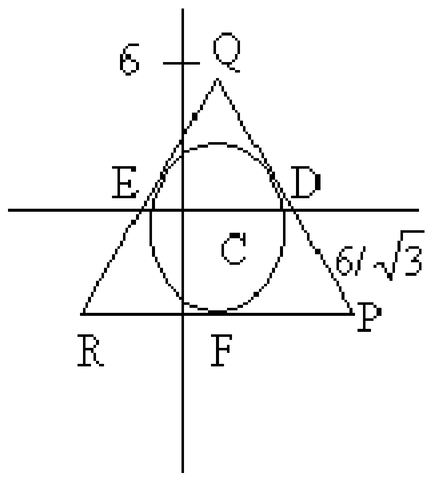
(C) $\left(\frac{\sqrt{3}}{2}, \frac{3}{2}\right), \left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$ (D) $\left(\frac{3}{2}, \frac{\sqrt{3}}{2}\right), \left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$

20. Equations of the sides QR, RP are

(A) $y = \frac{2}{\sqrt{3}}x + 1,\ y = -\frac{2}{\sqrt{3}}x - 1$ (B) $y = \frac{1}{\sqrt{3}}x, y = 0$

(C) $y = \frac{\sqrt{3}}{2}x + 1, y = -\frac{\sqrt{3}}{2}x - 1$ (D) $y = \sqrt{3}x, y = 0$

Solution 18-20:



S(PQ) = $-\sqrt{3}$ => S(PR) = 0 & S(RQ) = $\sqrt{3}$

$D \equiv \left(\frac{3\sqrt{3}}{2}, \frac{3}{2}\right)$

given

Required circle is member of the family of circles touching PQ at D.

$\Rightarrow \left(x - \frac{3\sqrt{3}}{2}\right)^2 + \left(y - \frac{3}{2}\right)^2 + k\left[\sqrt{3x} + y - 6\right] = 0$

Radius of required circle =1

$\Rightarrow \frac{3}{4}(k-3)^2 + \frac{(k-3)^2}{4} - (9-3k) = 1$

$k = \pm 1 \qquad \Rightarrow C \equiv (\sqrt{3}, 1)$

Solve to get $\qquad or\ C \equiv (2\sqrt{3}, 2)$

Accept $(\sqrt{3}, 1)$ as centre of C because sign of $(x, y) = \sqrt{3}x + y - 6$ should be less than D
{Parametric trim can be used to find centre}

Solution 18: (D)

Equation of required circle : $(x - \sqrt{3})^2 + (y - 1)^2 = 1$

Solution 19: (A)

As Δ is equilateral, $x_F = \sqrt{3}$ Replace in circle to get $y_F = 0$

$\Rightarrow F \equiv (\sqrt{3}, 0)$

Possible choices can be A or B

Ans. Would be A as $y_E > y_C$

Solution 20: (D)

$\because$ equation of PR is of type y = k & slope of QR is $\sqrt{3}$

Paragraph for Question Nos. 21 to 23

Consider the functions defined implicitly by the equation $y^3 - 3y + x = 0$ on various intervals in the real line.

If $x \in (-\infty, -2) \cup (2, \infty)$, the equation implicitly defines a unique real valued differentiable function y = f(x).

If $x \in (-2, 2)$, the equation implicitly defines a unique real valued differentiable function y = g(x) satisfying g (0) =0.

21. If $f(-10\sqrt{2}) = 2\sqrt{2}$, then $f''(-10\sqrt{2}) =$

(A) $\frac{4\sqrt{2}}{7^3 3^2}$ (B) $-\frac{4\sqrt{2}}{7^3 3^3}$ (C) $\frac{4\sqrt{2}}{7^3 3^2}$ (D) $-\frac{4\sqrt{2}}{7^3 3}$

22. The area of the region bounded by the curve y=f(x), the axis, and the lines x = a and x = b, where $-\infty < a < b < -2$, is

(A) $\int_a^b \frac{x}{3[-(f(x))^2 - 1]} dx + bf(b) - af(a)$

(B) $$-\int_a^b \frac{x}{3\left[(f(x))^2-1\right]}dx + bf(b) - af(a)$$

(C) $$\int_a^b \frac{x}{3\left[(f(x))^2-1\right]}dx - bf(b) + af(a)$$

(D) $$-\int_a^b -\frac{x}{3\left[(f(x))^2-1\right]}dx - bf(b) + af(a)$$

23. $\int_{-1}^{1} g'(x)dx =$

(A) 2g(-1) (B) 0 (C) -2g(1) (D) 2g(1)

Solution 21: (B)

$y^3 - 3y + x = 0$ ----- (1)

$$(3y^2-3)\frac{dy}{dx}+1=0 \quad \Rightarrow \frac{dy}{dx}\Big]_{-10\sqrt{2}} = \frac{1}{3-3y^2} = -\frac{1}{}$$

Different again

$$6y\left(\frac{dy}{dx}\right)^2 + (3y^2-3)\frac{d^2y}{dx^2} = 0$$

$$\Rightarrow \frac{d^2y}{dx^2} = \frac{6y\left(\frac{dy}{dx}\right)^2}{3-3y^2} = \frac{6(2\sqrt{2})}{3(1-8)} \times \frac{1}{21^2} = -\frac{4\sqrt{2}}{3^2 7^3}$$

Solution 22: (A)

$$\int_a^b y\,dx = y\int x\frac{dy}{dx}$$

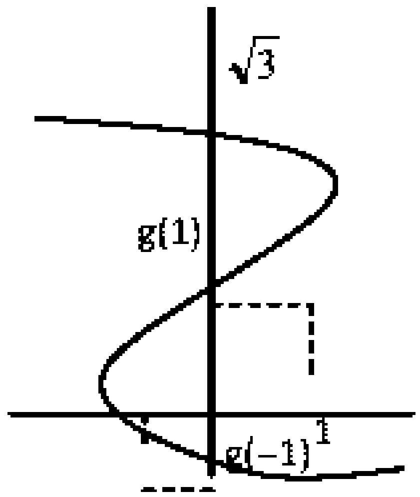
$$yx\Big|_a^b - \int x\frac{dy}{dx}dx$$

$$bf(b) - af(a) + \int \frac{x\,dx}{3[f^2(x)-1]}\{-u\sin g\ (1)\}$$

Solution 23:

$y^3 - 3y + x = 0$ Draw graph

{first draw $y = 3x - x^3$ & then reflect in $y = x$ line}



g(1)= − g(−1)

$$\int_{-1}^{1} g'(x)dx = g(x)\Big|_{-1}^{1} = g(1) - g(-1) = 2g(1)$$