MATHEMATICS

1. From the set of all possible permutations of the letters of the word 'IISER', one permutation is selected at random, and an unbiased coin is tossed. The probability that the permutation selected is IISER, given the condition that the head has already appeared on the coin is:

   A. $\frac{1}{30}$.
   B. $\frac{1}{60}$.
   C. $\frac{1}{72}$.
   D. $\frac{1}{120}$.

2. The equation $\int_{\pi/2}^{x} \sin^2 y \; dy = 0$ has

   A. No solution.
   B. One solution.
   C. Two solutions.
   D. Three solutions.

3. If $\vec{v}$ and $\vec{w}$ are two non-zero vectors in $\mathbb{R}^3$ then which of the following is a possibility?

   A. $\vec{v}.\vec{w} = 0$ and $\vec{v} \times \vec{w} = \vec{0}$.
   B. $\vec{v}.\vec{w} \neq 0$ and $\vec{v} \times \vec{w} = \vec{v}$.
   C. $\vec{v}.\vec{w} = 10$ and $\vec{v} \times \vec{w} = \vec{v} + \vec{w}$.
   D. $\vec{v}.\vec{w} = 10$ and $\vec{v} \times \vec{w} = \vec{0}$.

4. Let $f(z) = \text{Re}\left(\frac{z-2}{z-i}\right)$, where z is a complex number, i is a square root of -1 and 'Re' signifies the real part of a complex number. If $f(z) = 0$ then the locus of z is a circle with:

   A. Centre $(1, 2)$ and radius $\sqrt{5}$.
   B. Centre $(2, 1)$ and radius $\frac{\sqrt{5}}{2}$.
   C. Centre $(1, \frac{1}{2})$ and radius $\frac{\sqrt{5}}{2}$.
   D. Centre $(\frac{1}{2}, 1)$ and radius $\sqrt{5}$.

5. Let a, b, c be in geometric progression and $\log_e(a) - \log_e(2b)$, $\log_e(2b) - \log_e(3c)$, $\log_e(3c) - \log_e(a)$ be in arithmetic progression. Then with a, b, c as the sides, we can form:

   A. An acute-angled triangle.
   B. An obtuse-angled triangle.
   C. A right-angled triangle.
   D. No triangle.

6. The number of positive integral solutions of $x^2 - y^2 = 9876543210$ is:

   A. 0.
   B. 45.
   C. 90.
   D. 91.

7. Let g' denote the derivative of a differentiable function g. Take $f(x) = x|x|$. Then:

   A. At every point f' exists and is continuous.

B. At every point f' exists but is not continuous.

C. At some points f' does not exist.

D. At every point f'' exists but is not continuous.

8. Consider the system of linear equations given by the matrix multiplication $AX = B$ where

$$A = \begin{pmatrix} 1 & 2 & 1 \\ 2 & 4 & 5 \\ 3 & 6 & 1 \end{pmatrix} \text{ and } B = \begin{pmatrix} 1 \\ -4 \\ 7 \end{pmatrix}.$$

Then the system has:

A. No solution.

B. One solution.

C. Finitely many solutions.

D. Infinitely many solutions.

9. Suppose that under the constraints $3x_1 + 2x_2 \leq 12$, $x_2 \leq 1 + x_1$ and $x_2 \leq 2$, the function $y = x_1 + 3x_2$ has the minimum value y_1 and the maximum value y_2. Then (y_1, y_2) equals:

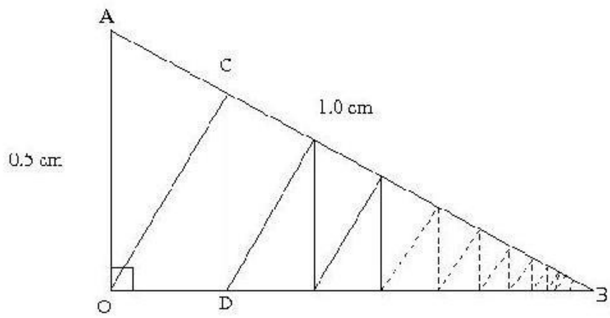
A. $(0, 7)$

B. $(0, \infty)$.

C. $(7, 11)$.

D. $(0, 11)$.

10. Let AOB be a right-angled triangle with $\angle AOB = 90°$ and $AB = 1.0\ cm$, $AO = 0.5\ cm$. A perpendicular is drawn from O on AB to intersect AB at C and from C to OB to intersect OB at D. The process is continued as shown in the figure. The length $|AO| + |OC| + |CD| + \cdots$ of the zig-zag figure made from both types of perpendiculars is:



A. $2\sqrt{3} - 2$

B. $2 + 2\sqrt{3}$.

C. $2 + \sqrt{3}$.

D. $2 - \sqrt{3}$.