DSB – 230 (MAT-UG)

Second B.Sc. Degree Examination, August/September 2008
(Directorate of Correspondence Course)
Paper – II : MATHEMATICS

Time : 3 Hours | Max. Marks : 90

*Note : Answer **any SIX** full questions of the following choosing at least **ONE** from **each** Part.*

PART – A

1. a) i) Find order and degree of the differential equation $\frac{d^3y}{dx^3} = \sqrt{\frac{dy}{dx}}$.

   ii) Solve : $y' = 1 + e^{x-y}$ **(2+2)**

   b) Solve : $\left(x \tan\left(\frac{y}{x}\right) - y \sec^2\left(\frac{y}{x}\right)\right)dx + x \sec^2\left(\frac{y}{x}\right)dy = 0$. **5**

   c) Find integrating factor and then solve $(x^3 - 2y^2)dx + 2xydy = 0$ when $x = 1$, $2y = 1$. **6**

2. a) i) Solve : $p = \log(px - y)$

   ii) Solve : $(p - 1)(xp + y) = 0$. **(2+2)**

   b) Solve : $xyp^2 + p(3x^2 - 2y^2) - 6xy = 0$. **5**

   c) Find the orthogonal trajectories of a family of coaxial circles $x^2 + y^2 + 2gx + c = 0$ where g is a parameter and c constant. **6**

PART – B

3. a) i) Solve : $(D^3 - 9D^2 + 23D - 15)y = 0$ where $D = \frac{d}{dx}$.

   ii) Solve : $(D^2 - 9)y = \cos 3x$ where $D = \frac{d}{dx}$. **(2+2)**

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b) Solve : $(D^2 + 3D + 2)y = e^{2x} \sin x$ where $D = \frac{d}{dx}$. **5**

c) Solve : $(x^3D^3 + 3x^2D^2 + xD + 1)y = x \log x$. **6**

4. a) i) Evaluate $\lim\limits_{x \to \frac{\pi}{2}} (\sec x - \tan x)$.

ii) Evaluate $\lim\limits_{x \to 0} \frac{e^{ax} - e^{-ax}}{\log(1 + bx)}$. **(2+2)**

b) State and prove Lagrange's Mean Value Theorem. **5**

c) Obtain the Maclaurin's expansion of log(sec x). **6**

PART – C

5. a) i) If $(ab)^2 = a^2b^2 \ \forall \ a, b \in G$. Then prove that G is abelian.

ii) Find the number of distinct generators of the cyclic group of order 24. **(2+2)**

b) If H is a subgroup of a group G and $g \in G$, then prove that the set $K = \{ghg^{-1} : h \in H\}$ is a subgroup of G. **5**

c) State and prove the Lagrange's theorem for a group. **6**

6. a) i) Prove that $|a| - |b| \le |a - b|$.

ii) Solve the inequality $2x - 3 < 5x + 3 < 2x + 3$. **(2+2)**

b) Find the order of the permutation

$$\phi = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \\ 5 & 9 & 6 & 3 & 1 & 4 & 2 & 10 & 8 & 7 \end{pmatrix}$$

Also find whether ϕ is even or odd. **5**

c) Find the envelope of the family of lines $x \cos^3 \alpha + y \sin^3 \alpha = a$ where α is a parameter. **6**

PART – D

7. a) i) If $\lim\limits_{n\to\infty} \{x_n\} = l$ and $\lim\limits_{n\to\infty} \{y_n\} = m$ then show that $\lim\limits_{n\to\infty} \{x_n + y_n\} = l + m$.

ii) Discuss the convergence of the sequence whose n^{th} term is $\dfrac{\log n}{n}$. **(2+2)**

b) Prove that the sequence whose n^{th} term is $\dfrac{3n+4}{2n+1}$.

i) is monotonically decreasing

ii) is bounded

iii) converges to $\dfrac{3}{2}$. **5**

c) Show that the sequence $\{a_n\}$ where $a_{n+1} = \sqrt{2 + a_n}$, $a_1 = \sqrt{2}$ is convergent to the positive root $x^2 - x - 2 = 0$. **6**

8. a) i) Show that an absolutely convergent series is convergent.

ii) Test the convergence of the series

$$\frac{1}{1.2} + \frac{1}{2.3} + \frac{1}{3.4} + \ldots + \infty.$$ **(2+2)**

b) Examine the convergence of the series

$$\left(\frac{2^2}{1^2} - \frac{2}{1}\right)^{-1} + \left(\frac{3^3}{2^3} - \frac{3}{2}\right)^{-2} + \left(\frac{4^4}{3^4} - \frac{4}{3}\right)^{-3} + \ldots + \infty.$$ **5**

c) Find the sum to infinity of the series

$$1 + \frac{1+3}{2!} + \frac{1+3+3^2}{3!} + \frac{1+3+3^2+3^3}{4!} + \ldots$$ **6**