# MATHEMATICS

***Time allowed : 3 hours*** ***Maximum Marks : 100***

***General Instructions:***

1. ***All*** *questions are compulsory.*
2. *The question paper consists of* ***29*** *questions divided into three sections, A, B and C. Section A comprises of* ***10*** *questions of one mark each, Section B comprises of* ***12*** *questions of* ***four*** *marks each and Section C comprises of* ***7*** *questions of* ***six*** *marks each.*
3. ***All*** *questions in Section A are to be answered in one word,* ***one*** *sentence or as per the exact requirement of the question.*
4. *There is no overall choice. However, internal choice has been provided in* ***4*** *questions of* ***four*** *marks each and* ***2*** *questions of* ***six*** *marks each. You have to attempt only* ***one*** *of the alternatives in all such questions.*
5. *Use of calculators is* ***not*** *permitted.*

## QUESTION PAPER CODE 65/1/1

## SECTION A

**Question numbers 1 to 10 carry one mark each.**

**1.** What is the range of the function $f(x) = \frac{|x-1|}{(x-1)}$ ?

**2.** What is the principal value of $\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)$ ?

**3.** If $A = \begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix}$, then for what value of $\alpha$ is A an identity matrix?

**4.** What is the value of the determinant $\begin{vmatrix} 0 & 2 & 0 \\ 2 & 3 & 4 \\ 4 & 5 & 6 \end{vmatrix}$ ?

**5.** Evaluate: $\int \frac{\log x}{x}\,dx$

**6.** What is the degree of the following differential equation?

$$5x\left(\frac{dy}{dx}\right)^2 - \frac{d^2y}{dx^2} - 6y = \log x$$

**7.** Write a vector of magnitude 15 units in the direction of vector

**8.** Write the vector equation of the following line:

$$\frac{x-5}{3} = \frac{y+4}{7} = \frac{6-z}{2}$$

**9.** If $\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}\begin{pmatrix} 3 & 1 \\ 2 & 5 \end{pmatrix} = \begin{pmatrix} 7 & 11 \\ k & 23 \end{pmatrix}$, then write the value of k.

**10.** What is the cosine of the angle which the vector $\sqrt{2}\,\hat{i} + \hat{j} + \hat{k}$ makes with y-axis?

## SECTION B

**Question numbers 11 to 22 carry 4 mark each.**

**11.** On a multiple choice examination with three possible answers (out of which only one is correct) for each of the five questions, what is the probability that a candidate would get four or more correct answers just by guessing?

**12.** Find the position vector of a point R which divides the line joining two points P and Q whose position vectors are $\left(2\vec{a} + \vec{b}\right)$ and $\left(\vec{a} - 3\vec{b}\right)$ respectively, externally in the ratio 1:2. Also, show that P is the mid point of the line segment RQ.

**13.** Find the Cartesian equation of the plane passing through the points A(0, 0, 0) and B(3, –1, 2) and parallel to the line $\frac{x-4}{1} = \frac{y+3}{-4} = \frac{z+1}{7}$

**14.** Using elementary row operations, find the inverse of the following matrix:

$$\begin{pmatrix} 2 & 5 \\ 1 & 3 \end{pmatrix}$$

**15.** Let Z be the set of all integers and R be the relation on Z defined as R = { (a, b) : a, b $\in$ Z, and (a – b) is divisible by 5.} Prove that R is an equivalence relation.

**16.** Prove the following:

$$\tan^{-1}\sqrt{x} = \frac{1}{2}\cos^{-1}\left(\frac{1-x}{1+x}\right), x \in (0, 1)$$

**OR**

Prove the following:

$$\cos^{-1}\left(\frac{12}{13}\right) + \sin^{-1}\left(\frac{3}{5}\right) = \sin^{-1}\left(\frac{56}{65}\right)$$

**17.** Show that the function f defined as follows, is continuous at x = 2, but not differentiable thereat:

$$f(x) = \begin{cases} 3x-2, & 0 < x \leq 1 \\ 2x^2 - x, & 1 < x \leq 2 \\ 5x-4, & x > 2 \end{cases}$$

**OR**

Find $\frac{dy}{dx}$, if $y = \sin^{-1}\left[x\sqrt{1-x} - \sqrt{x}\sqrt{1-x^2}\right]$

**18.** Evaluate: $\int e^x \left(\frac{\sin 4x - 4}{1 - \cos 4x}\right) dx$

**OR**

Evaluate: $\int \frac{1-x^2}{x\,(1-2x)} dx$

**19.** Evaluate: $\int_{\pi/6}^{\pi/3} \frac{\sin x + \cos x}{\sqrt{\sin 2x}} dx$

**20.** Find the points on the curve $y = x^3$ at which the slope of the tangent is equal to the y-coordinate of the point.

**21.** Find the general solution of the differential equation

$x \log x \cdot \frac{dy}{dx} + y = \frac{2}{x} \cdot \log x$

**OR**

Find the particular solution of the differential equation satisfying the given conditions:

$\frac{dy}{dx} = y \tan x$, given that y = 1 when x = 0.

22. Find the particular solution of the differential equation satisfying the given conditions:

$x^2 dy + (xy + y^2) dx = 0$ ; y = 1 when x = 1.

## SECTION - C

**Question number 23 to 29 carry 6 marks each.**

**23.** A small firm manufactures gold rings and chains. The total number of rings and chains manufactured per day is atmost 24. It takes 1 hour to make a ring and 30 minutes to make a chain. The maximum number of hours available per day is 16. If the profit on a ring is Rs. 300 and that on a chain is Rs. 190, find the number of rings and chains that should be manufactured per day, so as to earn the maximum profit. Make it as an L.P.P. and solve it graphically.

**24.** A card from a pack of 52 cards is lost. From the remaining cards of the pack, two cards are drawn at random and are found to both clubs. Find the probability of the lost card being of clubs.

**OR**

From a lot of 10 bulbs, which includes 3 defectives, a sample of 2 bulbs is drawn at random. Find the probability distribution of the number of defective bulbs.

**25.** The points A(4, 5, 10), B(2, 3, 4) and C(1, 2, –1) are three vertices of a parallelogram ABCD. Find the vector equations of the sides AB and BC and also find the coordinates of point D.

**26.** Using integration, find the area of the region bounded by the curve $x^2 = 4y$ and the line $x = 4y - 2$.

**OR**

Evaluate: $\int_0^{\pi} \frac{x \tan x}{\sec x + \tan x}\, dx$

**27.** Show that the right circular cylinder, open at the top, and of given surface area and maximum volume is such that its height is equal to the radius of the base.

**28.** Find the values of x for which $f(x) = [x(x-2)]^2$ is an increasing function. Also, find the points on the curve, where the tangent is parallel to x-axis.

**29.** Using properties of determinants, show the following:

$$\begin{vmatrix} (b+c)^2 & ab & ca \\ ab & (a+c)^2 & bc \\ ac & bc & (a+b)^2 \end{vmatrix} = 2abc\,(a+b+c)^3.$$

**QUESTION PAPER CODE 65/1**

**SECTION A**

**Questions number 1 to 10 carry 1 mark each.**

**1.** If $f : R \rightarrow R$ be defined by $f(x) = (3 - x^3)^{1/3}$, then find fof(x).

**2.** Write the principal value of $\sec^{-1}(-2)$.

**3.** What positive value of x makes the following pair of determinants equal? .

$$\begin{vmatrix} 2x & 3 \\ 5 & x \end{vmatrix}, \begin{vmatrix} 16 & 3 \\ 5 & 2 \end{vmatrix}$$

**4.** Evaluate:

$$\int \sec^2 (7 - 4x)\, dx$$

**5.** Write the adjoint of the following matrix:

$$\begin{pmatrix} 2 & -1 \\ 4 & 3 \end{pmatrix}$$

**6.** Write the value of the following integral:

$$\int_{-\pi/2}^{\pi/2} \sin^5 x \, dx$$

**7.** A is a square matrix of order 3 and $|A| = 7$. Write the value of $|\text{adj. } A|$.

**8.** Write tbe distance of the following plane from the origin:

$$2x - y + 2z + 1 = 0$$

**9.** Write a vector of magnitude 9 units in the direction of vector

$-2\hat{i} + \hat{j} + 2\hat{k}$.

**10.** Find $\lambda$ if $\left(2\hat{i} + 6\hat{j} + 14\hat{k}\right) \times \left(\hat{i} - \lambda\hat{j} + 7\hat{k}\right) = \vec{0}$.

## SECTION B

**Questions number 11 to 22 carry 4 marks each.**

11. A family has 2 children. Find the probability that both are boys, if it is known that

(i) at least one of the children is a boy,

(ii) the elder child is a boy.

**12.** Show that the relation S in the set A = $\{x \in Z : 0 \leq x \leq 12\}$ given by S = $\{(a, b): a, b \in Z, |a - b|$ is divisible by 4$\}$ is an equivalence relation. Find the set of all elements related to 1.

**13.** Prove the following:

$$\tan^{-1} x + \tan^{-1}\left(\frac{2x}{1-x^2}\right) = \tan^{-1}\left(\frac{3x - x^2}{1-3x^2}\right)$$

**OR**

Prove the following:

$$\cos\left[\tan^{-1}\{\sin(\cot^{-1} x)\}\right] = \sqrt{\frac{1+x^2}{2+x^2}}$$

**14.** Express the following matrix as the sum of a symmetric and a skew symmetric matrix, and verify your result :

$$\begin{pmatrix} 3 & -2 & -4 \\ 3 & -2 & -5 \\ -1 & 1 & 2 \end{pmatrix}$$

**15.** If $\vec{a} = \hat{i} + \hat{j} + \hat{k}$, $\vec{b} = 4\hat{i} - 2\hat{j} + 3\hat{k}$ and $\vec{c} = \hat{i} - 2\hat{j} + \hat{k}$, find a vector of magnitude 6 units which is parallel to the vector $2\vec{a} - \vec{b} + 3\vec{c}$.

**OR**

Let $\vec{a} = \hat{i} + 4\hat{j} + 2\hat{k}$, $\vec{b} = 3\hat{i} - 2\hat{j} + 7\hat{k}$ and $\vec{c} = 2\hat{i} - \hat{j} + 4\hat{k}$. Find a vector $\vec{d}$ which is perpendicular to both $\vec{a}$ and $\vec{b}$ and $\vec{c}\,.\,\vec{d} = 18$.

**16.** Find the points on the line $\frac{x+2}{3} = \frac{y+1}{2} = \frac{z-3}{2}$ at a distance of 5 units from the point P(l, 3, 3).

**OR**

Find the distance of the point P(6, 5, 9) from the plane determined by the points A(3, –1, 2), B(5, 2, 4) and C(–l, –1, 6).

**17.** Solve the following differential equation:

$$(x^2 - 1)\frac{dy}{dx} + 2xy = \frac{1}{x^2 - 1}; |x| \neq 1$$

**OR**

Solve the following differential equation:

$$\sqrt{1 + x^2 + y^2 + x^2y^2} + xy\frac{dy}{dx} = 0$$

**18.** Show that the differential equation $(x - y)\frac{dy}{dx} = x + 2y$, is homogeneous and solve it.

**19.** Evaluate the following :

$$\int \frac{x + 2}{\sqrt{(x - 2)(x - 3)}} dx$$

**20.** Evaluate the following :

$$\int_1^2 \frac{5x^2}{x^2 + 4x + 3} dx$$

**21.** If $y = e^{a \sin^{-1} x}$, $-1 \leq x \leq 1$, then show that

$$(1 - x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} - a^2y = 0.$$

**22.** If $y = \cos^{-1}\left(\frac{3x + 4\sqrt{1 - x^2}}{5}\right)$, find $\frac{dy}{dx}$.

## SECTION C

**Questions number 23 to 29 carry six marks each.**

23. Using properties of determinants, prove the following:

$$\begin{vmatrix} x & x^2 & 1+px^3 \\ y & y^2 & 1+py^3 \\ z & z^2 & 1+pz^3 \end{vmatrix} = (1 + pxyz)(x - y)(y - z)(z - x)$$

**OR**

Find the inverse of fhe following matrix using elementary operations:

$$A = \begin{pmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{pmatrix}$$

**24.** A bag contains 4 balls. Two balls are drawn at random, and are found to be white. What is the probability that all balls are white?

**25.** One kind of cake requires 300 g of flour and 15 g of fat, another kind of cake requires 150 g of flour and 30 g of fat. Find the maximum number of cakes which can be made from 7.5 kg of flour and 600 g of fat, assuming that there is no shortage of the other ingradients used in making the cakes. Make it as an L.P.P. and solve it graphically.

**26.** Find the coordinates of the foot of the perpendicular and the perpendicular distance of the point P(3, 2, 1) from the plane $2x - y + z + 1 = 0$. Find also, the image of the point in the plane.

**27.** Find the area of the circle $4x^2 + 4y^2 = 9$ which is interior to the parabola $x^2 = 4y$.

**OR**

Using integration, find the area of the triangle ABC, coordinates of whose vertices are A(4, 1), B(6, 6) and C(8, 4).

**28.** If the length of three sides of a trapezium other than the base is 10 cm each, find the area of the trapezium, when it is maximum.

**29.** Find the, intervals in which the following function is

(a) strictly increasing,

(b) strictly decreasing.

# Marking Scheme — Mathematics

***General Instructions :***

1. The Marking Scheme provides general guidelines to reduce subjectivity in the marking. The answers given in the Marking Scheme are suggested answers. The content is thus indicative. If a student has given any other answer which is different from the one given in the Marking Scheme, but conveys the meaning, such answers should be given full weightage.

2. Evaluation is to be done as per instructions provided in the marking scheme. It should not be done according to one's own interpretation or any other consideration — Marking Scheme should be strictly adhered to and religiously followed.

3. Alternative methods are accepted. Proportional marks are to be awarded.

4. In question(s) on differential equations, constant of integration has to be written.

5. If a candidate has attempted an extra question, marks obtained in the question attempted first should be retained and the other answer should be scored out.

6. A full scale of marks - 0 to 100 has to be used. Please do not hesitate to award full marks if the answer deserves it.

QUESTION PAPER CODE 65/1/1

**EXPECTED ANSWERS/VALUE POINTS**

**SECTION - A**

1. Range : $\{-1, 1\}$    2. $\frac{-\pi}{3}$    3. $\alpha = 0^{\circ}$

1×10 m

4. 8    5. $\frac{1}{2}(\log x)^2 + c$    6. 1

7. $5\hat{i} - 10\hat{j} + 10\hat{k}$    8. $\vec{r} = 5\hat{i} - 4\hat{j} + 6\hat{k} + \lambda \; 3\hat{i} + 7\hat{j} - 2\hat{k}$    9. k = 17

10. $\frac{1}{2}$

**SECTION - B**

11. Probability of selecting correct choice = $\frac{1}{3}$    1m

Probability of selecting wrong choice = $\frac{2}{3}$    ½ m

Probability distribution is given by $\left(\frac{2}{3} + \frac{1}{3}\right)^5$    1m

we want to compute P (4 correct answers)
+ P (5 correct answers)    ½ m

$$= \left(\frac{1}{3}\right)^5 + 5c_4 \left(\frac{1}{3}\right)^4 \left(\frac{2}{3}\right)$$

$$= \frac{11}{243}$$

1m

12. $P\left(2\vec{a}+\vec{b}\right)$

R 1 : 2 (external division) $\qquad\qquad$ $Q\left(\vec{a}-3\vec{b}\right)$

Position vector of R is $\dfrac{\left(\vec{a}-3\vec{b}\right)\times 1-\left(2\vec{a}+\vec{b}\right)(2)}{1-2}$ 1 m

$= 3\vec{a}+5\vec{b}$ 1 m

Mid - point of RQ is $\dfrac{3\vec{a}+5\vec{b}+\vec{a}-3\vec{b}}{2}$ 1 m

$= 2\vec{a}+\vec{b}$ ½ m

Which is position vector of P ½ m

13. Equation of plane passing through (0,0,0) is

$a(x-0)+b(y-0)+c(z-0) = 0 \quad \Rightarrow \therefore\ ax+by+cz = 0$ .............(i) 1 m

It passes through (3, –1, 2)

$\therefore\ 3a-b+2c=0$ ........................(ii) ½ m

line $\dfrac{x-4}{1} = \dfrac{y+3}{-4} = \dfrac{z+1}{7}$ is || to the plane (i)

$\Rightarrow\ a-4b+7c=0$ ............................................(iii) 1 m

From (ii) and (iii), $a=1$, $b=-19$ and $c=-11$ 1 m

Equation of plane is $x-19y-11z=0$ ½ m

14. Here $A = \begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix}$

Writing $A = IA \Rightarrow \begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$ ½ m

Applying $R_1 \to R_1 - R_2$, we get

$$\begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} A$$ 1 m

Applying $R_2 \to R_2 - R_1$, we get $\begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix} A$ 1 m

Applying $R_1 \to R_1 - 2R_2$, we get $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & -5 \\ -1 & 2 \end{bmatrix} A$ 1 m

$$\Rightarrow A^{-1} = \begin{bmatrix} 3 & -5 \\ -1 & 2 \end{bmatrix}$$ ½ m

15. $R = \{(a, b) : a, b \in Z \text{ and } (a-b) \text{ is divisible by } 5\}$

(i) $a - a = 0$ which is divisible by 5

$\therefore$ R is reflexive 1 m

(ii) $a - b$ is divisible by 5 and so is $b - a$

$\therefore$ R is symmetric 1 m

(iii) $a - c = (a - b) + (b - c)$

let $a - b = 5m$ and $b - c = 5n$

$\therefore$ $a - c = 5(m + n)$ $\Rightarrow$ $a - c$ is divisible by 5 1½ m

$\therefore$ R is transitive

$\therefore$ R is an equivalence - relation ½ m

16. Let $x = \tan^2\theta \Rightarrow \sqrt{x} = \tan\theta$ 1 m

$LHS = \tan^{-1}\left(\sqrt{x}\right) = \tan^{-1}\left(\tan\theta\right) = \theta$ 1 m

$RHS = \frac{1}{2}\cos^{-1}\left(\frac{1-\tan^2\theta}{1+\tan^2\theta}\right) = \frac{1}{2}\cos^{-1}\left(\cos 2\theta\right)$ 1 m

$= \frac{1}{2}\,2\theta = \theta$

$\Rightarrow$ LHS = RHS 1 m

OR

$\cos^{-1}\frac{12}{13} = \tan^{-1}\frac{5}{12}$ ½ m

$\sin^{-1}\frac{3}{5} = \tan^{-1}\frac{3}{4}$ ½ m

$\sin^{-1}\frac{56}{65} = \tan^{-1}\frac{56}{33}$ 1 m

$LHS = \tan^{-1}\frac{5}{12} + \tan^{-1}\frac{3}{4}$

$= \tan^{-1}\left[\frac{\frac{5+9}{12}}{1-\frac{5}{16}}\right] = \tan^{-1}\left(\frac{14}{\cancel{12}_{3}} \times \frac{\cancel{16}^{4}}{11}\right)$

$= \tan^{-1}\frac{56}{33} = RHS$ 2 m

17. $\lim_{h\to 0} f(2-h) = \lim_{h\to 0}\left[2(2-h)^2 - (2-h)\right] = 6$ .......................(i) ½ m

$\lim_{h\to 0} f(2+h) = \lim_{h\to 0}\left[5(2+h) - 4\right] = 6$ ................................(ii) ½ m

$f(2) = 8 - 2 = 6$ ........................................................(iii)

From (i), (ii), and (iii), $f(x)$ is continuous at $x = 2$ 1 m

$$RHD = \lim_{h\to 0}\left[\frac{\{5(2+h)-4\}-(6)\}}{h}\right] \neq LHD = \lim_{h\to 0}\left[\frac{\{(2h-3)(h-2)-6\}}{-h}\right] \text{ as } 5 \neq 7$$ ½+½ m

$\therefore$ f (x) is not differentiable there at 1 m

**OR**

$$y = \sin^{-1}\left[x\sqrt{1-x} - \sqrt{x}\sqrt{1-x^2}\right] \text{.............................(i)}$$

Let $x = \sin\alpha$ and $\sqrt{x} = \sin\theta$ 1 m

$\therefore$ (i) becomes $y = \sin^{-1}\left[\sin\alpha\cos\theta - \cos\alpha\sin\theta\right]$ ½ m

$= \sin^{-1}\left[\sin(\alpha-\theta)\right] = \alpha - \theta$ ½ m

$= \sin^{-1}x - \sin^{-1}\sqrt{x}$ 1 m

$\therefore \dfrac{dy}{dx} = \dfrac{1}{\sqrt{1-x^2}} - \dfrac{1}{2\sqrt{x}\sqrt{1-x}}$ 1 m

18. $I = \int e^x\left(\dfrac{\sin 4x - 4}{1-\cos 4x}\right)dx$

$= \int e^x\left[\dfrac{\sin 4x}{1-\cos 4x} - \dfrac{4}{1-\cos 4x}\right]dx$ 1 m

$= \int e^x\left[\dfrac{2\sin 2x\cos 2x}{2\cdot\sin^2 2x} - \dfrac{4}{2\sin^2 2x}\right]dx$ 1 m

$= \int e^x\left[\cot 2x - 2\,\text{cosec}^2\,2x\right]dx$ ½ m

This is of the form $= \int e^x\left[f(x)+f'(x)\right]dx$ ½ m

$\therefore I = e^x\cot 2x + c$ 1 m

**OR**

$$I = \int \frac{1-x^2}{x(1-2x)}\,dx = \frac{1}{2}\int \frac{2-2x^2}{x-2x^2}\,dx$$

$$= \frac{1}{2}\int \left[1 + \frac{2-x}{x(1-2x)}\right] dx$$ 1 m

$$= \frac{x}{2} + \frac{1}{2}\int \frac{2-x}{x(1-2x)}\,dx$$ ½ m

Let $\frac{2-x}{x(1-2x)} = \frac{A}{x} + \frac{B}{1-2x}$ . Getting A = 2, B = 3 1 m

$$\therefore\ I = \frac{x}{2} + \frac{1}{2}\int\left(\frac{2}{x} + \frac{3}{1-2x}\right) dx$$ ½ m

$$= \frac{x}{2} + \log|x| - \frac{3}{4}\log|1-2x| + c$$ 1 m

19. Let $\sin x - \cos x = t \Rightarrow (\cos x + \sin x)\,dx = dt$, Also, $\sin 2x = 1 - t^2$

When $x = \frac{\pi}{3}$, $t = \frac{\sqrt{3}-1}{2}$, when $x = \frac{\pi}{6}$, $t = \frac{1-\sqrt{3}}{2}$ 1 m

$\therefore$ Given integral becomes $I = \int_{\frac{1-\sqrt{3}}{2}}^{\frac{\sqrt{3}-1}{2}} \frac{dt}{\sqrt{1-t^2}}$ 1 m

$$= \left[\sin^{-1} t\right]_{\frac{1-\sqrt{3}}{2}}^{\frac{\sqrt{3}-1}{2}} = \sin^{-1}\left(\frac{\sqrt{3}-1}{2}\right) - \sin^{-1}\left(\frac{1-\sqrt{3}}{2}\right)$$ 2 m

or $2\sin^{-1}\left(\frac{\sqrt{3}-1}{2}\right)$

20. Equation of curve is $y = x^3 \Rightarrow \frac{dy}{dx} = 3x^2$ ......................(i) 1 m

(i) = y – coordinate of the point $\Rightarrow 3x^2 = y = x^3 \Rightarrow x^2(x-3) = 0$

$\Rightarrow x = 0,\ x = 3$ 1 m

When $x = 0, y = 0$, when $x = 3, y = 27$ 1 m

The points are (0,0), (3, 27) ½ + ½ m

21. The given differential equation can be written as

$$\frac{dy}{dx} + \frac{1}{x\log x}y = \frac{2}{x^2}$$ ½ m

$$\text{I.F.} = e^{\int\frac{1}{x\log x}dx} = e^{\log(\log x)} = \log x$$ 1 m

The solution is $y \cdot \log x = \int \frac{2}{x^2} \cdot \log x \ \ dx + c$ ½ m

or, $y \cdot \log x = 2\left[\log x \cdot \left(\frac{-1}{x}\right) + \int\frac{dx}{x^2}\right] + c = 2\left[\frac{-\log x}{x} - \frac{1}{x}\right] + c$ 1½ m

$$\Rightarrow \quad y \cdot \log x = -\frac{2}{x}\left[1 + \log x\right] + c$$ ½ m

**OR**

Given differential equation can be written as

$$\int\frac{dy}{y} = \int \tan x \ dx$$ 1 m

or, $\log y = \log \sec x + c$ 1 m

when, $x = 0,\ y = 1 \Rightarrow c = 0$

[Note : c = 1, if constant is taken as log c] 1 m

$\therefore \ \log y = \log \sec x$

or $y = \sec x$ 1 m

22. The given differential equation can be written as

$$\frac{dy}{dx} + \frac{xy + y^2}{x^2} = 0 \cdot \text{ Let } y = vx \Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$$ 1 m

$$\left.\begin{aligned} &\Rightarrow v + x\frac{dv}{dx} + \left(v + v^2\right) = 0 \\ &\Rightarrow x\frac{dv}{dx} = -\, v\left(2 + v\right) \end{aligned}\right\}$$ 1+½ m

$$\text{or } \frac{dv}{v\left(2+v\right)} = -\frac{dx}{x}$$

$$\text{or } \int\left(\frac{1}{v} - \frac{1}{2+v}\right)dx = -2\int\frac{dx}{x}$$ ½ m

$$\Rightarrow \log\frac{v}{v+2} = \log\frac{c}{x^2}$$

$$\text{or } \frac{y}{y+2x} = \frac{c}{x^2}$$ ½ m

when $x = 1, y = 1 \Rightarrow c = \frac{1}{3}$

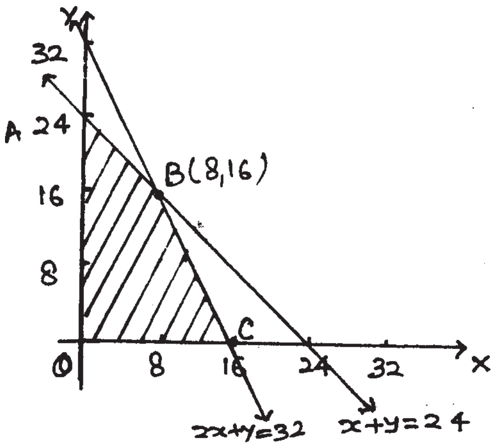
$\therefore$ The solution becomes

$y + 2x = 3x^2y$ ½ m

**SECTION - C**

23. Let x be the number of gold rings and y, the number of chains

The objective function is $Z = 300x + 190y$ 1 m




Constraints are :

$$\left.\begin{aligned} &x + y \leq 24 \\ &2x + y \leq 32 \\ &x \geq 0,\ y \geq 0 \end{aligned}\right\}$$ 2 m

Correct graph 2 m

Getting corners of feasible region as A (0, 24), B (8, 16) C (16, 0), O (0, 0)

$Z_{(0,\,0)} = 0$, $Z_A = 45\,60$, $Z_C = 4800$

$Z_B = 300 \times 8 + 190 \times 16 = 2400 + 3040 = 5440$

$\therefore$ Z is maximum at B (8. 16)

$\therefore$ For maximum profit, Rings = 8, chains = 16 — 1 m

24. Let $E_1$ be the event that lost card is that of clubs

$E_2$ be event that lost card is not of clubs

A : Two cards of clubs are drawn from remaining cards — 1 m

$$P(E_1) = \frac{1}{4}, \quad P(E_2) = \frac{3}{4}$$ ½ m

$$P\left(A/E_1\right) = {}^{12}c_2 / {}^{51}c_2 = \frac{\overset{2}{\cancel{12}} \times 11}{\underset{17}{\cancel{51}} \times \underset{25}{\cancel{50}}} = \frac{22}{425}$$ 1 m

$$P\left(A/E_2\right) = {}^{13}c_2 / {}^{51}c_2 = \frac{13 \times \overset{2}{\cancel{12}}}{\underset{17}{\cancel{51}} \times \underset{25}{\cancel{50}}} = \frac{26}{425}$$ 1 m

$$P\left(E_1/A\right) = \frac{P\left(A/E_1\right) \cdot P(E_1)}{\sum P(E_i) \cdot P\left(A/E_i\right)}$$ 1 m

$$= \frac{\frac{22}{425} \times \frac{1}{4}}{\frac{22}{425} \times \frac{1}{4} + \frac{26}{425} \times \frac{3}{4}} = \frac{11}{50}$$ 1 + ½ m

**OR**

Let X be the random variate giving number of defective bulbs, X can take values 0, 1, 2 — 1 m

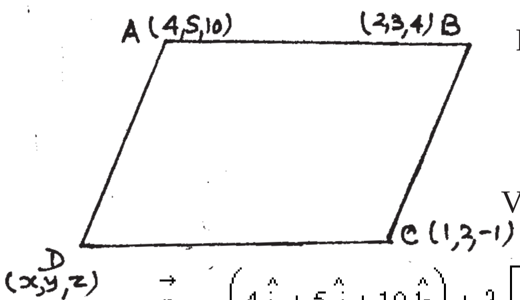
$$P(X=0) = \frac{{}^7c_2}{{}^{10}c_2} = \frac{7}{15},\ P(x=1) = \frac{{}^7c_1 \times {}^3c_1}{{}^{10}c_2} = \frac{7}{15},\ P(X=2) = \frac{{}^3c_2}{{}^{10}c_2} = \frac{1}{15}$$ 3 m

$\therefore$ Probability distribution of X is

| X | 0 | 1 | 2 |
|---|---|---|---|
| P (X) | $7/15$ | $7/15$ | $1/15$ |

2 m

25.




P.V. of A = $4\hat{i} + 5\hat{j} + 10\hat{k}$

and $B = 2\hat{i} + 3\hat{j} + 4\hat{k}$

Vector equation of A B is

$$\vec{r} = \left(4\hat{i} + 5\hat{j} + 10\hat{k}\right) + \lambda\left[\left(2\hat{i} + 3\hat{j} + 4\hat{k}\right) - \left(4\hat{i} + 5\hat{j} + 10\hat{k}\right)\right]$$

$$= 4\hat{i} + 5\hat{j} + 10\hat{k} + \lambda\left(-2\hat{i} - 2\hat{j} - 6\hat{k}\right)$$ 1+1 m

$$\text{or}\quad \vec{r} = 4\hat{i} + 5\hat{j} + 10\hat{k} + \lambda\left(\hat{i} + \hat{j} + 3\hat{k}\right)$$

Similarly, vector equation BC is — 1½ m

$$\vec{r} = \left(2\hat{i} + 3\hat{j} + 4\hat{k}\right) + \mu\left[-\hat{i} - \hat{j} - 5\hat{k}\right]$$

$$\text{or}\quad \vec{r} = 2\hat{i} + 3\hat{j} + 4\hat{k} + \mu\left(\hat{i} + \hat{j} + 5\hat{k}\right)$$

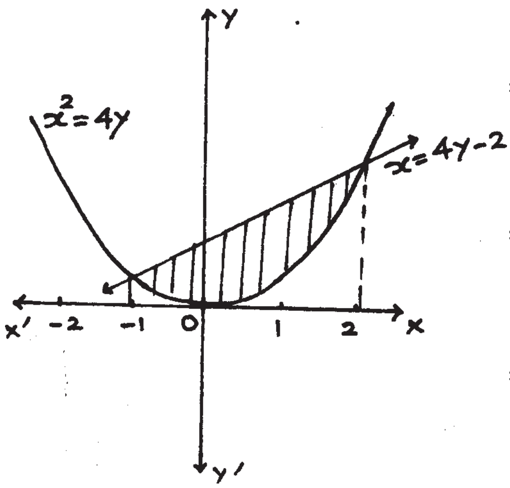
Mid-point of AC is $\frac{5}{2}\hat{i} + \frac{7}{2}\hat{j} + \frac{9}{2}\hat{k}$ — 1 m

Mid-point of BD is $\frac{x+2}{2}\hat{i} + \frac{3+y}{2}\hat{j} + \frac{4+z}{2}\hat{k}$ — 1 m

Coordinates of D are (3, 4, 5) — ½ m

26. Correct Figure — 1 m

Point of intersection of curve and line has x coordinates x = 2, x = – 1 — 1 m




Required area =

$$= \int_{-1}^{2} \frac{x+2}{4}\, dx - \int_{-1}^{2} \frac{x^2}{4}\, dx$$ 1+1 m

$$= \frac{1}{4}\left[\frac{x^2}{2} + 2x - \frac{x^3}{3}\right]_{-1}^{2}$$ 1 m

$$\left.\begin{aligned} &= \frac{1}{4}\left[\left(2+4-\frac{8}{3}\right) - \left(\frac{1}{2} - 2 + \frac{1}{3}\right)\right] \\ &= \frac{9}{8} \text{ sq. u.} \end{aligned}\right\}$$ 1 m

**OR**

$$= \pi^2 - 2\pi$$

$$I = \int_0^{\pi} \frac{x \tan x\, dx}{\sec x + \tan x} \Rightarrow I = \int_0^{\pi} \frac{(\pi - x)\tan x}{\sec x + \tan x}\, dx$$ 1 m

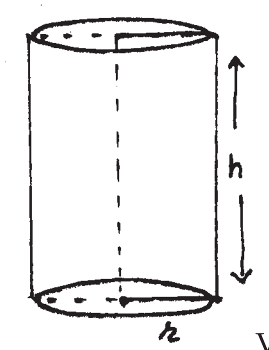
$$2I = \int_0^{\pi} \frac{\pi \tan x}{\sec x + \tan x} = \pi \int_0^{\pi} \frac{\sin x}{1 + \sin x}\, dx$$ ½ m

$$= \pi \int_0^{\pi} \left(1 - \frac{1}{1+\sin x}\right) dx = \pi \int_0^{\pi} \left(1 - \frac{1 - \sin x}{\cos^2 x}\right) dx$$ 2 m

$$= \pi \int_0^{\pi} \left(1 - \sec^2 x + \sec x \tan x\right) dx$$ 1 m

$$= \pi \left[x - \tan x + \sec x\right]_0^{\pi} = \pi\left[\pi - 1 - 1\right]$$ 1 m

$$\therefore\ I = \frac{\pi^2}{2} - \pi = \frac{\pi}{2}\left[\pi - 2\right]$$ ½ m

27. 


Let r be the radius of base of cylinder and h, be its height which is open at the top 1 m

s = Surface area = $2\pi r h + \pi r^2$

or $\dfrac{s - \pi r^2}{2\pi r} = h$ 1 m

V = Volume of cylinder = $\pi r^2 h$

$$= \pi r^2 \left(\frac{s - \pi r^2}{2\pi r}\right) = \frac{r}{2}\left(s - \pi r^2\right)$$

$$= \frac{rs}{2} - \frac{\pi}{2}\cdot r^3$$ 1½ m

$$\frac{dv}{dr} = \frac{s}{2} - \frac{\pi}{2}\cdot 3r^2$$

$$\frac{dv}{dr} = 0 \Rightarrow s = 3\pi r^2 = 2\pi r h + \pi r^2$$

$$\Rightarrow 2\pi r \cdot r = 2\pi r h \Rightarrow r = h$$ 1½ m

$\dfrac{d^2v}{dr^2} < 0 \Rightarrow$ Volume is maximum at r = h

$\therefore$ Radius of base of cylinder = its Height ½ + ½ m

28. $f(x) = [x(x-2)]^2$

$f'(x) = 4x(x-2)(x-1)$ 1½ m

$f'(x) = 0$ gives $x = 0,\ x = 1$ or $x = 2$

$\therefore$ Intervals are $(-\infty, 0), (0, 1), (1, 2), (2, \infty)$ 2 m

Increasing in $[0, 1]$ and $[2, \infty)$

$\Rightarrow$ or $0 \le x \le 1$ and $x \ge 2$ 1½ m

The point where tangents are parallel to x axis

are (0, 0), (1, 1), (2, 0) 1 m

29. $$\Delta = \begin{vmatrix} (b+c)^2 & ba & ca \\ ab & (c+a)^2 & bc \\ ac & bc & (a+b)^2 \end{vmatrix};$$ Applying $R_1 \to aR_1$, $R_2 \to bR_2$, $R_3 \to cR_3$ we get

$$= \frac{1}{abc}\begin{vmatrix} a(b+c)^2 & ba^2 & ca^2 \\ ab^2 & b(c+a)^2 & cb^2 \\ ac^2 & bc^2 & c(a+b)^2 \end{vmatrix}$$ 1 m

$$= \begin{vmatrix} (b+c)^2 & a^2 & a^2 \\ b^2 & (c+a)^2 & b^2 \\ c^2 & c^2 & (a+b)^2 \end{vmatrix}$$ ½ m

Applying $C_1 \to C_1 - C_3$, $C_2 \to C_2 - C_3$, we get

$$\Delta = \begin{vmatrix} (b+c)^2 - a^2 & 0 & a^2 \\ 0 & (c+a)^2 - b^2 & b^2 \\ c^2 - (a+b)^2 & c^2 - (a+b)^2 & (a+b)^2 \end{vmatrix}$$ 1 m

$$= (a+b+c)^2 \begin{vmatrix} b+c-a & 0 & a^2 \\ 0 & c+a-b & b^2 \\ c-a-b & c-a-b & (a+b)^2 \end{vmatrix}$$ ½ m

Applying $R_3 \to R_3 - (R_1 + R_2)$, we get

$$\Delta = (a+b+c)^2 \begin{vmatrix} b+c-a & 0 & a^2 \\ 0 & c+a-b & b^2 \\ -2b & -2a & 2ab \end{vmatrix}$$ 1 m

Applying $C_1 \to aC_1$ and $C_2 \to bC_2$ we get

$$\Delta = \frac{(a+b+c)^2}{a\,b}\begin{vmatrix} ab+ac-a^2 & 0 & a^2 \\ 0 & b(c+a-b) & b^2 \\ -2ba & -2ab & 2ab \end{vmatrix}$$ ½ m

Applying $C_1 \to C_1 + C_3$, $C_2 \to C_2 + C_3$ we get

$$\Delta = \frac{(a+b+c)^2}{a\,b}\begin{vmatrix} a(b+c) & a^2 & a^2 \\ b^2 & b(a+c) & b^2 \\ 0 & 0 & 2ab \end{vmatrix}$$ ½ m

$$= \frac{(a+b+c)^2}{a\,b} \times ab \times 2ab \begin{vmatrix} b+c & a & a \\ b & c+a & b \\ 0 & 0 & 1 \end{vmatrix}$$ ½ m

$= 2\,ab\,(a+b+c)^2\,[(b+c)(c+a) - ab]$

$= 2\,ab\,(a+b+c)^2\,[bc + c^2 + ab + ac - ab]$

$= 2\,abc\,(a+b+c)^3$ ½ m

QUESTION PAPER CODE 65/1

**EXPECTED ANSWERS/VALUE POINTS**

**SECTION - A**

Marks

1-10. 1. $x$  2. $\frac{2\pi}{3}$  3. $x=4$  4. $-\frac{1}{4}\tan(7-4x)+c$  5. $\begin{pmatrix} 3 & 1 \\ -4 & 2 \end{pmatrix}$ 1x10 = 10 m

6. zero  7. 49  8. $\frac{1}{3}$  9. $-6\hat{i}+3\hat{j}+6\hat{k}$  10. $-3$

**SECTION - B**

11. Let event A is that the family has two boys

(i) event B: At least one is a boy

P(both boys, given that at least one is a boy) = P(A/B) ½ m

$$= \frac{P(A \cap B)}{P(B)} = \frac{P\{(B, B)\}}{P\{(B, G), (G, B), (B, B),\}}$$ ½ + ½ m

$$= \frac{1/4}{3/4} = \frac{1}{3}$$ ½ m

(ii) event C: the elder child is a boy

P(both boys, given that at elder child is a boy) = P(A/C)

$$= \frac{P(A \cap C)}{P(C)} = \frac{P\{(B, B)\}}{P\{(B, G), (B, B),\}}$$ 1 m

$$= \frac{1/4}{2/4} = \frac{1}{2}$$ 1 m

12. (i) For all a $\in$ A, (a, a) $\in$ S ($\because$ a – a = 0 is divisible by 4)

$\therefore$ S is reflexive in A 1 m

(ii) For all a, b $\in$ A, if (a, b) $\in$ S then $|a-b|$ is divisible by 4.

Hence $|b-a|$ is also divisible by 4 $\Rightarrow$ S is symmetric in A 1 m

(iii) $\forall$ a, b, c $\in$ A, Let (a, b) $\in$ S and (b, c) $\in$ S

i.e. $|a-b|$ is divisible by 4 and $|b-c|$ is divisible by 4

$\Rightarrow$ (a–b) = $\pm$ 4p, (b–c) = $\pm$ 4q, adding to get a–c = 4m $\Rightarrow$ (a, c) $\in$ S 1½ m

$\Rightarrow$ S is transitive in A

Hence S is an equivalence relation

Elements related to 1 are {1, 5, 9} ½ m

13. $$\text{LHS} = \tan^{-1}\left[\frac{x + \frac{2x}{1-x^2}}{1 - x\,\frac{2x}{1-x^2}}\right]$$ 2 m

$$= \tan^{-1}\left[\frac{x\,(1-x^2)+2x}{1-x^2-2x^2}\right]$$ 1 m

$$= \tan^{-1}\left[\frac{3x-x^3}{1-3x^2}\right] = \text{RHS.}$$ 1 m

**OR**

LHS $= \cos[\tan^{-1}\{\sin(\cot^{-1} x)\}]$

$$= \cos\left[\tan^{-1}\left\{\sin\left(\sin^{-1}\frac{1}{\sqrt{1+x^2}}\right)\right\}\right]$$ 1 m

$$= \cos\left[\tan^{-1}\left(\frac{1}{\sqrt{1+x^2}}\right)\right] = \cos\left[\cos^{-1}\frac{\sqrt{1+x^2}}{\sqrt{2+x^2}}\right]$$ 1+1 m

$$= \frac{\sqrt{1+x^2}}{\sqrt{2+x^2}} = \text{R.H.S}$$ 1 m

14. $A = \begin{pmatrix} 3 & -2 & -4 \\ 3 & -2 & -5 \\ -1 & 1 & 2 \end{pmatrix}$, then $A' = \begin{pmatrix} 3 & 3 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{pmatrix}$ 1 m

Writing $A = \frac{1}{2}(A+A') + \frac{1}{2}(A-A')$ ½ m

$$\frac{1}{2}(A+A') = \begin{pmatrix} 3 & 1/2 & -5/2 \\ 1/2 & -2 & -2 \\ -5/2 & -2 & 2 \end{pmatrix}$$ ½ m

$$\frac{1}{2}(A-A') = \begin{pmatrix} 0 & -5/2 & -3/2 \\ 5/2 & 0 & -3 \\ 3/2 & 3 & 0 \end{pmatrix}$$ ½ m

and $\begin{pmatrix} 3 & -2 & -4 \\ 3 & -2 & -5 \\ -1 & 1 & 2 \end{pmatrix} = \begin{pmatrix} 3 & 1/2 & -5/2 \\ 1/2 & -2 & -2 \\ -5/2 & -2 & 2 \end{pmatrix} + \begin{pmatrix} 0 & -5/2 & -3/2 \\ 5/2 & 0 & -3 \\ 3/2 & 3 & 0 \end{pmatrix}$ 1 m

Thus A = B + C

Where B is Symmetric matrix and C is skew symmetric matrix ½ m

15. $2\vec{a}-\vec{b}+3\vec{c} = (2\hat{i}+2\hat{j}+2\hat{k}) - (4\hat{i}-2\hat{j}+3\hat{k}) + (3\hat{i}-6\hat{j}+3\hat{k}) = \hat{i}-2\hat{j}+2\hat{k}$ 2 m

$|2\vec{a}-\vec{b}+3\vec{c}| = 3$ 1 m

$\therefore$ Required vector $= 2\hat{i}-4\hat{j}+4\hat{k}$ 1 m

**OR**

A vector perpendicutar to $\vec{a}$ and $\vec{b} = \vec{a} \times \vec{b} = 32\hat{i} - \hat{j} - 14\hat{k}$ ½+1 m

Let $\vec{d} = \lambda\,(32\hat{i} - \hat{j} - 14\hat{k})$ ½ m

$\therefore\ \vec{c} \cdot \vec{d} = 18 \Rightarrow \lambda\,(64+1-56) = 18 \Rightarrow \lambda = 2$ 1½ m

$\therefore\ \vec{d} = 64\hat{i} - 2\hat{j} - 28\hat{k}$ ½ m

16. Any point Q on the given line is Q $(3\lambda - 2, 2\lambda - 1, 2\lambda + 3)$ 1 m

$PQ^2 = (3\lambda - 3)^2 + (2\lambda - 4)^2 + (2\lambda)^2 = 17\lambda^2 - 18\lambda - 16\lambda + 25$ 1 m

$PQ^2 = (5)^2 \Rightarrow 17\lambda(\lambda - 2) = 0 \Rightarrow \lambda = 0 \text{ or } \lambda = 2$ 1 m

$\therefore$ The points are Q (–2, –1, 3) and R (4, 3, 7) ½+½ m

OR

Normal to the plane passing through A, B and C

is $\overrightarrow{AB} \times \overrightarrow{BC} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 2 \\ -6 & -3 & 2 \end{vmatrix} = 12\hat{i} - 16\hat{j} + 12\hat{k}$ or $3\hat{i} - 4\hat{j} + 3\hat{k}$ 1½ m

$\therefore$ Equation of plane is $\vec{r} \cdot (3\hat{i} - 4\hat{j} + 3\hat{k}) = 19$ or $3x - 4y + 3z - 19 = 0$ 1½ m

Distance of P(6, 5, 9) from the plane = $\dfrac{|18 - 20 + 27 - 19|}{\sqrt{(3)^2 + (-4)^2 + (3)^2}}$

$= \dfrac{6}{\sqrt{34}}$ 1 m

17. Given differential equation can be written as

$\dfrac{dy}{dx} + \dfrac{2x}{x^2 - 1} \cdot y = \dfrac{1}{(x^2 - 1)^2}$ 1 m

Which is of the form $\frac{dy}{dx} + P(x) \cdot y = Q(x)$

$$\int P(x)\,dx = \int \frac{2x}{x^2-1}\,dx = \log|x^2-1|$$ ½ m

$\therefore$ Integrating factor $= e^{\int P(x)\,dx} = (x^2-1)$ 1 m

$\therefore$ The solution is $(x^2-1)\cdot y = \int \frac{1}{(x^2-1)^2}(x^2-1)\,dx$ 1 m

$$(x^2-1)\cdot y = \frac{1}{2}\log\left|\frac{x-1}{x+1}\right| + c$$ ½ m

**OR**

Given differential equation can be written as

$$\sqrt{(1+x^2)}\cdot\sqrt{(1+y^2)} + xy\frac{dy}{dx} = 0$$ ½ m

$$\Rightarrow \frac{y}{\sqrt{1+y^2}}\,dy = -\frac{\sqrt{1+x^2}}{x}\,dx$$ ½ m

Integrating both sides, we get

$$\sqrt{1+y^2} = -\int\frac{\sqrt{1+x^2}}{x^2}\cdot x\,dx = -\int\frac{t^2\,dt}{t^2-1} \text{ where } (1+x^2)=t^2$$ 1 m

$$\Rightarrow \sqrt{1+y^2} = -\int\left(1+\frac{1}{t^2-1}\right)dt = -t-\frac{1}{2}\log\frac{t-1}{t+1}+c$$

$$= -\sqrt{1+x^2} - \frac{1}{2}\log\left|\frac{\sqrt{1+x^2}-1}{\sqrt{1+x^2}+1}\right| + c$$ 1+1 m

or $$\sqrt{1+y^2} + \sqrt{1+x^2} + \frac{1}{2}\log\left|\frac{\sqrt{1+x^2}-1}{\sqrt{1+x^2}+1}\right| = c$$

18. Given differential equation can be written as

$$\frac{dy}{dx} = \frac{x+2y}{x-y} = \frac{1+2\,y/x}{1-y/x} = f\left(y/x\right)$$

hence, the differential equation is homogeneous. 1 m

Taking $\frac{y}{x} = v$ or $y = vx \Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$ ½ m

$$\therefore\ v + x\frac{dv}{dx} = \frac{1+2v}{1-v} \text{ or } x\frac{dv}{dx} = \frac{1+2v}{1-v} - v = \frac{1+v+v^2}{1-v}$$

$$\Rightarrow \int \frac{v-1}{v^2+v+1}\,dv = -\int \frac{dx}{x}$$ 1 m

$$\Rightarrow \frac{1}{2}\int \frac{2v+1-3}{v^2+v+1}\,dv = -\log|x| + c$$

$$or\ \frac{1}{2}\log\left|v^2+v+1\right| - \frac{3}{2}\int \frac{dv}{\left(v+\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2} = -\log|x| + c$$

$$\Rightarrow \log\left|v^2+v+1\right| + \log x^2 = 2\sqrt{3}\ \tan^{-1}\left(\frac{2v+1}{\sqrt{3}}\right) + c$$ 1 m

$$\Rightarrow \log\left|y^2+xy+x^2\right| = 2\sqrt{3}\ \tan^{-1}\left(\frac{2y+x}{\sqrt{3}\,x}\right) + c$$ ½ m

19. Here $I = \int \frac{x+2}{\sqrt{x^2-5x+6}}\,dx = \frac{1}{2}\int \frac{2x-5+9}{\sqrt{x^2-5x+6}}\,dx$ 1 m

$$= \frac{1}{2}\int \frac{2x-5}{\sqrt{x^2-5x+6}}\,dx + \frac{9}{2}\int \frac{1}{\sqrt{\left(x-5/2\right)^2 - \left(\frac{1}{2}\right)^2}}\,dx$$ 1 m

$$= \sqrt{x^2-5x+6} + \frac{9}{2}\log\left|\left(x-\frac{5}{2}\right) + \sqrt{x^2-5x+6}\right| + c$$ 1+1 m

20. $I = \int_1^2 \frac{5x^2}{x^2+4x+3}\, dx = 5\int_1^2 1 - \frac{4x+3}{x^2+4x+3}\, dx$ 1 m

$= 5[x]_1^2 - 10\int_1^2 \frac{2x+4-\frac{5}{2}}{x^2+4x+3}\, dx$ ½ m

$= 5 - 10\left[\log\left|x^2+4x+3\right|\right]_1^2 + 25\int_1^2 \frac{1}{(x+2)^2-(1)^2}\, dx$ 1 m

$= 5 - 10\ \log\frac{15}{8} + 25\cdot\frac{1}{2}\left[\log\left|\frac{x+2-1}{x+2+1}\right|\right]_1^2$ 1 m

$= 5 - 10\ \log\frac{15}{8} + \frac{25}{2}\ \log\frac{6}{5}$ ½ m

Note: If solved using partial fractions, the answer be of the fo

$$5 + \frac{5}{2}\ \log\frac{3}{2} - \frac{45}{2}\ \log\left(\frac{5}{4}\right)$$

21. $\frac{dy}{dx} = e^{a\sin^{-1}x}\frac{a}{\sqrt{1-x^2}} = \frac{ay}{\sqrt{1-x^2}}$ 1 m

$\Rightarrow \sqrt{1-x^2}\cdot\frac{dy}{dx} = ay$ ......................... (i) ½ m

$\Rightarrow \sqrt{1-x^2}\cdot\frac{d^2y}{dx^2} - \frac{x}{\sqrt{1-x^2}}\cdot\frac{dy}{dx} = a\frac{dy}{dx}$ 1 m

$\Rightarrow \left(1-x^2\right)\frac{d^2y}{dx^2} - x\frac{dy}{dx} - a\sqrt{1-x^2}\ \frac{dy}{dx} = 0$ ½ m

$\Rightarrow \left(1-x^2\right)\frac{d^2y}{dx^2} - x\frac{dy}{dx} - a^2y = 0$ [Using (i)] 1 m

22. $y = \cos^{-1}\left[\frac{3}{5}x + \frac{4}{5}\sqrt{1-x^2}\right]$

$= \cos^{-1}\left[\frac{3}{5}\cdot\cos\theta + \frac{4}{5}\sin\theta\right]$ where $x = \cos\theta$ 1 m

$= \cos^{-1}\left[\cos\alpha\cdot\cos\theta + \sin\alpha\cdot\sin\theta\right]$, $\because$ if $\frac{3}{5} = \cos\alpha$, then $\frac{4}{5} = \sin\alpha$ 1 m

$= \cos^{-1}\left[\cos\left(\alpha-\theta\right)\right] = \alpha-\theta = \cos^{-1}\left(3/5\right) - \cos^{-1}x$ 1 m

$\Rightarrow \frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}}$ $\left[\text{Note: Answer can also be } -\frac{1}{\sqrt{1-x^2}}\right]$ 1 m

## SECTION - C

23. $\text{LHS} = \begin{vmatrix} x & x^2 & 1+px^3 \\ y & y^2 & 1+py^3 \\ z & z^2 & 1+pz^3 \end{vmatrix} = \begin{vmatrix} x & x^2 & 1 \\ y & y^2 & 1 \\ z & z^2 & 1 \end{vmatrix} + p\begin{vmatrix} x & x^2 & x^3 \\ y & y^2 & y^3 \\ z & z^2 & z^3 \end{vmatrix}$ 1 m

$= \begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix} + pxyz\begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix}$ ½ m

$= (1+pxyz)\begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix}$ ½ m

$= (1+pxyz)\begin{vmatrix} 1 & x & x^2 \\ 0 & y-x & y^2-x^2 \\ 0 & z-x & z^2-x^2 \end{vmatrix}$ $\begin{matrix} R_2 \to R_2 - R_1 \\ R_3 \to R_3 - R_1 \end{matrix}$ 1 m

$$= (1+pxyz)(x-y)(z-x)\begin{vmatrix} 1 & x & x^2 \\ 0 & -1 & -(x+y) \\ 0 & 1 & z+x \end{vmatrix}$$ ½ m

$$= (1+pxyz)(x-y)(z-x)\begin{vmatrix} 1 & x & x^2 \\ 0 & 0 & z-y \\ 0 & 1 & z+x \end{vmatrix} \quad R_2 \to R_2 + R_3$$ 1 m

$$= (1+pxyz)(x-y)(y-z)(z-x)\begin{vmatrix} 1 & x & x^2 \\ 0 & 0 & -1 \\ 0 & 1 & z+x \end{vmatrix}$$ ½ m

$$= (1+pxyz)(x-y)(y-z)(z-x)\cdot 1 = \text{R.H.S.}$$ 1 m

**OR**

Writing $\begin{pmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} A$ 1 m

$R_2 \to R_2 + R_1 \Rightarrow \begin{pmatrix} 1 & 2 & -2 \\ 0 & 5 & -2 \\ 0 & -2 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} A$ 1 m

$R_2 \to R_2 + 2R_3 \Rightarrow \begin{pmatrix} 1 & 2 & -2 \\ 0 & 1 & 0 \\ 0 & -2 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 2 \\ 0 & 0 & 1 \end{pmatrix} A$ 1 m

$R_3 \to R_3 + 2R_2 \Rightarrow \begin{pmatrix} 1 & 2 & -2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 2 \\ 2 & 2 & 5 \end{pmatrix} A$ 1 m

$$R_1 \to R_1 + 2R_3 \Rightarrow \begin{pmatrix} 1 & 2 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 5 & 4 & 10 \\ 1 & 1 & 2 \\ 2 & 2 & 5 \end{pmatrix} A$$ ½ m

$$R_1 \to R_1 - 2R_2 \Rightarrow \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 3 & 2 & 6 \\ 1 & 1 & 2 \\ 2 & 2 & 5 \end{pmatrix} A$$ ½ m

$$\text{Hence } A^{-1} = \begin{pmatrix} 3 & 2 & 6 \\ 1 & 1 & 2 \\ 2 & 2 & 5 \end{pmatrix}$$ 1 m

24. $E_1$ : Bag contains 2 white balls and 2 non whites

$E_2$ : Bag contains 3 white balls and 1 non whites 1 m

$E_3$ : Bag contains 4 white balls

A : Getting two white balls

$$P(E_1) = \frac{1}{3},\ P(E_2) = \frac{1}{3},\ P(E_3) = \frac{1}{3}$$ ½ m

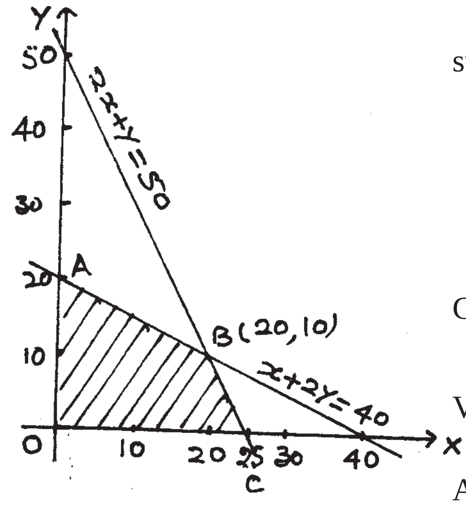
$$P(A/E_1) = \frac{2c_2}{4c_2} = \frac{1}{6},\ P(A/E_2) = \frac{3c_2}{4c_2} = \frac{1}{2},\ P(A/E_3) = 1$$ 1½ m

$$P(E_3/A) = \frac{P(E_3)\cdot P(A/E_3)}{P(E_1)\ P(A/E_1) + P(E_2)\ P(A/E_2) + P(E_3)\ P(A/E_3)}$$ 1 m

$$= \frac{\frac{1}{3}\cdot 1}{\frac{1}{3}\cdot\frac{1}{6} + \frac{1}{3}\cdot\frac{1}{2} + \frac{1}{3}\cdot 1}$$ 1 m

$$= \frac{6}{10} = \frac{3}{5}$$ 1 m

25. Let x cakes of first type and y cakes of second type are made




Maximise $S = x + y$ 1 m

subject to $300x + 150y \leq 7500$ or $2x + y \leq 50$

$15x + 30y \leq 600$ or $x + 2y \leq 40$ 2 m

$x \geq 0, y \geq 0$

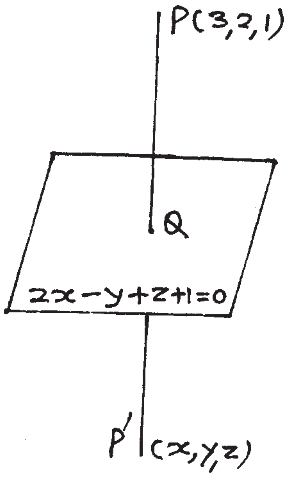
Correct graph 2 m

Vertices of feasible region are

A (0, 20), B (20, 10) C (25, 0)

Maximum cakes = 20 + 10 = 30 1 m

26. Let Q be the foot of perpendicular from P to the plane




$\therefore$ Equation of PQ is $\dfrac{x-3}{2} = \dfrac{y-2}{-1} = \dfrac{z-1}{1}$ 1 m

Any point on this line is $(2\lambda+3, -\lambda+2, \lambda+1)$ ½ m

If this point is Q, then it must satisfy the equation of plane

$\therefore\ 2(2\lambda+3) - (-\lambda+2) + (\lambda+1) + 1 = 0$ 1 m

$\Rightarrow\ \lambda = -1$ ½ m

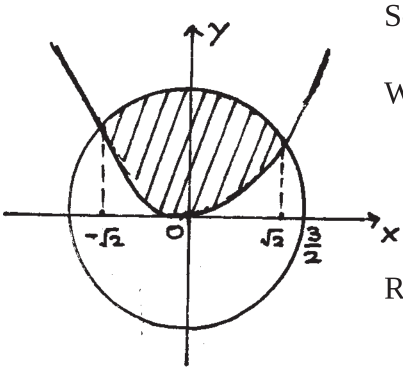
$\therefore$ coordinates of foot of perpendicularare are Q (1, 3, 0) 1 m

Perpendicular distance = PQ = $\sqrt{4+1+1} = \sqrt{6}$ units 1 m

Let $P'(x, y, z)$ be the image, then $\dfrac{x+3}{2} = 1, \dfrac{y+2}{2} = 3, \dfrac{z+1}{2} = 0$

$\therefore\ P'$ is $(-1, 4, -1)$ 1 m

27. Correct Figure 1 m




Solving $4x^2 + 4y^2 = 9$ and $x^2 = 4y$

We get $x = \pm\sqrt{2}$ (as points of intersection)

or $y = \frac{1}{2}$ ½ m

Required area

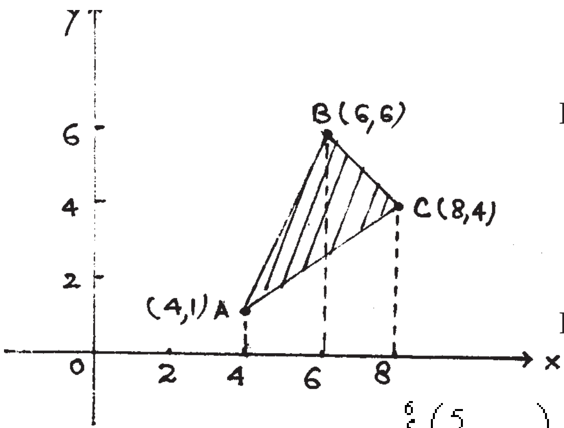
$$= 2\left[\int_0^{\sqrt{2}} \sqrt{\frac{9}{4} - x^2}\, dx - \int_0^{\sqrt{2}} \frac{1}{4}x^2\, dx\right]$$ 2 m

$$= 2\left[\frac{x}{2}\sqrt{\frac{9}{4} - x^2} + \frac{9}{8}\sin^{-1}\frac{2x}{3} - \frac{x^3}{12}\right]_0^{\sqrt{2}}$$ 1½ m

$$= 2\left(\frac{\sqrt{2}}{2}\cdot\frac{1}{2} + \frac{9}{8}\sin^{-1}\frac{2\sqrt{2}}{3} - \frac{2\sqrt{2}}{12}\right)$$ ½ m

$$= \left(\frac{\sqrt{2}}{6} + \frac{9}{4}\sin^{-1}\frac{2\sqrt{2}}{3}\right) \text{sq. units}$$ ½ m

**OR**




Equations of AB, BC and AC respectively are

$$y = \frac{5}{2}x - 9,\ y = 12 - x,\ y = \frac{3}{4}x - 2$$ 1½ m

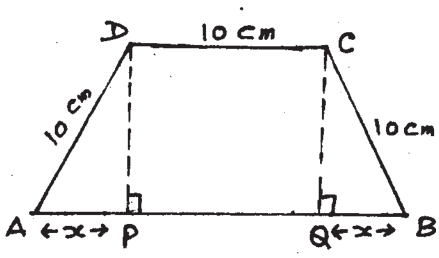
Required area

$$= \int_4^6 \left(\frac{5}{2}x - 9\right) dx + \int_6^8 (12 - x)\, dx - \int_4^8 \left(\frac{3}{4}x - 2\right) dx$$ 2 m

$$= \left[\frac{5x^2}{4} - 9x\right]_4^6 + \left[12x - \frac{x^2}{2}\right]_6^8 - \left[\frac{3x^2}{8} - 2x\right]_4^8$$ 1½ m

$= \quad (7 + 10 - 10)$ sq units 1 m

$= \quad 7$ sq units

28. 


Let ABCD be the given trapezium

with AD = DC = BC = 10 cm.

Draw DP $\perp$ AB and CQ AB

and let AP = x cm = > QB = x cm 1 m

$$\text{Area of trapezium, A} = \frac{1}{2}\left[10+(10+2x)\right]\sqrt{100-x^2}$$ 1 m

$$A = (x+10)\sqrt{100-x^2}$$

Let $S = (x + 10)^2 \cdot (100 - x^2) \Rightarrow \dfrac{ds}{dx} = -2x\,(x + 10)^2 + 2\,(x + 10)\,(100 - x^2)$

$= 2\,(x + 10)^2 \cdot (-x + 10 - x)$ 1 m

$= 2\,(x + 10)^2 \cdot (10 - 2x)$

$\dfrac{ds}{dx} = 0 \Rightarrow x = 5$ [rejecting $x = -10$] 1 m

$\dfrac{d^2s}{dx^2} = -4\,(x + 10)^2 + 4\,(x + 10)\;(10 - 2x) = -900\;(-\text{ve})$ 1 m

$\therefore$ Maximum Area $A = 15\sqrt{75}\ \text{cm}^2$ *or* $75\sqrt{3}\ \text{cm}^2$ 1 m

29. Full marks to be given to every candidate for this question. 6 m