DO NOT OPEN THIS TEST BOOKLET UNTIL YOU ARE ASKED TO DO SO

T.B.C. : P-PFC-K-OC   **Test Booklet Series**

Serial No. 129477

TEST BOOKLET

MATHEMATICS

<div style="border:1px solid;display:inline-block;">**A**</div>

Time Allowed : Two Hours and Thirty Minutes   **Maximum Marks : 300**

INSTRUCTIONS

1. IMMEDIATELY AFTER THE COMMENCEMENT OF THE EXAMINATION, YOU SHOULD CHECK THAT THIS TEST BOOKLET *DOES NOT* HAVE ANY UNPRINTED OR TORN OR MISSING PAGES OR ITEMS, ETC. IF SO, GET IT REPLACED BY A COMPLETE TEST BOOKLET.

2. ENCODE CLEARLY THE TEST BOOKLET SERIES **A, B, C** OR **D** AS THE CASE MAY BE IN THE APPROPRIATE PLACE IN THE ANSWER SHEET.

3. You have to enter your Roll Number on the Test Booklet in the Box provided alongside. *DO NOT* write *anything else* on the Test Booklet.

4. This Test Booklet contains **120** items (questions). Each item is printed both in **Hindi** and **English**. Each item comprises four responses (answers). You will select the response which you want to mark on the Answer Sheet. In case you feel that there is more than one correct response, mark the response which you consider the best. In any case, choose *ONLY ONE* response for each item.

5. You have to mark all your responses *ONLY* on the separate Answer Sheet provided. See directions in the Answer Sheet.

6. All items carry equal marks.

7. Before you proceed to mark in the Answer Sheet the response to various items in the Test Booklet, you have to fill in some particulars in the Answer Sheet as per instructions sent to you with your Admission Certificate.

8. After you have completed filling in all your responses on the Answer Sheet and the examination has concluded, you should hand over to the Invigilator *only the Answer Sheet.* You are permitted to take away with you the Test Booklet.

9. Sheets for rough work are appended in the Test Booklet at the end.

10. **Penalty for wrong answers :**
    THERE WILL BE PENALTY FOR WRONG ANSWERS MARKED BY A CANDIDATE IN THE OBJECTIVE TYPE QUESTION PAPERS.

    (i) There are four alternatives for the answer to every question. For each question for which a wrong answer has been given by the candidate, **one-third (0·33)** of the marks assigned to that question will be deducted as penalty.

    (ii) If a candidate gives more than one answer, it will be treated as a **wrong answer** even if one of the given answers happens to be correct and there will be same penalty as above to that question.

    (iii) If a question is left blank, i.e., no answer is given by the candidate, there will be **no penalty** for that question.

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ध्यान दें : अनुदेशों का हिन्दी रूपान्तर इस पुस्तिका के पिछले पृष्ठ पर छपा है।

1. The relation

$R = \{(1, 1), (2, 2), (3, 3), (1, 2), (2, 3), (1, 3)\}$

on a set $A = \{1, 2, 3\}$ is

(a) reflexive, transitive but not symmetric

(b) reflexive, symmetric but not transitive

(c) symmetric, transitive but not reflexive

(d) reflexive but neither symmetric nor transitive

2. If

$$\begin{bmatrix} 1 & -3 & 2 \\ 2 & -8 & 5 \\ 4 & 2 & \lambda \end{bmatrix}$$

is not an invertible matrix, then what is the value of λ?

(a) -1

(b) 0

(c) 1

(d) 2

3. What is the number of words that can be formed from the letters of the word 'UNIVERSAL', the vowels remaining always together?

(a) 720

(b) 1440

(c) 17280

(d) 21540

4. The arithmetic mean of two numbers exceeds their geometric mean by 2 and the geometric mean exceeds their harmonic mean by 1·6. What are the two numbers?

(a) 16, 4

(b) 81, 9

(c) 256, 16

(d) 625, 25

5. What is the conjugate of $\left(\dfrac{1+2i}{2+i}\right)^2$?

(a) $\dfrac{7}{25} + i\dfrac{24}{25}$

(b) $-\dfrac{7}{25} - i\dfrac{24}{25}$

(c) $-\dfrac{7}{25} + i\dfrac{24}{25}$

(d) $\dfrac{7}{25} - i\dfrac{24}{25}$

6. If the equations $x^2 + kx + 64 = 0$ and $x^2 - 8x + k = 0$ have real roots, then what is the value of k?

(a) 4

(b) 8

(c) 12

(d) 16

7. What is $\left(\dfrac{\sqrt{3}+i}{\sqrt{3}-i}\right)^6$ equal to?

(a) −1

(b) 0

(c) 1

(d) 2

8. If

$$A = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}, \ B = \begin{bmatrix} i & 0 \\ 0 & -i \end{bmatrix}, \ C = \begin{bmatrix} 0 & -i \\ -i & 0 \end{bmatrix}$$

then which one of the following is **not** correct?

(a) $A^2 = B^2$

(b) $B^2 = C^2$

(c) $AB = C$

(d) $AB = BA$

9. If $\log_3[\log_3[\log_3 x]] = \log_3 3$, then what is the value of x?

(a) 3

(b) 27

(c) 3^9

(d) 3^{27}

10. If ω is a complex cube root of unity, then what is $\omega^{10} + \omega^{-10}$ equal to?

(a) 2

(b) −1

(c) −2

(d) 1

11. If the roots of the equation

$$(a^2 + b^2)x^2 - 2b(a+c)x + (b^2 + c^2) = 0$$

are equal, then which one of the following is correct?

(a) $2b = a + c$

(b) $b^2 = ac$

(c) $b + c = 2a$

(d) $b = ac$

12. What is the maximum number of straight lines that can be drawn with any four points in a plane such that each line contains at least two of these points?

(a) 2

(b) 4

(c) 6

(d) 12

13. If

$$x + iy = \begin{vmatrix} 6i & -3i & 1 \\ 4 & 3i & -1 \\ 20 & 3 & i \end{vmatrix}$$

then what is $x - iy$ equal to?

(a) $3 + i$

(b) $1 + 3i$

(c) $3i$

(d) 0

14. If $|A| = 8$, where A is a square matrix of order 3, then what is $|\text{adj } A|$ equal to?

(a) 16

(b) 24

(c) 64

(d) 512

15. What is the binary number equivalent of the decimal number 32·25?

(a) 100010·10

(b) 100000·10

(c) 100010·01

(d) 100000·01

16. If A and B are two disjoint sets, then which one of the following is correct?

(a) $A - B = A - (A \cap B)$

(b) $B - A' = B \cap A$

(c) $A \cap B = (A - B) \cap B$

(d) All of the above

17. Consider the following statements in respect of a square matrix A and its transpose A^T :

1. $A + A^T$ is always symmetric.

2. $A - A^T$ is always anti-symmetric.

Which of the statements given above is/are correct?

(a) 1 only

(b) 2 only

(c) Both 1 and 2

(d) Neither 1 nor 2

18. If α and β are the roots of the equation $x^2 - 2x + 4 = 0$, then what is the value of $\alpha^3 + \beta^3$?

(a) 16

(b) −16

(c) 8

(d) −8

19. What is the value of $(-1 + i\sqrt{3})^{48}$?

(a) 1

(b) 2

(c) 2^{24}

(d) 2^{48}

20. If a matrix A is such that

$$3A^3 + 2A^2 + 5A + I = 0$$

then what is A^{-1} equal to?

(a) $-(3A^2 + 2A + 5)$

(b) $3A^2 + 2A + 5I$

(c) $3A^2 - 2A - 5I$

(d) $-(3A^2 + 2A + 5I)$

21. What is the sum of all the coefficients in the expansion of $(1 + x)^n$?

(a) 2^n

(b) $2^n - 1$

(c) 2^{n-1}

(d) $2(n - 1)$

22. Let N denote the set of natural numbers and $A = \{n^2 : n \in N\}$ and $B = \{n^3 : n \in N\}$. Which one of the following is correct?

(a) $A \cup B = N$

(b) The complement of $(A \cup B)$ is an infinite set

(c) $A \cap B$ must be a finite set

(d) $A \cap B$ must be a proper subset of $\{m^6 : m \in N\}$

23. Which of the following are the two roots of the equation $(x^2 + 2)^2 + 8x^2 = 6x(x^2 + 2)$?

(a) $1 \pm i$

(b) $2 \pm i$

(c) $1 \pm \sqrt{2}$

(d) $2 \pm i\sqrt{2}$

24. What is the coefficient of x^4 in the expansion of $\left(\dfrac{1-x}{1+x}\right)^2$?

(a) -16

(b) 16

(c) 8

(d) -8

25. If α and β are the roots of the equation $x^2 + x + 1 = 0$, then which of the following are the roots of the equation $x^2 - x + 1 = 0$?

(a) α^7 and β^{13}

(b) α^{13} and β^7

(c) α^{20} and β^{20}

(d) None of the above

26. A square is drawn by joining mid-points of the sides of a square. Another square is drawn inside the second square in the same way and the process is continued indefinitely. If the side of the first square is 16 cm, then what is the sum of the areas of all the squares?

(a) 256 sq. cm

(b) 512 sq. cm

(c) 1024 sq. cm

(d) 512/3 sq. cm

27. Let A and B be matrices of order 3×3. If $AB = 0$, then which of the following can be concluded?

(a) $A = 0$ and $B = 0$

(b) $|A| = 0$ and $|B| = 0$

(c) Either $|A| = 0$ or $|B| = 0$

(d) Either $A = 0$ or $B = 0$

28. The sum of an infinite geometric progression is 6. If the sum of the first two terms is 9/2, then what is the first term?

(a) 1

(b) 5/2

(c) 3 or 3/2

(d) 9 or 3

29. If A is a square matrix, then what is $\operatorname{adj} A^T - (\operatorname{adj} A)^T$ equal to?

(a) $2|A|$

(b) $2|A|I$

(c) Null matrix

(d) Unit matrix

30. Which one of the following is correct?

(a) $\sin 1^\circ > \sin 1$

(b) $\sin 1^\circ < \sin 1$

(c) $\sin 1^\circ = \sin 1$

(d) $\sin 1^\circ = \dfrac{\pi}{180}\sin 1$

31. One of the angles of a triangle is $1/2$ radian and the other is $99°$. What is the third angle in radian measure?

(a) $\dfrac{9\pi - 10}{\pi}$

(b) $\dfrac{90\pi - 100}{7\pi}$

(c) $\dfrac{90\pi - 10}{\pi}$

(d) None of the above .

32. Two poles are 10 m and 20 m high. The line joining their tops makes an angle of $15°$ with the horizontal. What is the approximate distance between the poles?

(a) 35·3 m

(b) 37·3 m

(c) 41 m

(d) 44 m

33. What is the value of
$$\sin^{-1}\frac{4}{5} + 2\tan^{-1}\frac{1}{3}\ ?$$

(a) $\dfrac{\pi}{3}$

(b) $\dfrac{\pi}{2}$

(c) $\dfrac{\pi}{4}$

(d) $\dfrac{\pi}{6}$

34. What is
$$\left(\frac{\sec 18°}{\sec 144°} + \frac{\operatorname{cosec} 18°}{\operatorname{cosec} 144°}\right)$$
equal to?

(a) $\sec 18°$

(b) $\operatorname{cosec} 18°$

(c) $-\sec 18°$

(d) $-\operatorname{cosec} 18°$

35. If α and β are positive angles such that $\alpha + \beta = \dfrac{\pi}{4}$, then what is
$$(1 + \tan\alpha)(1 + \tan\beta)$$
equal to?

(a) 0

(b) 1

(c) 2

(d) 3

36. What is the value of
$$(\sin 50° - \sin 70° + \sin 10°)\ ?$$

(a) 1

(b) $\dfrac{1}{\sqrt{2}}$

(c) $\dfrac{\sqrt{3}}{2}$

(d) 0

37. If

$$\cos A + \cos B = m$$

$$\sin A + \sin B = n$$

where $m, n \neq 0$, then what is $\sin(A + B)$ equal to?

(a) $\dfrac{mn}{m^2 + n^2}$

(b) $\dfrac{2mn}{m^2 + n^2}$

(c) $\dfrac{m^2 + n^2}{2mn}$

(d) $\dfrac{mn}{m + n}$

38. If $y = \sec^2 \theta + \cos^2 \theta$, where $0 < \theta < \dfrac{\pi}{2}$, then which one of the following is correct?

(a) $y = 0$

(b) $0 \leq y \leq 2$

(c) $y \geq 2$

(d) None of the above

39. If $\tan A = 3/4$ and $\tan B = -12/5$, then how many values can $\cot(A - B)$ have depending on the actual values of A and B?

(a) 1

(b) 2

(c) 3

(d) 4

40. What is the value of $\sin 15° \sin 75°$?

(a) $1/4$

(b) $1/8$

(c) $1/16$

(d) 1

41. What is the equation to the straight line joining the origin to the point of intersection of the lines

$$\frac{x}{a} + \frac{y}{b} = 1 \quad \text{and} \quad \frac{x}{b} + \frac{y}{a} = 1 \text{ ?}$$

(a) $x + y = 0$

(b) $x + y + 1 = 0$

(c) $x - y = 0$

(d) $x + y + 2 = 0$

42. What is the equation to circle which touches both the axes and has centre on the line $x + y = 4$?

(a) $x^2 + y^2 - 4x + 4y + 4 = 0$

(b) $x^2 + y^2 - 4x - 4y + 4 = 0$

(c) $x^2 + y^2 + 4x - 4y - 4 = 0$

(d) $x^2 + y^2 + 4x + 4y - 4 = 0$

43. If the position vector of a point P with respect to origin O is $\hat{i} + 3\hat{j} - 2\hat{k}$ and that of a point Q is $3\hat{i} + \hat{j} - 2\hat{k}$, then what is the position vector of the bisector of the angle POQ?

(a) $\hat{i} - \hat{j} - \hat{k}$

(b) $\hat{i} + \hat{j} - \hat{k}$

(c) $\hat{i} + \hat{j} + \hat{k}$

(d) None of the above

44. If the straight lines $x - 2y = 0$ and $kx + y = 1$ intersect at the point $\left(1, \dfrac{1}{2}\right)$, then what is the value of k?

(a) 1

(b) 2

(c) 1/2

(d) −1/2

45. Let a, b and c be the distinct non-negative numbers. If the vectors $a\hat{i} + a\hat{j} + c\hat{k}$, $\hat{i} + \hat{k}$, $c\hat{i} + c\hat{j} + b\hat{k}$ lie on a plane, then which one of the following is correct?

(a) c is the arithmetic mean of a and b

(b) c is the geometric mean of a and b

(c) c is the harmonic mean of a and b

(d) c is equal to zero

46. The two planes $ax + by + cz + d = 0$ and $ax + by + cz + d_1 = 0$, where $d \neq d_1$, have

(a) one point only in common

(b) three points in common

(c) infinite points in common

(d) no points in common

47. A point P moves such that the difference of its distances from two given points $(c, 0)$ and $(-c, 0)$ is constant. What is the locus of the point P?

(a) Circle

(b) Ellipse

(c) Hyperbola

(d) Parabola

48. What is the distance of the origin from the plane $2x + 6y - 3z + 7 = 0$?

(a) 1

(b) 2

(c) 3

(d) 6

49. $PQRS$ is a parallelogram, where $\overrightarrow{PQ} = 3\hat{i} + 2\hat{j} - m\hat{k}$, $\overrightarrow{PS} = \hat{i} + 3\hat{j} + \hat{k}$ and the area of the parallelogram is $\sqrt{90}$. What is the value of m?

(a) 1

(b) –1

(c) 2

(d) –2

50. What is the vector equally inclined to the vectors $\hat{i} + 3\hat{j}$ and $3\hat{i} + \hat{j}$?

(a) $\hat{i} + \hat{j}$

(b) $2\hat{i} - \hat{j}$

(c) $2\hat{i} + \hat{j}$

(d) None of the above

51. $ABCD$ is a quadrilateral. Forces $\overrightarrow{AB}$, $\overrightarrow{CB}$, $\overrightarrow{CD}$ and $\overrightarrow{DA}$ act along its sides. What is their resultant?

(a) $2\overrightarrow{CD}$

(b) $2\overrightarrow{DA}$

(c) $2\overrightarrow{BC}$

(d) $2\overrightarrow{CB}$

52. Under which of the following conditions does a general second-degree equation

$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0 \,(a \neq 0)$

represent a circle?

(a) $h = g, \ a = b$

(b) $h = g = f, \ a = b$

(c) $h = 0, \ a = b$

(d) $h = 0, \ g^2 + f^2 - c = a + b$

53. What is the area of a triangle whose vertices are at $(3, -1, 2)$, $(1, -1, -3)$ and $(4, -3, 1)$?

(a) $\dfrac{\sqrt{165}}{2}$

(b) $\dfrac{\sqrt{135}}{2}$

(c) 4

(d) 2

54. What is the value of b such that the scalar product of the vector $\hat{i} + \hat{j} + \hat{k}$ with the unit vector parallel to the sum of the vectors $2\hat{i} + 4\hat{j} - 5\hat{k}$ and $b\hat{i} + 2\hat{j} + 3\hat{k}$ is unity?

(a) –2

(b) –1

(c) 0

(d) 1

55. If the latus rectum of an ellipse is equal to half its minor axis, then what is its eccentricity?

(a) $\dfrac{1}{2}$

(b) $\sqrt{3}$

(c) $\dfrac{\sqrt{3}}{2}$

(d) $\dfrac{1}{\sqrt{2}}$

56. If $A = \{2, 3\}$, $B = \{4, 5\}$, $C = \{5, 6\}$, then what is the number of elements in $A \times (B \cap C)$?

(a) 2

(b) 4

(c) 6.

(d) 8

57. What is the slope of the line perpendicular to the line

$$\frac{x}{4} + \frac{y}{3} = 1 ?$$

(a) $\dfrac{3}{4}$

(b) $-\dfrac{3}{4}$

(c) $-\dfrac{4}{3}$

(d) $\dfrac{4}{3}$

58. ABC is a triangle in which $BC = 10$ cm, $CA = 6$ cm and $AB = 8$ cm. Which one of the following is correct?

(a) ABC is an acute-angled triangle

(b) ABC is an obtuse-angled triangle

(c) ABC is a right-angled triangle

(d) None of the above

59. If the AM and GM between two numbers are in the ratio $m : n$, then what is the ratio between the two numbers?

(a) $\dfrac{m + \sqrt{m^2 - n^2}}{m - \sqrt{m^2 - n^2}}$

(b) $\dfrac{m + n}{m - n}$

(c) $\dfrac{m^2 - n^2}{m^2 + n^2}$

(d) $\dfrac{m^2 + n^2 - mn}{m^2 + n^2 + mn}$

60. Consider the function $f : R \to \{0, 1\}$ such that

$$f(x) = \begin{cases} 1 & \text{if } x \text{ is rational} \\ 0 & \text{if } x \text{ is irrational} \end{cases}$$

Which one of the following is correct?

(a) The function is one-one into

(b) The function is many-one into

(c) The function is one-one onto

(d) The function is many-one onto

61. If $f(x) = 2x + 7$ and $g(x) = x^2 + 7$, $x \in R$, then what are the values of x for which $f \circ g(x) = 25$?

(a) $-1, 1$

(b) $-2, 2$

(c) $-\sqrt{2}, \sqrt{2}$

(d) None of the above

62. What does the differential equation

$$y\frac{dy}{dx} + x = k \quad (k \text{ is any constant})$$

represent?

(a) A family of circles having centre on the y-axis

(b) A family of circles having centre on the x-axis

(c) A family of circles touching the x-axis

(d) A family of ellipses

63. What is the area of the region bounded by the curve

$$f(x) = 1 - \frac{x^2}{4}, \quad x \in [-2, 2]$$

and the x-axis?

(a) 8/3 square units

(b) 4/3 square units

(c) 2/3 square unit

(d) 1/3 square unit

64. What is

$$\lim_{x \to 0} \frac{a^x - b^x}{x}$$

equal to?

(a) $\ln(ab)$

(b) $\dfrac{\ln a}{\ln b}$

(c) $\ln\left(\dfrac{a}{b}\right)$

(d) $\ln\left(\dfrac{b}{a}\right)$

65. If $\sqrt{x} + \sqrt{y} = 2$, then what is $\dfrac{dy}{dx}$ at $y = 1$ equal to?

(a) 5

(b) 2

(c) 4

(d) −1

66. For a point of inflection of $y = f(x)$, which one of the following is correct?

(a) $\dfrac{dy}{dx}$ must be zero

(b) $\dfrac{d^2y}{dx^2}$ must be zero

(c) $\dfrac{dy}{dx}$ must be non-zero

(d) $\dfrac{d^2y}{dx^2}$ must be non-zero

67. If the function

$$f(x) = \frac{x(x-2)}{x^2 - 4}, \ x \neq \pm 2$$

is continuous at $x = 2$, then what is $f(2)$ equal to?

(a) 0

(b) 1/2

(c) 1

(d) 2

68. If $x = \cos(2t)$ and $y = \sin^2 t$, then what is $\dfrac{d^2y}{dx^2}$ equal to?

(a) 0

(b) $\sin(2t)$

(c) $-\cos(2t)$

(d) −1/2

69. What is the value of p for which the function

$$f(x) = p\sin x + \frac{\sin 3x}{3}$$

has an extremum at $x = \dfrac{\pi}{3}$?

(a) 0

(b) 1

(c) −1

(d) 2

70. What is the value of the integral

$$\int_{-1}^{1} |x|\,dx\,?$$

(a) 1

(b) 0

(c) 2

(d) −1

71. What is the differential equation to family of parabolas having their vertices at the origin and foci on the x-axis?

(a) $y = 2xy'$

(b) $x = 2yy'$

(c) $xy = y'$

(d) $x = yy'$

72. At how many points is the function $f(x) = [x]$ discontinuous?

(a) 1

(b) 2

(c) 3

(d) Infinite

73. What is

$$\int \frac{e^x (1+x)}{\cos^2 (xe^x)} dx$$

equal to?

(a) $xe^x + c$

(b) $\cos(xe^x) + c$

(c) $\tan(xe^x) + c$

(d) $x\cosec(xe^x) + c$

where c is a constant of integration.

74. If at any instant t, for a sphere, r denotes the radius, S denotes the surface area and V denotes the volume, then what is $\frac{dV}{dt}$ equal to?

(a) $\frac{1}{2}S\frac{dr}{dt}$

(b) $\frac{1}{2}r\frac{dS}{dt}$

(c) $r\frac{dS}{dt}$

(d) $\frac{1}{2}r^2\frac{dS}{dt}$

75. If

$$f(x) = \frac{2}{3}x + \frac{3}{2}, \ x \in R$$

then what is $f^{-1}(x)$ equal to?

(a) $\frac{3}{2}x + \frac{2}{3}$

(b) $\frac{3}{2}x - \frac{9}{4}$

(c) $\frac{2}{3}x - \frac{4}{9}$

(d) $\frac{2}{3}x - \frac{2}{3}$

76. What is the interval over which the function $f(x) = 6x - x^2$, $x > 0$ is increasing?

(a) (0, 3)

(b) (3, 6)

(c) (6, 9)

(d) None of the above

77. What is $\int \sqrt{x}\, e^{\sqrt{x}}\, dx$ equal to?

(a) $2e^{\sqrt{x}}(x - 2\sqrt{x} + 2) + c$

(b) $2e^{\sqrt{x}}(x + 2\sqrt{x} + 2) + c$

(c) $2e^{\sqrt{x}}(x + 2\sqrt{x} - 2) + c$

(d) $2e^{\sqrt{x}}(x - 2\sqrt{x} - 2) + c$

where c is a constant of integration.

78. What is the degree of the differential equation

$$\left(1 + \frac{dy}{dx}\right)^4 = \left(\frac{d^2y}{dx^2}\right)^2 \ ?$$

(a) 1

(b) 2

(c) 4

(d) 8

79. What is the general solution of

$$(1 + e^x)y\,dy = e^x dx \ ?$$

(a) $y^2 = \ln[c^2(e^x + 1)^2]$

(b) $y = \ln[c(e^x + 1)]$

(c) $y^2 = \ln[c(e^x + 1)]$

(d) None of the above

where c is a constant of integration.

80. What is the value of

$$\int_{\pi/6}^{\pi/4} \frac{dx}{\sin x \cos x} \ ?$$

(a) $2\ln\sqrt{3}$

(b) $\ln\sqrt{3}$

(c) $2\ln 3$

(d) $4\ln 3$

81. If f and g are two increasing functions such that $f \circ g$ is defined, then which one of the following is correct?

(a) $f \circ g$ is always an increasing function

(b) $f \circ g$ is always a decreasing function

(c) $f \circ g$ is neither an increasing nor a decreasing function

(d) None of the above

82. What is the value of

$$\int_1^2 e^x \left(\frac{1}{x} - \frac{1}{x^2} \right) dx \ ?$$

(a) $e\left(\frac{e}{2} - 1\right)$

(b) $e(e - 1)$

(c) $e - \frac{1}{e}$

(d) 0

83. Which one of the following is the differential equation to family of circles having centre at the origin?

(a) $(x^2 - y^2)\frac{dy}{dx} = 2xy$

(b) $(x^2 + y^2)\frac{dy}{dx} = 2xy$

(c) $\frac{dy}{dx} = (x^2 + y^2)$

(d) $x\,dx + y\,dy = 0$

84. What is the area under the curve $f(x) = xe^x$ above the x-axis and between the lines $x = 0$ and $x = 1$?

(a) 1/2 square unit

(b) 1 square unit

(c) 3/2 square units

(d) 2 square units

85. What does the solution of the differential equation

$$x\frac{dy}{dx} = y$$

represent?

(a) Family of straight lines through the origin

(b) Family of circles with their centres at the origin

(c) Family of parabolas with their vertices at the origin

(d) Family of straight lines having slope 1 and not passing through the origin

86. What is the coefficient of x^{17} in the expansion of

$$\left(3x - \frac{x^3}{6} \right)^9 \ ?$$

(a) 189/8

(b) 567/2

(c) 21/16

(d) None of the above

87. What is the derivative of $\sin^2 x$ with respect to $\cos^2 x$?

(a) $\tan^2 x$

(b) $\cot^2 x$

(c) -1

(d) 1

88. Consider the following statements :

1. $f(x) = |x - 3|$ is continuous at $x = 0$.

2. $f(x) = |x - 3|$ is differentiable at $x = 0$.

Which of the statements given above is/are correct?

(a) 1 only

(b) 2 only

(c) Both 1 and 2

(d) Neither 1 nor 2

89. What is $(1111)_2 + (1001)_2 - (1010)_2$ equal to?

(a) $(111)_2$

(b) $(1100)_2$

(c) $(1110)_2$

(d) $(1010)_2$

90. What is $\dfrac{\sin\theta + 1}{\cos\theta}$ equal to?

(a) $\dfrac{\sin\theta + \cos\theta - 1}{\sin\theta + \cos\theta + 1}$

(b) $\dfrac{\sin\theta + \cos\theta + 1}{\sin\theta + \cos\theta - 1}$

(c) $\dfrac{\sin\theta - \cos\theta - 1}{\sin\theta + \cos\theta + 1}$

(d) $\dfrac{\sin\theta - \cos\theta + 1}{\sin\theta + \cos\theta - 1}$

91. From the top of a lighthouse 120 m above the sea, the angle of depression of a boat is 15°. What is the distance of the boat from the lighthouse?

(a) 400 m

(b) 421 m

(c) 444 m

(d) 460 m

92. What is the value of

$$\frac{\sin\theta + \cos\theta - \tan\theta}{\sec\theta + \csc\theta - \cot\theta}$$

when $\theta = \dfrac{3\pi}{4}$?

(a) 0

(b) 1

(c) −1

(d) None of the above

93. What is the value of $\sin 292\frac{1}{2}°$?

(a) $\dfrac{1}{3}\sqrt{2 + \sqrt{3}}$

(b) $-\dfrac{1}{3}\sqrt{2 - \sqrt{3}}$

(c) $\dfrac{1}{2}\sqrt{2 + \sqrt{2}}$

(d) $-\dfrac{1}{2}\sqrt{2 + \sqrt{2}}$

94. What is $\int \sec^n x \tan x \, dx$ equal to?

(a) $\dfrac{\sec^n x}{n} + c$

(b) $\dfrac{\sec^{n-1} x}{n-1} + c$

(c) $\dfrac{\tan^n x}{n} + c$

(d) $\dfrac{\tan^{n-1} x}{n-1} + c$

where c is a constant of integration.

95. What is the area bounded by the curve $y = x^2$ and the line $y = 16$?

(a) $32/3$

(b) $64/3$

(c) $256/3$

(d) $128/3$

96. What is the number of terms in the expansion of $(a+b+c)^n$, $n \in N$?

(a) $n+1$

(b) $n+2$

(c) $n(n+1)$

(d) $(n+1)(n+2)/2$

97. What is the number of signals that can be sent by 6 flags of different colours taking one or more at a time?

(a) 21

(b) 63

(c) 720

(d) 1956

98. In how many ways can a committee consisting of 3 men and 2 women be formed from 7 men and 5 women?

(a) 45

(b) 350

(c) 700

(d) 4200

99. If

$$\vec{a} = \hat{i} - \hat{k}$$
$$\vec{b} = x\hat{i} + \hat{j} + (1-x)\hat{k}$$
$$\vec{c} = y\hat{i} + x\hat{j} + (1+x-y)\hat{k}$$

then $\vec{a} \cdot (\vec{b} \times \vec{c})$ depends on

(a) x only

(b) y only

(c) both x and y

(d) neither x nor y

100. What is the probability of having 53 Sundays or 53 Mondays in a leap year?

(a) 2/7

(b) 3/7

(c) 4/7

(d) 5/7

101. Three digital numbers are formed using the digits 0, 2, 4, 6, 8. A number is chosen at random out of these numbers. What is the probability that the number has the same digits?

(a) 1/16

(b) 1/25

(c) 16/25

(d) 1/645

102. What is the mean deviation of the data 2, 9, 9, 3, 6, 9, 4 ?

(a) 2·23

(b) 2·57

(c) 3·23

(d) 3·57

103. A set of n values $x_1, x_2, ..., x_n$ has standard deviation σ. What is the standard deviation of n values $x_1 + k, x_2 + k, ..., x_n + k$?

(a) σ

(b) $\sigma + k$

(c) $\sigma - k$

(d) $k\sigma$

104. The two lines of regression are $8x - 10y = 66$ and $40x - 18y = 214$, and variance of x series is 9. What is the standard deviation of y series?

(a) 3

(b) 4

(c) 6

(d) 8

105. The standard deviation of some consecutive integers is found to be 2. Which of the following statements best describes the nature of the consecutive integers?

(a) The integers are any set of eight consecutive integers

(b) The integers are any set of eight consecutive positive integers

(c) The integers are any set of seven consecutive integers

(d) None of the above

106. Consider the following data :

	Factory-A	Factory-B
Mean wage of workers	Rs 540	Rs 620
Standard deviation of wages	Rs 40·50	Rs 31

What is the variability in the wages of the workers in Factory-A ?

(a) 100% more than the variability in the wages of the workers in Factory-B

(b) 50% more than the variability in the wages of the workers in Factory-B

(c) 50% less than the variability in the wages of the workers in Factory-B

(d) 150% more than the variability in the wages of the workers in Factory-B

107. Let $U = \{1, 2, 3, ..., 20\}$. Let A, B, C be the subsets of U. Let A be the set of all numbers which are perfect squares, B be the set of all numbers which are multiples of 5 and C be the set of all numbers which are divisible by 2 and 3.

Consider the following statements :

1. A, B, C are mutually exclusive.
2. A, B, C are mutually exhaustive.
3. The number of elements in the complement set of $A \cup B$ is 12.

Which of the statements given above are correct?

(a) 1 and 2 only

(b) 1 and 3 only

(c) 2 and 3 only

(d) 1, 2 and 3

108. A lot of 4 white and 4 red balls is randomly divided into two halves. What is the probability that there will be 2 red and 2 white balls in each half ?

(a) 18/35

(b) 3/35

(c) 1/2

(d) None of the above

109. The distributions X and Y with total number of observations 36 and 64, and mean 4 and 3 respectively are combined. What is the mean of the resulting distribution $X + Y$?

(a) 3·26

(b) 3·32

(c) 3·36

(d) 3·42

110. Consider the following data :

x	5	7	8	4	6
y	2	4	3	2	4

What is the regression equation of y on x ?

(a) $y = 0·6 + 0·4x$

(b) $y = 0·7 + 0·3x$

(c) $y = 6 + 5x$

(d) $y = 4 + 9x$

111. What is the value of

$$\lim_{x \to 0} \frac{\cos(ax) - \cos(bx)}{x^2}\ ?$$

(a) $a - b$

(b) $a + b$

(c) $\dfrac{b^2 - a^2}{2}$

(d) $\dfrac{b^2 + a^2}{2}$

For the next **three (3)** items which follow :

The frequency distribution of life of 90 TV tubes whose median life is 17 months is as follows :

Life of TV tubes (in months)	No. of TV tubes
0–5	3
5–10	12
10–15	x
15–20	35
20–25	y
25–30	4

112. What is the lower limit of the median class?

(a) 10

(b) 15

(c) 20

(d) 25

113. What is the missing frequency y?

(a) 20

(b) 16

(c) 15

(d) 12

114. What is the cumulative frequency of the modal class?

(a) 31

(b) 35

(c) 66

(d) Cannot be determined as the given data is insufficient

115. If the area of a triangle with vertices $(-3, 0)$, $(3, 0)$ and $(0, k)$ is 9 square units, then what is the value of k ?

(a) 3

(b) 6

(c) 9

(d) 12

116. Consider the following statements :

If A and B are independent events, then

1. A and $\overline{B}$ are independent

2. $\overline{A}$ and B are independent

3. $\overline{A}$ and $\overline{B}$ are independent

Which of the above statements is/are correct?

(a) 3 only

(b) 1 and 2 only

(c) 1, 2 and 3

(d) None of the statements 1, 2 and 3 is correct

117. If $x = k(\theta + \sin\theta)$ and $y = k(1 + \cos\theta)$, then what is the derivative of y with respect to x at $\theta = \pi/2$?

(a) -1

(b) 0

(c) 1

(d) 2

118. What is the value of

$$1 + i^2 + i^4 + i^6 + \cdots + i^{100}$$

where $i = \sqrt{-1}$?

(a) 0

(b) 1

(c) -1

(d) None of the above

119. What is the value of

$$\begin{vmatrix} 1 & \omega & 2\omega^2 \\ 2 & 2\omega^2 & 4\omega^3 \\ 3 & 3\omega^3 & 6\omega^4 \end{vmatrix}$$

where ω is the cube root of unity?

(a) 0

(b) 1

(c) 2

(d) 3

120. If the cardinality of a set A is 4 and that of a set B is 3, then what is the cardinality of the set $A \Delta B$?

(a) 1

(b) 5

(c) 7

(d) Cannot be determined as the sets A and B are not given