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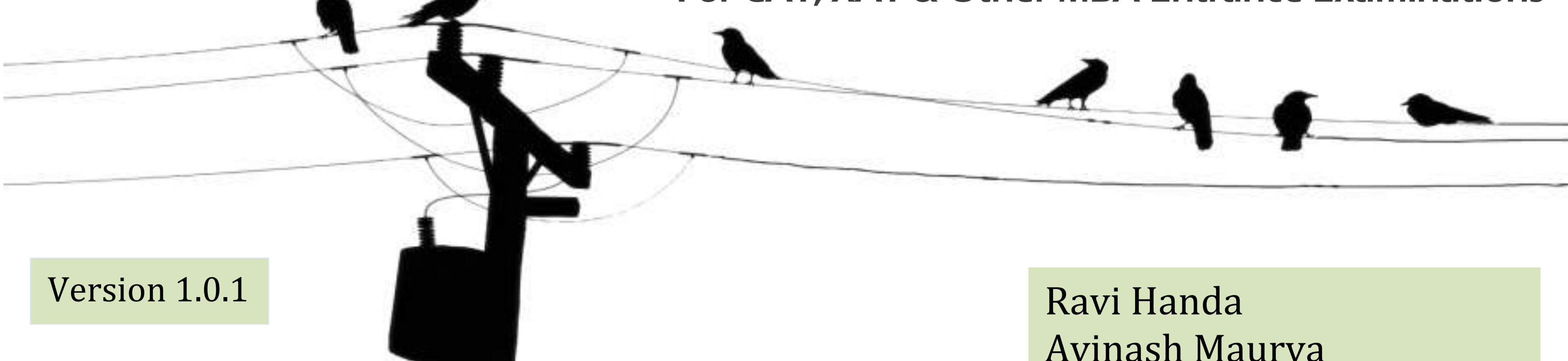
MATH FORMULAS & FUNDAS

For CAT, XAT & Other MBA Entrance Examinations



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WHAT WE DO?

VIDEOS

The pool of 50 videos on our website covers the entire Math syllabus for MBA Entrance Exams (CAT/ XAT / MAT and others) [Absolutely FREE !!]

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Preface

On more than one occasion, students have come to me asking for some sort of list of important formulas for CAT, XAT and other MBA Entrance Examinations. The best solution that I could suggest to them was to look at any good MBA-prep book or material from a coaching institute. More often than not, they have the formulas at the beginning or the end or scattered throughout the chapter. Till now, there exists no comprehensive list / book of formulas and fundas.

So, Avinash and I thought of making one. Once the idea gathered some momentum various management training portals came on board to help us distribute it via their channels. We hope that our effort will not be in vain and it would prove beneficial for students.

We would love to hear what you think of the eBook. For that you can use the feedback page or contact us directly. Our contact details are given at the end of the eBook.

Ravi Handa & Avinash Maurya

How to use the eBook effectively

Best View Details:-

For Adobe Reader: Follow the path given below;
Toolbar>>View>>Full Screen Mode
Or Ctrl+L
Note: Click on the Topic Names in Contents to go to their respective pages. Contents are hyperlinked to different pages in the same document.

Bookmark Details:-

Bookmarks (on left) are given to navigate between the topics faster and more easily.

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Home page:-

Click on "Home" (at the bottom of every page) to go to the main page titled HOME. All topics under HOME are hyperlinked to the content page of that topic.

Suppose, one wants to go from Algebra to Geometry. Click "Home" and then click Geometry. This will take one from any page in Algebra to the content page of Geometry.

NUMBER SYSTEM

Glossary

Natural Numbers: 1, 2, 3, 4.....

Whole Numbers: 0, 1, 2, 3, 4.....

Integers: ....-2, -1, 0, 1, 2 .....

Rational Numbers: Any number which can be expressed as a ratio of two integers for example a p/q format where 'p' and 'q' are integers. Proper fraction will have ($p<q$) and improper fraction will have ($p>q$)

Factors: A positive integer 'f' is said to be a factor of a given positive integer 'n' if f divides n without leaving a remainder. e.g. 1, 2, 3, 4, 6 and 12 are the factors of 12.

Prime Numbers: A prime number is a positive number which has no factors besides itself and unity.

Composite Numbers: A composite number is a number which has other factors besides itself and unity.

Factorial: For a natural number 'n', its factorial is defined as: n! = 1 x 2 x 3 x 4 x .... x n (Note: 0! = 1)

Absolute value: Absolute value of x (written as |x|) is the distance of 'x' from 0 on the number line. |x| is always positive. $|x| = x$ *for* $x > 0$ OR $-x$ *for* $x < 0$

Funda: The product of 'n' consecutive natural numbers is always divisible by n!

Funda: Square of any natural number can be written in the form of 3n or 3n+1. Also, square of any natural number can be written in the form of 4n or 4n+1.

Funda: Square of a natural number can only end in 0, 1, 4, 5, 6 or 9. Second last digit of a square of a natural number is always even except when last digit is 6. If the last digit is 5, second last digit has to be 2.

Funda: Any prime number greater than 3 can be written as 6k $\pm$1.

Funda: Any two digit number 'pq' can effectively be written as 10p+q and a three digit number 'pqr' can effectively be written as 100p+10q+r.

Laws of Indices

$$a^m \times a^n = a^{m+n}$$

$$a^m \div a^n = a^{m-n}$$

$$(a^m)^n = a^{mn}$$

$$a^{\left(\frac{1}{m}\right)} = \sqrt[m]{a}$$

$$a^{-m} = \frac{1}{a^m}$$

$$a^{\left(\frac{m}{n}\right)} = \sqrt[m]{a^n}$$

$$a^0 = 1$$

Funda: If $a^m = a^n$, then m = n

Funda: If $a^m = b^m$ and $m \neq 0$;
Then $a = b$ if m is Odd
Or $a = \pm b$ if m is Even

Last digit of a^n

n(Right) a(Down)	1	2	3	4	Cyclicity
0	0	0	0	0	1
1	1	1	1	1	1
2	2	4	8	6	4
3	3	9	7	1	4
4	4	6	4	6	2
5	5	5	5	5	1
6	6	6	6	6	1
7	7	9	3	1	4
8	8	4	2	6	4
9	9	1	9	1	2

Funda: The fifth power of any number has the same units place digit as the number itself.

HCF and LCM

For two numbers, HCF x LCM = product of the two.

HCF of Fractions = $\frac{HCF\ of\ Numerator}{LCM\ of\ Denominator}$

LCM of Fractions = $\frac{LCM\ of\ Numerator}{HCF\ of\ Denominator}$

Funda: If a, b and c give remainders p, q and r respectively, when divided by the same number H, then H is HCF of (a-p), (b-q), (c-r)

Funda: If the HCF of two numbers ‘a’ and ‘b’ is H, then, the numbers (a+b) and (a-b) are also divisible by H.

Funda: If a number N always leaves a remainder R when divided by the numbers a, b and c, then N = LCM (or a multiple of LCM) of a, b and c + R.

Relatively Prime or Co-Prime Numbers: Two positive integers are said to be relatively prime to each other if their highest common factor is 1.

Factor Theory

If N = $x^a y^b z^c$ where x, y, z are prime factors. Then,

Number of factors of N = **P** = (a + 1)(b + 1)(c + 1)

Sum of factors of N = $\frac{x^{a+1}-1}{x-1}\ X\ \frac{y^{b+1}-1}{y-1}\ X\ \frac{z^{c+1}-1}{z-1}$

Number of ways N can be written as product of two factors = P/2 or (P+1)/2 if P is even or odd respectively

The number of ways in which a composite number can be resolved into two co-prime factors is 2^{m-1}, where m is the number of different prime factors of the number.

Number of numbers which are less than N and co-prime to $\emptyset(N) = N\left(1-\frac{1}{a}\right)\left(1-\frac{1}{b}\right)\left(1-\frac{1}{c}\right)$ {Euler’s Totient}

Funda: If N = $(2)^a(y)^b(z)^c$ where x, y, z are prime factors
Number of even factors of N = (a)(b+1)(c+1)
Number of odd factors of N = (b+1)(c+1)

Divisibility Rules

A number is divisible by:

2, 4 & 8 when the number formed by the last, last two, last three digits are divisible by 2,4 & 8 respectively.

3 & 9 when the sum of the digits of the number is divisible by 3 & 9 respectively.

11 when the difference between the sum of the digits in the odd places and of those in even places is 0 or a multiple of 11.

6, 12 & 15 when it is divisible by 2 and 3, 3 and 4 & 3 and 5 respectively.

7, if the number of tens added to five times the number of units is divisible by 7.

13, if the number of tens added to four times the number of units is divisible by 13.

19, if the number of tens added to twice the number of units is divisible by 19.

Algebraic Formulae

$\mathbf{a^3 \pm b^3 = (a \pm b)(a^2 + ab + b^2)}$. Hence, $a^3 \pm b^3$ *is divisible by* $(a \pm b)$ *and* $(a^2 \pm ab + b^2)$.

$\mathbf{a^n - b^n = (a - b)(a^{n-1} + a^{n-2}b + a^{n-3}b^2 + \ldots + b^{n-1})}$[for all n]. Hence, $a^n - b^n$ *is divisible by* $a - b$ *for all* n.

$\mathbf{a^n - b^n = (a + b)(a^{n-1} - a^{n-2}b + a^{n-3}b2 \ldots - b^{n-1})}$[n-even] Hence, $a^n - b^n$ *is divisible by* $a + b$ *for even* n.

$\mathbf{a^n + b^n = (a + b)(a^{n-1} - a^{n-2}b + a^{n-3}b^2 + \ldots + b^{n-1})}$[n-odd] Hence, $a^n + b^n$ *is divisible by* $a + b$ *for odd* n.

$\mathbf{a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - ac - bc)}$ Hence, $a^3 + b^3 + c^3 = 3abc$ *if* $a + b + c = 0$

For ex., check divisibility of 312 by 7, 13 & 19

- ⇨ **For 7:** 31 + 2 x 5 = 31 + 10 = 41 ***Not divisible***
- ⇨ **For 13:** 31 + 2 x 4 = 31 + 8 = 39 ***Divisible***.
- ⇨ **For 19:** 31 + 2 x 2 = 31 + 4 = 35 ***Not divisible***.

Remainder / Modular Arithmetic

$$\boldsymbol{Rem\left[\frac{a*b*c\ldots}{d}\right] = Rem\left[\frac{a}{d}\right] * Rem\left[\frac{b}{d}\right] * Rem\left[\frac{c}{d}\right]\ldots}$$

$$\boldsymbol{Rem\left[\frac{a+b+c\ldots}{d}\right] = Rem\left[\frac{a}{d}\right] + Rem\left[\frac{b}{d}\right] + Rem\left[\frac{c}{d}\right]\ldots}$$

Case 1 – When the dividend (M) and divisor (N) have a factor in common (k)

⇨ $\boldsymbol{Rem\left[\frac{M}{N}\right] = Rem\left[\frac{ka}{kb}\right] = k\ Rem\left[\frac{a}{b}\right]}$

Example: $Rem\left[\frac{3^{15}}{15}\right] = 5\ Rem\left[\frac{3^{14}}{5}\right] = 5*4 = 20 \equiv 5$

Case 2 – When the divisor can be broken down into smaller co-prime factors.

⇨ $Rem\left[\frac{M}{N}\right] = Rem\left[\frac{M}{a*b}\right]$ *{HCF (a,b) = 1}*

⇨ Let $Rem\left[\frac{M}{a}\right] = r_1$ & $Rem\left[\frac{M}{b}\right] = r_2$

⇨ $\boldsymbol{Rem\left[\frac{M}{N}\right] = axr_2 + byr_1}$ *{Such that ax+by = 1}*

Example: $Rem\left[\frac{7^{15}}{15}\right] = Rem\left[\frac{7^{15}}{3*5}\right]$

⇨ $Rem\left[\frac{7^{15}}{3}\right] = 1$ & $Rem\left[\frac{7^{15}}{5}\right] = Rem\left[\frac{2^{15}}{5}\right] = 3$

⇨ $Rem\left[\frac{7^{15}}{15}\right] = 3*x*3 + 5*y*1$

{Such that 3x+5y=1}

⇨ Valid values are x = -3 and y = 2

⇨ $Rem\left[\frac{7^{15}}{15}\right] = 9x + 5y = -17 \equiv 13$

Case 3 – Remainder when $f(x) = ax^n + bx^{n-1} + cx^{n-2}\ldots$ is divided by $(x-a)$ the remainder is $f(a)$

Funda: If f(a) = 0, (x-a) is a factor of f(x)

Continued >>

Remainder Related Theorems

Euler's Theorem:

Number of numbers which are less than N = $a^p * b^q * c^r$ and co-prime to it are

⇨ $\emptyset(N) = N\left(1-\frac{1}{a}\right)\left(1-\frac{1}{b}\right)\left(1-\frac{1}{c}\right)$

If M and N are co-prime ie HCF(M,N) = 1

⇨ $\boldsymbol{Rem\left[\frac{M^{\emptyset(n)}}{N}\right] = 1}$

Example: $Rem\left[\frac{7^{50}}{90}\right] = ?$

⇨ $\emptyset(90) = 90\left(1-\frac{1}{2}\right)\left(1-\frac{1}{3}\right)\left(1-\frac{1}{5}\right)$

⇨ $\emptyset(90) = 90 * \frac{1}{2} * \frac{2}{3} * \frac{4}{5} = 24$

⇨ $Rem\left[\frac{7^{24}}{90}\right] = 1 = Rem\left[\frac{7^{48}}{90}\right]$

⇨ $Rem\left[\frac{7^{50}}{90}\right] = Rem\left[\frac{7^{2}}{90}\right] * Rem\left[\frac{7^{48}}{90}\right] = 49 * 1 = 49$

Fermat's Theorem:

If N is a prime number and *M and N are co-primes*

⇨ $\boldsymbol{Rem\left[\frac{M^{N}}{N}\right] = M}$

⇨ $\boldsymbol{Rem\left[\frac{M^{N-1}}{N}\right] = 1}$

Example: $Rem\left[\frac{6^{31}}{31}\right] = 6\ \&\ Rem\left[\frac{6^{30}}{31}\right] = 1$

Wilson's Theorem

If N is a prime number

⇨ $\boldsymbol{Rem\left[\frac{(N-1)!}{N}\right] = N-1}$

⇨ $\boldsymbol{Rem\left[\frac{(N-2)!}{N}\right] = 1}$

Example: $Rem\left[\frac{30!}{31}\right] = 30\ \&\ Rem\left[\frac{29!}{31}\right] = 1$

Base System Concepts

Decimal	Binary	Hex
0	0000	0
1	0001	1
2	0010	2
3	0011	3
4	0100	4
5	0101	5
6	0110	6
7	0111	7
8	1000	8
9	1001	9
10	1010	A
11	1011	B
12	1100	C
13	1101	D
14	1110	E
15	1111	F

Converting from base 'n' to decimal

⇨ $(pqrst)_n = pn^4 + qn^3 + rn^2 + sn + t$

Converting from decimal to base 'n'

The example given below is converting from 156 to binary. For this we need to keep dividing by 2 till we get the quotient as 0.

```
2)156    0
 2)78    0
 2)39    1
 2)19    1
  2)9    1
  2)4    0
  2)2    0
  2)1    1
    0
```


Starting with the bottom remainder, we read the sequence of remainders upwards to the top. By that, we get $156_{10} = 10011100_2$

Funda: $(pqrst)_n \times n^2 = (pqrst00)_n$

$(pqrst)_n \times n^3 = (pqrst000)_n$

ARITHMETIC

Averages

Simple Average = $\frac{\text{Sum of elements}}{\text{Number of elements}}$

Weighted Average = $\frac{w_1x_1 + w_2x_2 + \cdots + w_nx_n}{w_1 + w_2 + \cdots + w_n}$

Arithmetic Mean = $(a_1 + a_2 + a_3 \ldots.a_n) / n$

Geometric Mean = $\sqrt[n]{a_1a_2\cdots a_n}$

Harmonic Mean = $\frac{n}{\frac{1}{x_1} + \frac{1}{x_2} + \cdots + \frac{1}{x_n}}$

For two numbers a and b

- ⇨ AM = (a + b)/2
- ⇨ GM = $\sqrt{a.b}$
- ⇨ HM = $\frac{2ab}{a+b}$

Funda: AM ≥ GM ≥ HM is always true. They will be equal if all elements are equal to each other. If I have just two values then GM^2 = AM x HM

Funda: The sum of deviation (D) of each element with respect to the average is 0

⇨ $D = (x_1 - x_{avg}) + (x_2 - x_{avg}) + (x_3 - x_{avg}) \ldots + (x_1 - x_{avg}) = 0$

Funda: $x_{avg} = x_{assumed\ avg} + \frac{Deviation}{No.of\ elements}$

Median of a finite list of numbers can be found by arranging all the observations from lowest value to highest value and picking the middle one.

Mode is the value that occurs most often

Percentages

Fractions and their percentage equivalents:

Fraction	%age	Fraction	%age
1/2	50%	1/9	11.11%
1/3	33.33%	1/10	10%
1/4	25%	1/11	9.09%
1/5	20%	1/12	8.33%
1/6	16.66%	1/13	7.69%
1/7	14.28%	1/14	7.14%
1/8	12.5%	1/15	6.66%

Funda: r% change can be nullified by $\frac{\mathbf{100r}}{\mathbf{100+r}}$% change in another direction. Eg: An increase of 25% in prices can be nullified by a reduction of [100x25/(100+25)] = 20% reduction in consumption.

Funda: If a number 'x' is successively changed by a%, b%, c%...

⇨ Final value = $x\left(1+\frac{a}{100}\right)\left(1+\frac{b}{100}\right)\left(1+\frac{c}{100}\right)$ ...

Funda: The net change after two successive changes of a% and b% is $\left(a+b+\frac{ab}{100}\right)\%$

Interest

Amount = Principal + Interest

Simple Interest = PNR/100

Compound Interest = $P(1+\frac{r}{100})^n - P$

Population formula $P' = P(1 \pm \frac{r}{100})^n$

Depreciation formula = Initial Value x $(1 - \frac{r}{100})^n$

Funda: SI and CI are same for a certain sum of money (P) at a certain rate (r) per annum for the first year. The difference after a period of two years is given by

$$\Rightarrow \Delta = \frac{PR^2}{100^2}$$

Growth and Growth Rates

Absolute Growth = Final Value – Initial Value

Growth rate for one year period =

$$\frac{\text{Final value} - \text{Initial Value}}{\text{Initial Value}} \times 100$$

$$\text{SAGR or AAGR} = \frac{\text{Final value} - \text{Initial Value}}{\text{Initial Value}} \times 100$$

$$\text{CAGR} = \left(\frac{\text{Final value} - \text{Initial Value}}{\text{Initial Value}}\right)^{\frac{1}{no.of\ years}} - 1$$

Funda: If the time period is more than a year, CAGR < AAGR. This can be used for approximating the value of CAGR instead of calculating it.

Profit and Loss

$$\%\text{Profit / Loss} = \frac{\text{Selling Price} - \text{Cost Price}}{\text{Initial Value}} \times 100$$

In case false weights are used while selling,

$$\%\ \text{Profit} = \left(\frac{\text{Claimed Weigth} - \text{Actual Weight}}{\text{Initial Value}} - 1\right) x\ 100$$

$$\text{Discount}\ \% = \frac{\text{Marked Price} - \text{Selling Price}}{\text{Selling Price}} \times 100$$

Funda: Effective Discount after successive discount of a% and b% is $\left(a + b - \frac{ab}{100}\right)$. Effective Discount when you buy x goods and get y goods free is $\frac{y}{x+y} \times 100$.

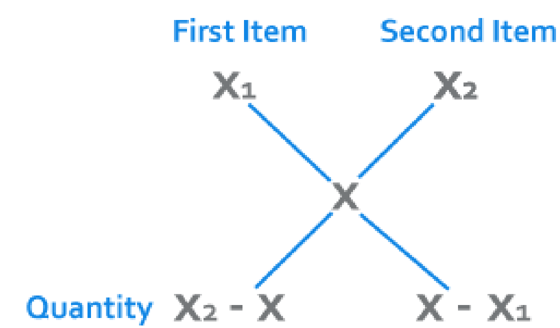
Mixtures and Alligation

Successive Replacement – Where *a* is the original quantity, *b* is the quantity that is replaced and *n* is the number of times the replacement process is carried out, then

$$\frac{\text{Quantity of original entity after n operation}}{\text{Quantity of mixture}} = \left(\frac{a-b}{a}\right)^n$$

Alligation – The ratio of the weights of the two items mixed will be inversely proportional to the deviation of attributes of these two items from the average attribute of the resultant mixture

$$\Rightarrow \frac{\text{Quantity of first item}}{\text{Quantity of second item}} = \frac{x_2 - x}{x - x_1}$$



Ratio and Proportion

Compounded Ratio of two ratios a/b and c/d is ac/bd,
Duplicate ratio of a : b is $a^2 : b^2$
Triplicate ratio of a : b is $a^3 : b^3$
Sub-duplicate ratio of a : b is $\sqrt{a} : \sqrt{b}$
Sub-triplicate ratio of a : b is $\sqrt[3]{a} : \sqrt[3]{b}$
Reciprocal ratio of a : b is b : a

Componendo and Dividendo

If $\frac{a}{b} = \frac{c}{d}$ & $a \neq b$ then $\frac{a+b}{a-b} = \frac{c+d}{c-d}$
Four (non-zero) quantities of the same kind a,b,c,d are said to be in proportion if a/b = c/d.

The non-zero quantities of the same kind a, b, c, d.. are said to be in continued proportion if a/b = b/c = c/d.

Proportion

a, b, c, d are said to be in proportion if $\frac{a}{b} = \frac{c}{d}$

a, b, c, d are said to be in continued proportion if

$$\frac{a}{b} = \frac{b}{c} = \frac{c}{d}$$

Funda: If a/b = c/d = e/f = k

$$\Rightarrow \frac{a+c+e}{b+d+f} = k$$

$$\Rightarrow \frac{pa+qc+re}{pb+qd+rf} = k$$

$$\Rightarrow \frac{p^n a+q^n c+r^n e}{p^n b+q^n d+r^n f} = k^n$$

Given two variables *x* and *y*, *y is* ***(directly) proportional*** *to x* (*x and y* ***vary directly****,* or *x and y are in* ***direct variation***) if there is a non-zero constant *k* such that y = kx. It is denoted by $y \propto x$

Two variables are **inversely proportional** (or **varying inversely**, or in **inverse variation**, or in **inverse proportion** or **reciprocal proportion**) if there exists a non-zero constant *k* such that y = k/x.

Time Speed and Distance

Speed = Distance / Time

1 kmph = 5/18 m/sec; 1 m/sec = 18/5 kmph

$$\text{Speed}_{\text{Avg}} = \frac{\textit{Total Distance Covered}}{\textit{Total Time Taken}} = \frac{d_1 + d_2 + d_3 \ldots d_n}{t_1 + t_2 + t_3 \ldots t_n}$$

If the distance covered is constant then the average speed is Harmonic Mean of the values ($s_1, s_2, s_3 \ldots s_n$)

⇨ $\text{Speed}_{\text{Avg}} = \dfrac{n}{1/s_1 + 1/s_2 + 1/s_3 \ldots 1/s_n}$

⇨ $\text{Speed}_{\text{Avg}} = \dfrac{2 s_1 s_2}{s_1 + s_2}$ (for two speeds)

If the time taken is constant then the average speed is Arithmetic Mean of the values ($s_1, s_2, s_3 \ldots s_n$)

⇨ $\text{Speed}_{\text{Avg}} = \dfrac{s_1 + s_2 + s_3 \ldots s_n}{n}$

⇨ $\text{Speed}_{\text{Avg}} = \dfrac{s_1 + s_2}{2}$ (for two speeds)

Funda: Given that the distance between two points is constant, then

⇨ If the speeds are in *Arithmetic Progression*, then the times taken are in *Harmonic Progression*

⇨ If the speeds are in *Harmonic Progression*, then the times taken are in *Arithmetic Progression*

For Trains, time taken = $\dfrac{\textit{Total length to be covered}}{\textit{Relative Speed}}$

For Boats,

$\text{Speed}_{\text{Upstream}} = \text{Speed}_{\text{Boat}} - \text{Speed}_{\text{River}}$

$\text{Speed}_{\text{Downstream}} = \text{Speed}_{\text{Boat}} + \text{Speed}_{\text{River}}$

$\text{Speed}_{\text{Boat}} = (\text{Speed}_{\text{Downstream}} + \text{Speed}_{\text{Upstream}}) / 2$

$\text{Speed}_{\text{River}} = (\text{Speed}_{\text{Downstream}} - \text{Speed}_{\text{Upstream}}) / 2$

For Escalators, The difference between escalator problems and boat problems is that escalator can go either up or down.

Races & Clocks

Linear Races

Winner's distance = Length of race

Loser's distance = Winner's distance – (beat distance + start distance)

Winner's time = Loser's time – (beat time + start time)

Deadlock / dead heat occurs when beat time = 0 or beat distance = 0

Circular Races

Two people are running on a circular track of length L with speeds a and b in the same direction

- ⇨ Time for 1st meeting = $\frac{L}{a-b}$
- ⇨ Time for 1st meeting at the starting point = LCM $\left(\frac{L}{a}, \frac{L}{b}\right)$

Two people are running on a circular track of length L with speeds a and b in the opposite direction

- ⇨ Time for 1st meeting = $\frac{L}{a+b}$
- ⇨ Time for 1st meeting at the starting point = LCM $\left(\frac{L}{a}, \frac{L}{b}\right)$

Three people are running on a circular track of length L with speeds a, b and c in the same direction

- ⇨ Time for 1st meeting = LCM $\left(\frac{L}{a-b}, \frac{L}{a-c}\right)$
- ⇨ Time for 1st meeting at the starting point = LCM $\left(\frac{L}{a}, \frac{L}{b}, \frac{L}{c}\right)$

Clocks To solve questions on clocks, consider a circular track of length 360°. The minute hand moves at a speed of 6° per min and the hour hand moves at a speed of ½° per minute.

Funda: Hands of a clock coincide (or make 180°) 11 times in every 12 hours. Any other angle is made 22 times in every 12 hours.

Time and Work

If a person can do a certain task in t hours, then in 1 hour he would do 1/t portion of the task.

A does a particular job in 'a' hours and B does the same job in 'b' hours, together they will take $\frac{ab}{a+b}$ hours

A does a particular job in 'a' hours more than A and B combined whereas B does the same job in 'b' hours more than A and B combined, then together they will take $\sqrt{ab}$ hours to finish the job.

Funda: A does a particular job in 'a' hours, B does the same job in 'b' hours and C does the same job in 'c' hours, then together they will take $\frac{abc}{ab+bc+ca}$ hours.

Funda: If A does a particular job in 'a' hours and A&B together do the job in 't' hours, the B alone will take $\frac{at}{a-t}$ hours.

Funda: If A does a particular job in 'a' hours, B does the same job in 'b' hours and ABC together do the job in 't' hours, then

- ⇨ C alone can do it in $\frac{abt}{ab-at-bt}$ hours
- ⇨ A and C together can do it in $\frac{bt}{b-t}$ hours
- ⇨ B and C together can do it in $\frac{at}{a-t}$ hours

Funda: If the objective is to fill the tank, then the *Inlet pipes* do **positive work** whereas the *Outlet pipes* do **negative work**. If the objective is to empty the tank, then the *Outlet pipes* do **positive work** whereas the *Inlet Pipes* do **negative work**.

Algebra

Quadratic & Other Equations

Inequalities

Logarithm

Quadratic and Other Equations

For a quadratic equation, $ax^2 + bx + c = 0$, its roots

- ⇨ $\alpha \text{ or } \beta = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$
- ⇨ Sum of roots = $\alpha + \beta = \frac{-b}{a}$
- ⇨ Product of roots = $\alpha\beta = \frac{c}{a}$

Discriminant $\Delta = b^2 - 4ac$

Condition	Nature of Roots
$\Delta < 0$	Complex Conjugate
$\Delta = 0$	Real and equal
$\Delta > 0$ and a perfect square	Rational and unequal
$\Delta > 0$ and not a perfect square	Irrational and unequal

Funda: If c = a, then roots are reciprocal of each other

Funda: If b =0, then roots are equal in magnitude but opposite in sign.

Funda: Provided a, b and c are rational

- ⇨ If one root is p + iq, other root will be p – iq
- ⇨ If one root is p + $\sqrt{q}$, other root will be p – $\sqrt{q}$

Cubic equation $ax^3+bx^2+cx+d = 0$

- ⇨ Sum of the roots = - b/a
- ⇨ Sum of the product of the roots taken two at a time = c/a
- ⇨ Product of the roots = -d/a

Biquadratic equation $ax^4+bx^3+cx^2+dx+e = 0$

- ⇨ Sum of the roots = - b/a
- ⇨ Sum of the product of the roots taken three at a time = c/a
- ⇨ Sum of the product of the roots taken two at a time = -d/a
- ⇨ Product of the roots = e/a

Inequalities

If a > b and c > 0,

- ⇨ a + c > b + c
- ⇨ a - c > b - c
- ⇨ ac > bc
- ⇨ a/c > b/c

If a, b ≥ 0, then $a^n > b^n$ and $1/a^n < 1/b^n$, where n is positive.

a < b and x > 0, then $\frac{a+x}{b+x} > \frac{a}{b}$

a > b and x > 0, then $\frac{a+x}{b+x} < \frac{a}{b}$

Modular Inequalities

|x- y| = |y - x|
|x. y| = |x| . |y|
|x+ y| < |x| + |y|
|x+ y| > |x| - |y|

Quadratic Inequalities

(x – a) (x – b) > 0 {a < b}

⇨ (x < a) U (x > b)

(x – a) (x – b) < 0 {a > b}

⇨ a < x < b

For any set of positive numbers: AM≥GM≥HM

⇨ $(a_1+a_2+ \ldots +a_n)/n \geq (a_1.a_2. \ldots .a_n)^{1/n}$

If a and b are positive quantities, then

⇨ $\frac{a+b}{2} \geq \sqrt{ab}$

If a,b,c,d are positive quantities, then

⇨ $\frac{a}{b}+\frac{b}{c}+\frac{c}{d}+\frac{d}{a} \geq 4$

⇨ $a^4 + b^4 + c^4 + d^4 \geq 4abcd$

Continued >>

If a,b,c …. k are n positive quantities and m is a natural number, then

$$\Rightarrow \frac{a^m+b^m+c^m \ldots .+k^m}{n} > \left(\frac{a+b+c \ldots +k}{n}\right)^m$$

$$\left(\frac{a+b+c+\cdots+k}{n}\right)^n > a.b.c.d \ldots .k$$

Funda: $\frac{a^m+b^m}{2} > \left(\frac{a+b}{2}\right)^m \quad [m \leq 0 \; or \; m \geq 1]$

$\frac{a^m+b^m}{2} < \left(\frac{a+b}{2}\right)^m \quad [0 < m < 1]$

Funda: For any positive integer n, $2 \leq \left(1+\frac{1}{n}\right)^n \leq 3$

Funda: $a^m b^n c^p$……..will be greatest when $\frac{a}{m} = \frac{b}{n} = \frac{c}{p}$

Funda: If a > b and both are natural numbers, then

⇨ $a^b < b^a$ {Except $3^2 > 2^3$ & $4^2 = 2^4$}

Funda: $(n!)^2 \geq n^n$

Funda: If the sum of two or more positive quantities is constant, their product is greatest when they are equal and if their product is constant then their sum is the least when the numbers are equal.

⇨ If x + y = k, then xy is greatest when x = y

⇨ If xy = k, then x + y is least when x = y

Logarithm

$Log(ab) = Log(a) + Log(b)$

$Log\left(\frac{a}{b}\right) = Log(a) - Log(b)$

$Log(a^n) = n\,Log(a)$

$Log_b(a) = \frac{log_c(a)}{log_c(b)}$

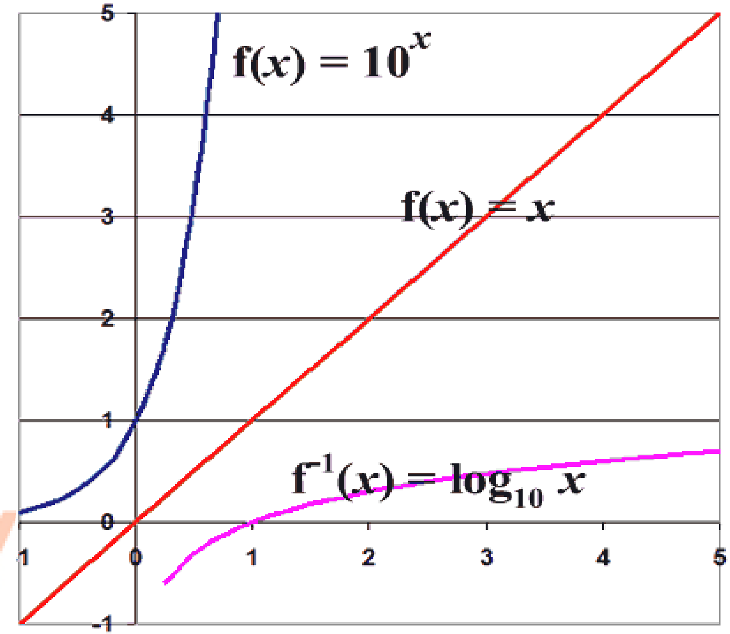
$Log_b\, b = 1$

$Log_b\, 1 = 0$

$Log_b\, b^x = x$

Ln x means $\log_e x$

$x = b^{\log_b x}$



GEOMETRY

Lines & Angles

Triangles

Polygons

Circles

Solid Figures

Co-ordinate Geometry

Trigo nometry

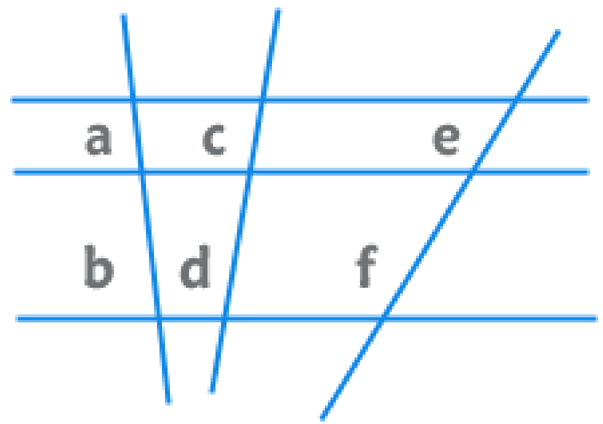
Lines and Angles

Sum of the angles in a straight line is 180°

Vertically opposite angles are congruent (equal).

If any point is equidistant from the endpoints of a segment, then it must lie on the **perpendicular bisector**

When two **parallel lines** are intersected by a **transversal**, **corresponding angles** are equal, **alternate angles** are equal and **co-interior angles** are supplementary. (*All acute angles formed are equal to each other and all obtuse angles are equal to each other*)



Funda: The ratio of intercepts formed by a transversal intersecting three parallel lines is equal to the ratio of corresponding intercepts formed by any other transversal.

$$\Rightarrow \frac{a}{b} = \frac{c}{d} = \frac{e}{f}$$

Triangles

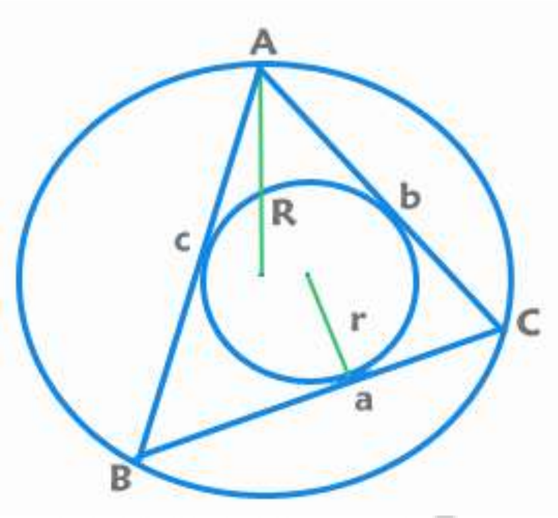
Sum of interior angles of a triangle is 180° and sum of exterior angles is 360°.

Exterior Angle = Sum of remote interior angles.

Sum of two sides is always greater than the third side and the difference of two sides is always lesser than the third side.

Side opposite to the biggest angle is longest and the side opposite to the smallest angle is the shortest.

Area of a triangle:



= ½ x Base x Height

= ½ x Product of sides x Sine of included angle

$=\sqrt{s(s-a)(s-b)(s-c)}$; here s is the semi perimeter [s = (a+b+c)/2]

= r x s [r is radius of incircle]

$=\dfrac{abc}{4R}$ [R is radius of circumcircle]

Continued >>

A **Median** of a triangle is a line segment joining a vertex to the midpoint of the opposing side. The three medians intersect in a single point, called the **Centroid** of the triangle. Centroid divides the median in the ratio of 2:1

An **Altitude** of a triangle is a straight line through a vertex and perpendicular to the opposite side or an extension of the opposite side. The three altitudes intersect in a single point, called the **Orthocenter** of the triangle.

A **Perpendicular Bisector** is a line that forms a right angle with one of the triangle's sides and intersects that side at its midpoint. The three perpendicular bisectors intersect in a single point, called the **Circumcenter** of the triangle. It is the center of the circumcircle which passes through all the vertices of the triangle.

An **Angle Bisector** is a line that divides the angle at one of the vertices in two equal parts. The three angle bisectors intersect in a single point, called the **Incenter** of the triangle. It is the center of the incircle which touches all sides of a triangle.

Funda: *Centroid* and *Incenter* will always lie inside the triangle.

- For an **acute angled triangle**, the *Circumcenter* and the *Orthocenter* will lie inside the triangle.
- For an **obtuse angled triangle**, the *Circumcenter* and the *Orthocenter* will lie outside the triangle.
- For a **right angled triangle** the *Circumcenter* will lie at the midpoint of the hypotenuse and the *Orthocenter* will lie at the vertex at which the angle is 90°.

Funda: The *orthocenter*, *centroid*, and *circumcenter* always lie on the same line known as **Euler Line**.

- The orthocenter is twice as far from the centroid as the circumcenter is.
- If the triangle is **Isosceles** then the incenter lies on the **same line**.
- If the triangle is **equilateral**, all four are the **same point**.

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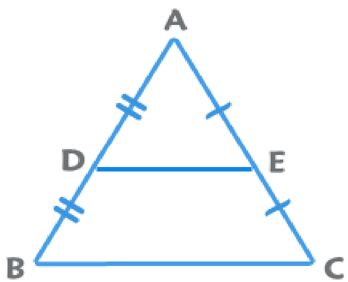
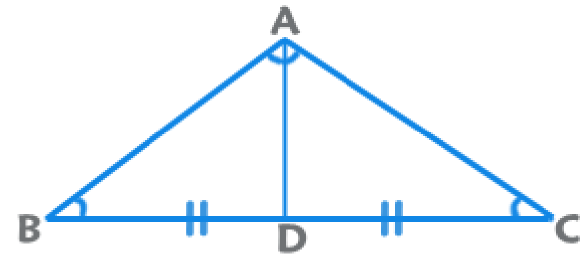
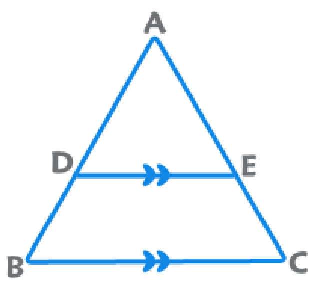
Theorems



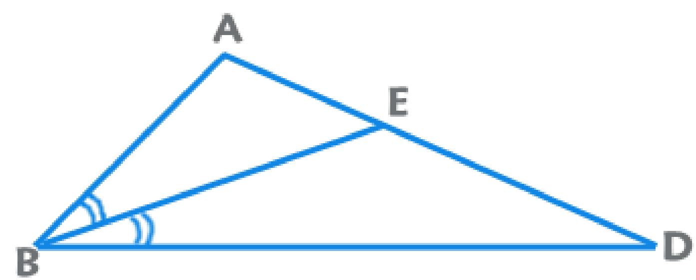
Mid Point Theorem: The line joining the midpoint of any two sides is parallel to the third side and is half the length of the third side.



Apollonius' Theorem: $AB^2 + AC^2 = 2\,(AD^2 + BD^2)$



Basic Proportionality Theorem: If DE || BC, then AD/DB = AE/EC



Interior Angle Bisector Theorem: AE/ED = BA/BD

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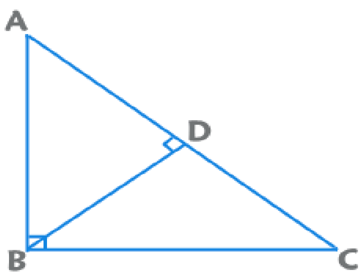
Special Triangles

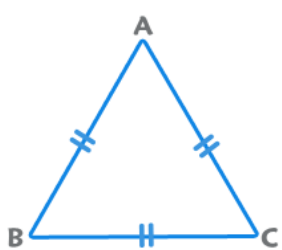
Right Angled Triangle:



$\Delta ABC \approx \Delta ADB \approx \Delta BDC$

$BD^2 = AD \times DC$ and $AB \times BC = BD \times DC$

Equilateral Triangle:



All angles are equal to 60°. All sides are equal also.

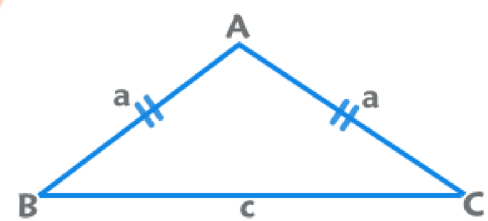
Height = $\frac{\sqrt{3}}{2}$ x Side

Area = $\frac{\sqrt{3}}{4}$ X Side^2

Inradius = 1/3 Height

Circumradius = 2/3 Height.

Isosceles Triangle:



Angles equal to opposite sides are equal.

Area = $\frac{a}{4}\sqrt{4c^2 - a^2}$

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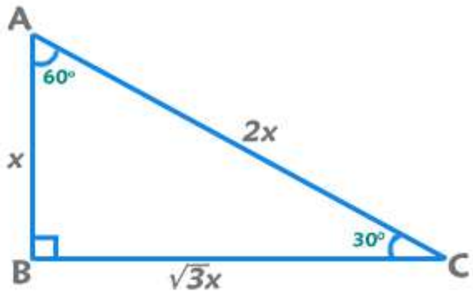
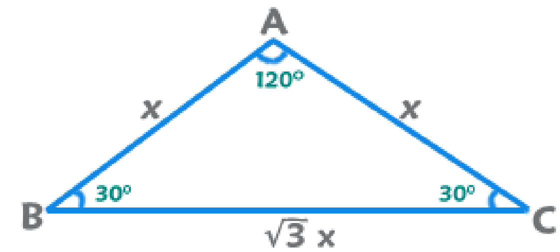
30°-60°-90° Triangle



Area = $\frac{\sqrt{3}}{2} * x^2$

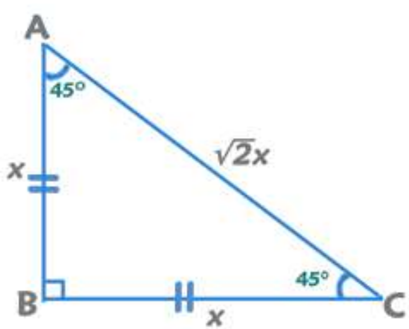
30°-30°-120° Triangle



Area = $\frac{\sqrt{3}}{4} * x^2$

Continued >>

45°-45°-90° Triangle



Area = $x^2/2$

Similarity of Triangles

Two triangles are similar if their corresponding angles are congruent and corresponding sides are in proportion.

Tests of similarity: (AA / SSS / SAS)

For similar triangles, if the sides are in the ratio of a:b

- ⇨ Corresponding **heights** are in the ratio of a:b
- ⇨ Corresponding **medians** are in the ratio of a:b
- ⇨ **Circumradii** are in the ratio of a:b
- ⇨ **Inradii** are in the ratio of a:b
- ⇨ **Perimeters** are in the ratio of a:b
- ⇨ **Areas** are in the ratio $a^2 : b^2$

Congruency of Triangles

Two triangles are congruent if their corresponding sides and angles are congruent.

Tests of congruence: (SSS / SAS / AAS / ASA)

All ratios mentioned in similar triangle are now 1:1

Polygons

Sum of interior angles = (n-2) x 180° = (2n-4) x 90°
Sum of exterior angles = 360°

Number of diagonals = $^nC_2 - n = \frac{n(n-3)}{2}$

Number of triangles which can be formed by the vertices = $^nC_3$

Regular Polygon:

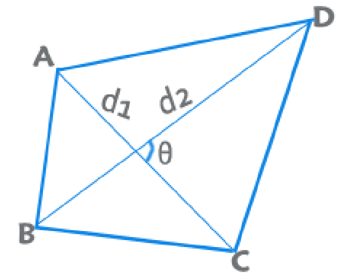
If all sides and all angles are equal, it is a regular polygon.

All regular polygons can be inscribed in or circumscribed about a circle.

Area = ½ x Perimeter x Inradius *{Inradius is the perpendicular from centre to any side}*

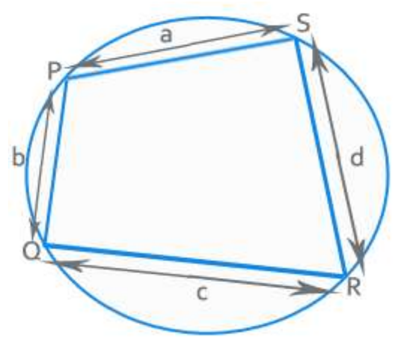
Each Interior Angle = $\frac{(n-2)180°}{n}$; Exterior = 360°/n

Quadrilaterals:



Sum of the interior angles = Sum of the exterior angles = 360°
Area for a quadrilateral is given by ½ $d_1 d_2 \text{Sin}\theta$.

Cyclic Quadrilateral



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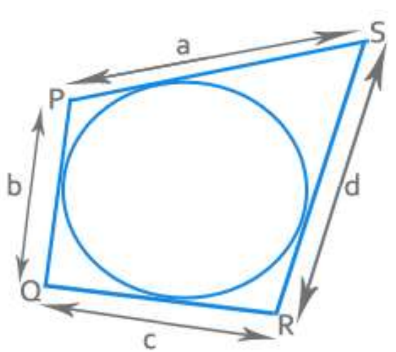
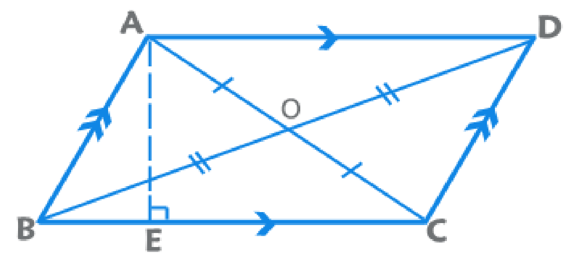
If all vertices of a quadrilateral lie on the circumference of a circle, it is known as a cyclic quadrilateral.
Opposite angles are supplementary
Area = $\sqrt{(s-a)(s-b)(s-c)(s-d)}$ where s is the semi perimeter $s = \frac{a+b+c+d}{2}$

Funda: Sum or product of opposite sides = Product of diagonals



Funda: If a circle can be inscribed in a quadrilateral, its area is given by = $\sqrt{abcd}$

Parallelogram



Opposite sides are parallel and congruent.

Opposite angles are congruent and consecutive angles are supplementary.

Diagonals of a parallelogram bisect each other.

Perimeter = 2(Sum of adjacent sides);
Area = Base x Height = AD x BE

Funda: A parallelogram inscribed in a circle is always a *Rectangle*. A parallelogram circumscribed about a circle is always a *Rhombus*.

Continued >>

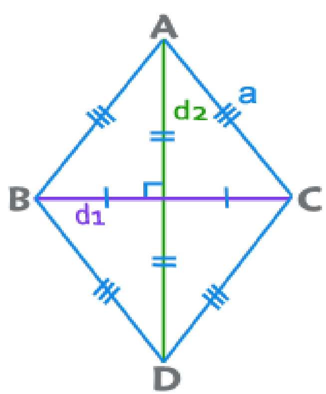
Funda: Each diagonal divides a parallelogram in two triangles of equal area.

Funda: Sum of squares of diagonals = Sum of squares of four sides

⇨ $AC^2 + BD^2 = AB^2 + BC^2 + CD^2 + DA^2$

Funda: A *Rectangle* is formed by intersection of the four angle bisectors of a parallelogram.

Rhombus



A parallelogram with all sides equal is a Rhombus. Its diagonals bisect at 90°.

Perimeter = 4a;   Area = ½ $d_1 d_2$;   Area = d x $\sqrt{a^2 - (\frac{d}{2})^2}$

Rectangle

A parallelogram with all angles equal (90°) is a Rectangle. Its diagonals are congruent.

Perimeter = 2(l+b);   Area = lb

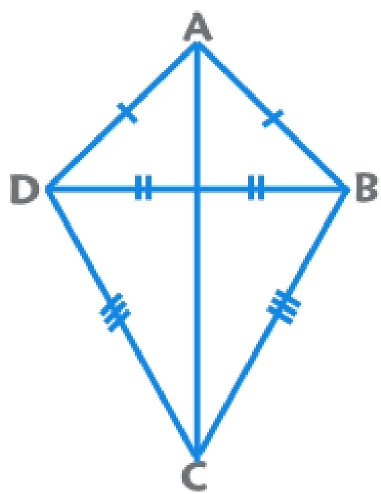
Square

A parallelogram with sides equal and all angles equal is a square. Its diagonals are congruent and bisect at 90°.

Perimeter = 4a; Area = a^2; Diagonals = $a\sqrt{2}$

Funda: From all quadrilaterals with a given area, the square has the least perimeter. For all quadrilaterals with a given perimeter, the square has the greatest area.

Kite

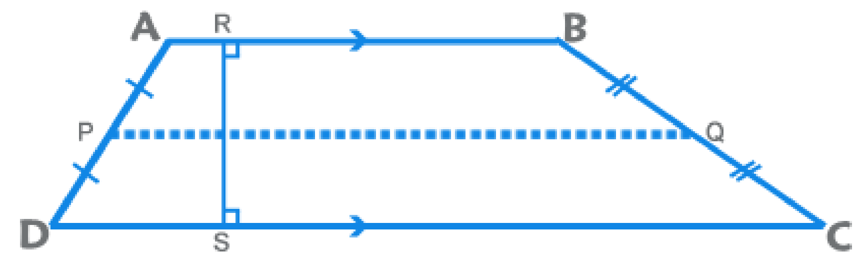


Two pairs of adjacent sides are congruent.

The longer diagonal bisects the shorter diagonal at 90°.

Area = Product of Diagonals / 2

Trapezium / Trapezoid



A quadrilateral with exactly one pair of sides parallel is known as a Trapezoid. The parallel sides are known as bases and the non-parallel sides are known as lateral sides.

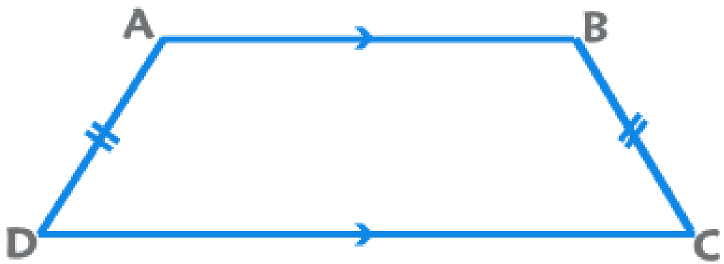
Area = ½ x (Sum of parallel sides) x Height

Median, the line joining the midpoints of lateral sides, is half the sum of parallel sides.

Funda: Sum of the squares of the length of the diagonals = Sum of squares of lateral sides + 2 Product of bases.

⇨ $AC^2 + BD^2 = AD^2 + BC^2 + 2 \times AB \times CD$

Isosceles Trapezium



The non-parallel sides (lateral sides) are equal in length. Angles made by each parallel side with the lateral sides are equal.

Funda: If a trapezium is inscribed in a circle, it has to be an isosceles trapezium. If a circle can be inscribed in a trapezium, Sum of parallel sides = Sum of lateral sides.

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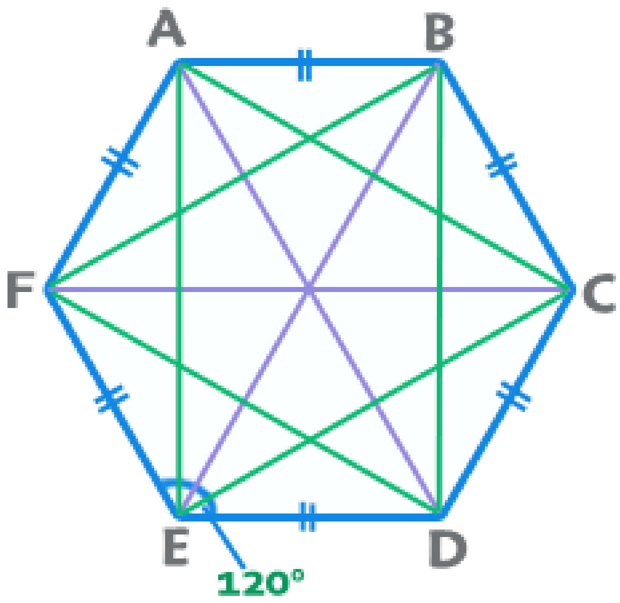
Hexagon (Regular)



Perimeter = 6a; Area = $\frac{3\sqrt{3}}{2}$ x a^2

Sum of Interior angles = 720°.

Each Interior Angle = 120°. Exterior = 60°

Number of diagonals = 9 {3 big and 6 small}

Length of big diagonals (3) = 2a

Length of small diagonals (6) = $\sqrt{3}$ a

Funda: A regular hexagon can be considered as a combination of six equilateral triangles. All regular polygons can be considered as a combination of 'n' isosceles triangles.

Area of a **Pentagon** = 1.72 a^2

Area of an **Octagon** = $2(\sqrt{2} + 1)\ a^2$

Circles

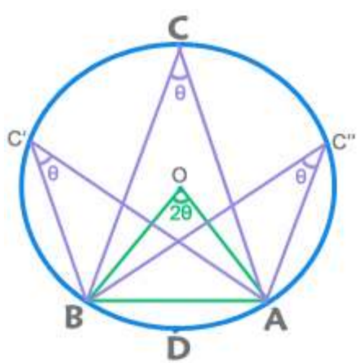
Diameter = 2r; Circumference = $2\pi r$; Area = πr^2

Chords equidistant from the centre of a circle are equal.

A line from the centre, perpendicular to a chord, bisects the chord.

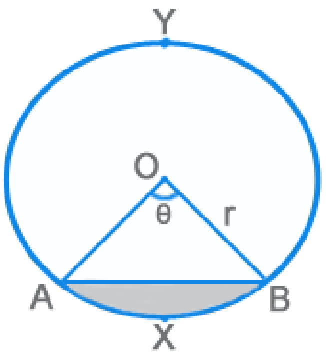
Equal chords subtend equal angles at the centre.

The diameter is the longest chord of a circle.



A chord /arc subtends equal angle at any point on the circumference and double of that at the centre.

Chords / Arcs of equal lengths subtend equal angles.



Chord AB divides the circle into two parts: Minor Arc AXB and Major Arc AYB.

Measure of arc AXB = $\angle$ AOB = θ

Length (arc AXB) = $\frac{\theta}{360°} \times 2\pi r$

Area (sector OAXB) = = $\frac{\theta}{360°} \times \pi r^2$

Area of Minor Segment = Shaded Area in above figure

⇨ Area of Sector OAXB - Area of Δ OAB

⇨ $r^2 \left[\frac{\pi\theta}{360°} - \frac{\sin\theta}{2} \right]$

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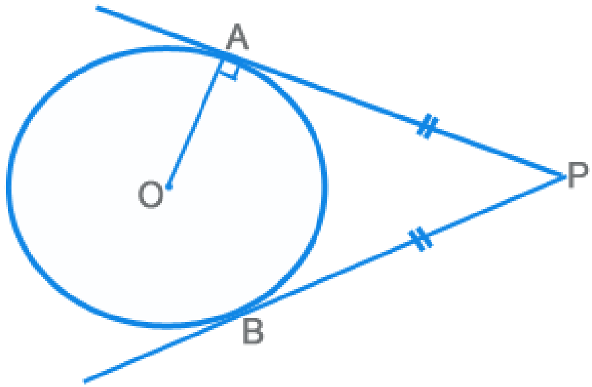
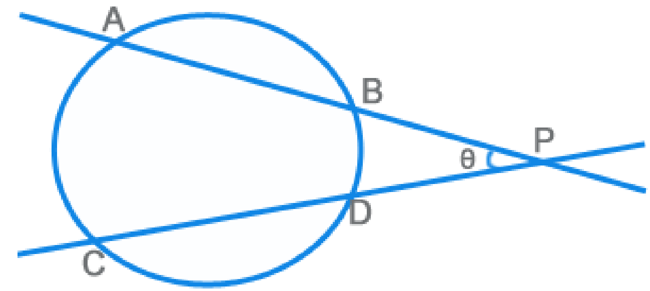
Properties of Tangents, Secants and Chords



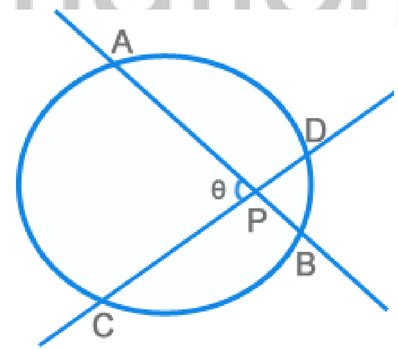


PA x PB = PC x PD

θ = ½ [m(Arc AC) – m(Arc BD)]

The radius and tangent are perpendicular to each other.

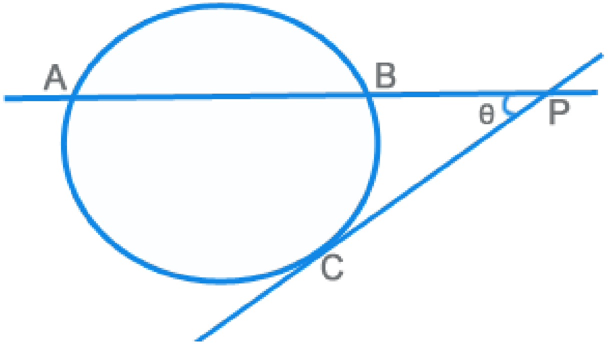
There can only be two tangents from an external point, which are equal in length **PA = PB**



PA x PB = PC x PD
θ = ½ [m(Arc AC) + m(Arc BD)]

Continued >>

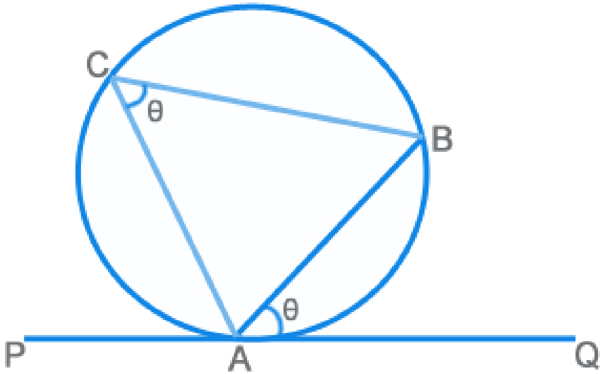
Properties *(contd.)*



PA x PB = PC^2

θ = ½ [m(Arc AC) - m(Arc BC)]

Alternate Segment Theorem



The angle made by the chord AB with the tangent at A (PQ) is equal to the angle that it subtends on the opposite side of the circumference.

⇨ ∠ BAQ = ∠ ACB

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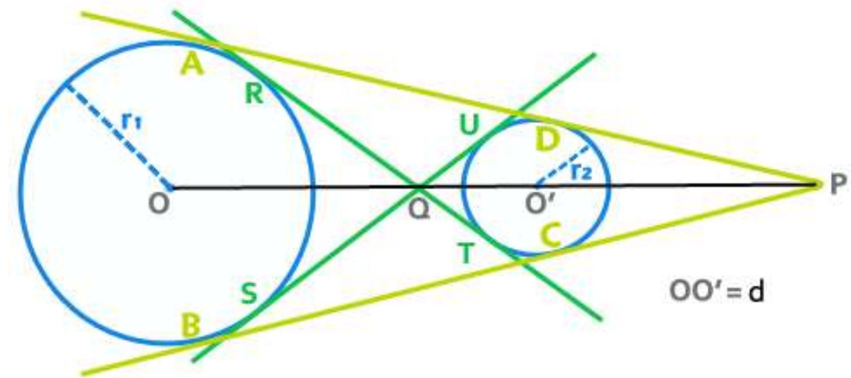
Common Tangents

Two Circles	No. of Common Tangents	Distance Between Centers (d)
One is completely inside other	0	< r1 - r2
Touch internally	1	= r1 - r2
Intersect	2	r1 - r2 < d < r1 + r2
Touch externally	3	= r1 + r2
One is completely outside other	4	> r1 + r2



Length of the Direct Common Tangent (DCT)

⇨ AD = BC = $\sqrt{d^2 - (r1 - r2)^2}$

Length of the Transverse Common Tangent (TCT)

⇨ RT = SU = $\sqrt{d^2 - (r1 + r2)^2}$

Funda: The two centers(O and O'), point of intersection of DCTs (P)and point of intersection of TCTs (Q) are collinear. Q divides OO' in the ratio $r_1 : r_2$ internally whearea P divides OO' in the ratio $r_1 : r_2$ externally.

Solid Figures

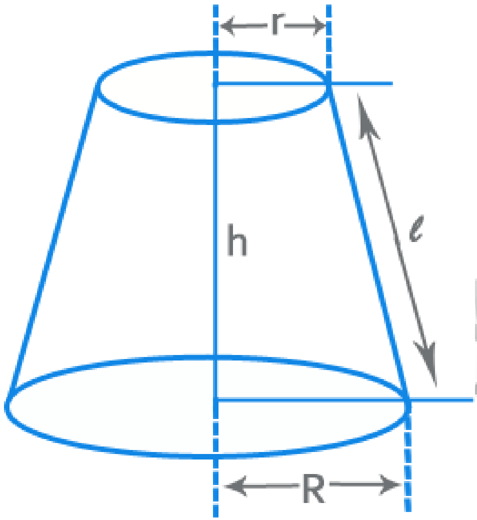
	Volume	Total Surface Area	Lateral / Curved Surface Area
Cube	$Side^3$	$6 \times Side^2$	$4 \times Side^2$
Cuboid	L x B x H	2(LB + LH + BH)	2 (LH + BH)
Cylinder	$\pi r^2 h$	$2\pi r\,(r + h)$	$2\pi rh$
Cone	$(1/3)\,\pi r^2 h$	$\pi r\,(r + L)$	πrl {where $L = \sqrt{r^2 + h^2}$}
Sphere	$(4/3)\,\pi r^3$	$4\,\pi r^2$	$4\,\pi r^2$
Hemisphere	$(2/3)\,\pi r^3$	$3\,\pi r^2$	$2\,\pi r^2$

Funda: There are 4 body diagonals in a cube / cuboid of length ($\sqrt{3}$ x side) and $\sqrt{l^2 + b^2 + h^2}$ respectively.

Continued >>

Frustum / Truncated Cone

It can be obtained by cutting a cone with a plane parallel to the circular base.

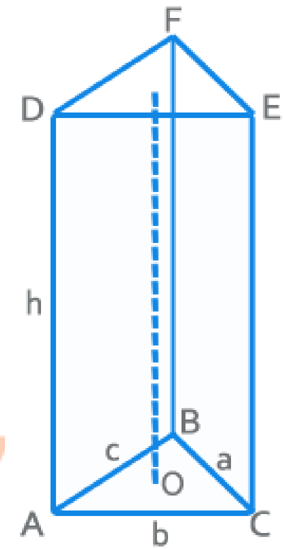


Volume = 1/3 πh ($R^2 + r^2$ + Rr)

Lateral Surface Area = π (R+r) L

Total Surface Area = π (R+r) L + π (R^2+r^2)

Prism



It is a solid with rectangular vertical faces and bases as congruent polygons (of n sides). It will have '2n' Vertices; 'n+2' Faces and '3n' Sides / Edges.

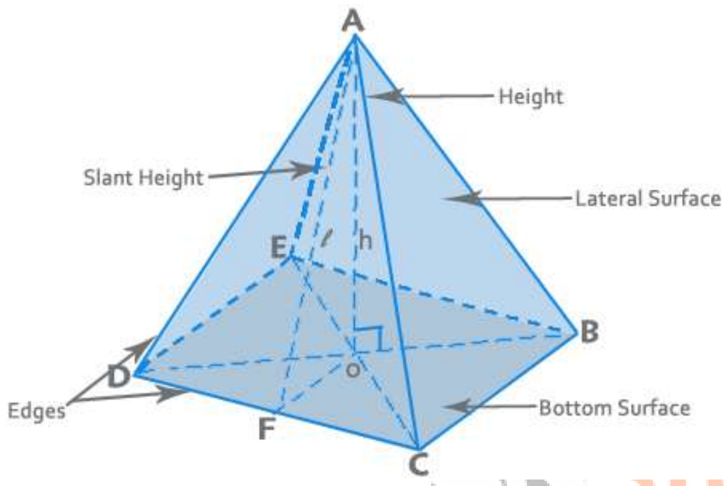
Lateral Surface Area = Perimeter x Height

Total Surface Area = Perimeter x Height + 2 $\text{Area}_{\text{Base}}$

Volume = $\text{Area}_{\text{Base}}$ x Height

Continued >>

Pyramid



It is a figure in which the outer surfaces are triangular and converge at a point known as the apex, which is aligned directly above the centre of the base.

Lateral Surface Area = ½ x Perimeter x Slant Height

Total Surface Area = ½ x Perimeter x Slant Height + $Area_{Base}$

Volume = ½ x $Area_{Base}$ x Height

Funda: If a sphere is inscribed in a cube of side a, the radius of the sphere will be a/2. If a sphere is circumscribed about a cube of side a, the radius of the sphere will be $\sqrt{3}$ a /2.

Funda: If a largest possible sphere is inscribed in a cylinder of radius 'a' and height h, its radius r will be

⇨ r = h/2 {If 2a > h}
⇨ r = a {If 2a < h}

Funda: If a largest possible sphere is inscribed in a cone of radius r and slant height equal to 2r, then the radius of sphere = r/$\sqrt{3}$

Funda: If a cube is inscribed in a hemisphere of radius r, then the edge of the cube = r $\sqrt{\frac{2}{3}}$

Co-ordinate Geometry

Distance between two points P(x_1,y_1) and Q(x_2,y_2) is given by = $\sqrt{(x_2 - x_1)^2 - (y_2 - y_1)^2}$

If a point R (x,y) divides P(x_1,y_1) and Q(x_2,y_2) internally in the ratio of m:n, the coordinates of R ie (x,y) are given by

$$= \frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}$$

If a point R (x,y) divides P(x_1,y_1) and Q(x_2,y_2) externally in the ratio of m:n, the coordinates of R ie (x,y) are given by

$$= \frac{mx_2 - nx_1}{m-n}, \frac{my_2 - ny_1}{m-n}$$

Funda: The X axis divides the line joining P(x_1,y_1) and Q(x_2,y_2) in the ratio of $y_1 : y_2$

Funda: The Y axis divides the line joining P(x_1,y_1) and Q(x_2,y_2) in the ratio of $x_1 : x_2$

Slope(m) of a line is the tangent of the angle made by the line with the positive direction of the X-Axis.
For a general equation ax + by + c = 0; slope (m) = -a/b.
For a line joining two points, P (x_1,y_1) and Q(x_2,y_2), the slope(m) is = $\frac{y_2 - y_1}{x_2 - x_1}$

Slope(m)	Type of line	Angle with X-Axis
> 0 (+ive)	Rising	Acute
0	Parallel to X-Axis	0^0
< 0 (-ive)	Falling	Obtuse
∞	Parallel to Y-Axis	90^0

Equation of a line parallel to X-axis is y = a {Of X-Axis is y = 0}
Equation of a line parallel to Y-Axis is x = a {Of Y-Axis is x = 0}

The intercept of a line is the distance between the point where it cuts the X-Axis or Y-Axis and the origin. Y-Intercept is often denoted with the letter 'c'.

Continued >>

Equation of a line

General form: ax + by + c = 0

Slope Intercept Form: Slope is m, y-intercept is c

⇨ $y = mx + c$

Slope Point Form: Slope is m, point is x_1, y_1

⇨ $y - y_1 = m(x - x_1)$

Two Point Form: Two points are x_1, y_1 and x_2, y_2

⇨ $y - y_1 = \left[\frac{y_2 - y_1}{x_2 - x_1}\right](x - x_1)$

Two Intercept Form: X-intercept is a, Y-intercept is b.

⇨ $\frac{x}{a} + \frac{y}{b} = 1$ OR $bx + ay = ab$

Acute angle between two lines with slope m_1 and m_2 is given by

⇨ $\text{Tan } \theta = \left| \frac{m_1 - m_2}{1 + m_1 m_2} \right|$

⇨ For parallel lines, θ = 0°; $m_1 = m_2$

⇨ For parallel lines, θ = 90°; $m_1 m_2 = -1$

Distance of a point P (x_1, y_1) **from a line** ax + by + c = 0

⇨ $d = \left| \frac{ax_1 + by_1 + c}{\sqrt{a^2 + b^2}} \right|$

⇨ From origin, $d = \left| \frac{c}{\sqrt{a^2 + b^2}} \right|$

Distance between two parallel lines, $ax + by + c_1 = 0$ and $ax + by + c_2 = 0$

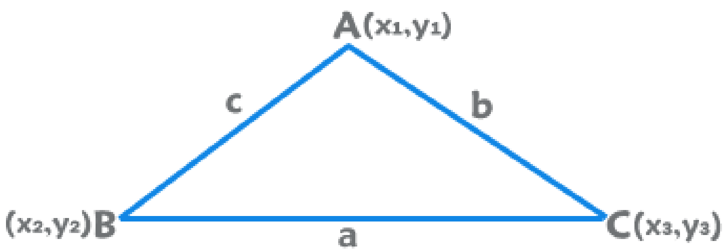
⇨ $d = \left| \frac{c_1 - c_2}{\sqrt{a^2 + b^2}} \right|$

Funda: If we know three points $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_2, y_2)$ of a parallelogram, the fourth point is given by

⇨ $(x_1 + x_3 - x_2,\ y_1 + y_3 - y_2)$

Continued >>

Triangle



The vertices are P (x_1,y_1), Q(x_2,y_2) and R(x_3,y_3)

Incenter = $\{\frac{ax_1+bx_2+cx_3}{a+b+c}, \frac{ay_1+by_2+cy_3}{a+b+c}\}$

Centroid = $\{\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{a+b+c}\}$

Area = ½ [$x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)$]

Circle

General Equation: $x^2 + y^2 + 2gx + 2fy + c = 0$

⇨ Centre is (-g, -f) and radius = $\sqrt{g^2 + f^2 - c}$

Centre is (h, k) and radius is r

⇨ $\sqrt{(x-h)^2 + (y-k)^2} = r$

Centre is origin and radius is r

⇨ $x^2 + y^2 = r^2$

Trigonometry

$\sin\theta = \dfrac{Perpendicular}{Hypotenuse} = \dfrac{AB}{AC}$

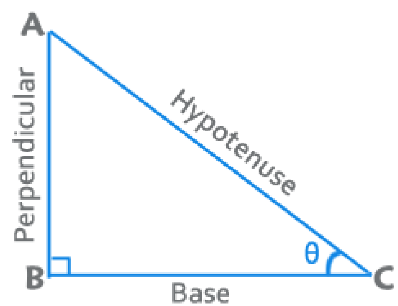
$\cos\theta = \dfrac{Base}{Hypotenuse} = \dfrac{BC}{AC}$

$\tan\theta = \dfrac{Perpendicular}{Base} = \dfrac{AB}{BC}$

$\text{cosec}\,\theta = \dfrac{1}{\sin\theta}$

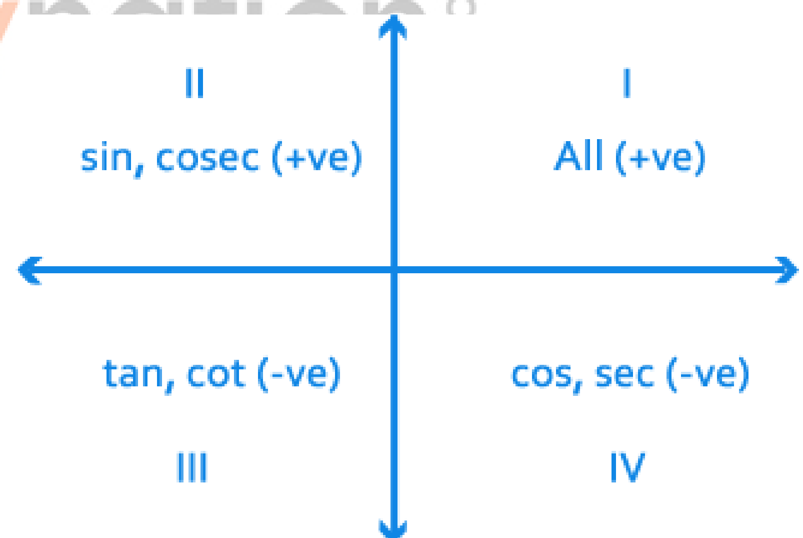
$\sec\theta = \dfrac{1}{\cos\theta}$

$\cot\theta = \dfrac{1}{\tan\theta}$



θ	Sin θ	Cos θ	Tan θ
0^0	0	1	0
30^0	$1/2$	$\sqrt{3}/2$	$1/\sqrt{3}$
45^0	$1/\sqrt{2}$	$1/\sqrt{2}$	1
60^0	$\sqrt{3}/2$	$1/2$	$\sqrt{3}$
90^0	1	0	∞

Signs of T-ratios in Different Quadrants:



Some Basic Identities:

$\sin^2\theta + \cos^2\theta = 1$

$1 + \tan^2\theta = \sec^2\theta$

$1 + \cot^2\theta = \text{cosec}^2\,\theta$

Continued >>

Addition Formulae

Sin (A + B) = sinA cos B + cosA sinB

Cos (A + B) = cos A cos B - sin A sin B

Tan (A + B) - $\frac{tanA+tanB}{1-tanAtanB}$

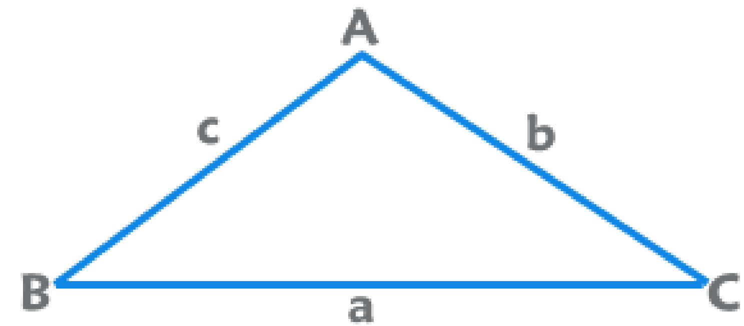
Subtraction Formulae

sin (A - B) = sin A cos B - cos A sin B

cos (A - B) = cos A cos B + sin A sin B

tan (A - B) = $\frac{tanA-tanB}{1+tanAtanB}$

Trigonometric Rules

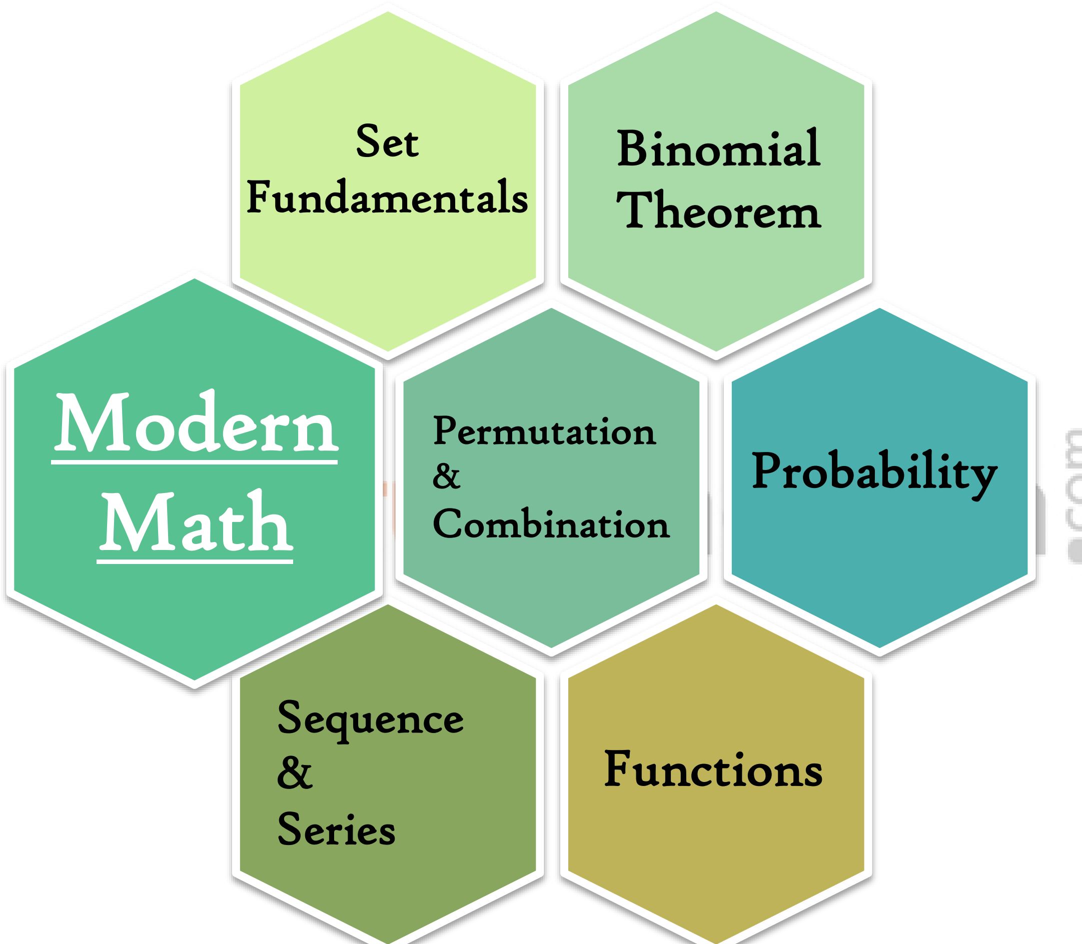


Sine Rule: $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c} = \frac{1}{2R}$

Cosine Rule: Cos A = $\frac{b^2 + c^2 - a^2}{2bc}$

Cos B = $\frac{a^2 + c^2 - b^2}{2ac}$

Cos C = $\frac{a^2 + b^2 - c^2}{2ab}$


Set Fundamentals
Binomial Theorem
Modern Math
Permutation & Combination
Probability
Sequence & Series
Functions

Set Fundamentals

The number of elements in a set is called its **cardinal number** and is written as n(A). A set with cardinal number 0 is called a null set while that with cardinal number ∞ is called an infinite set.

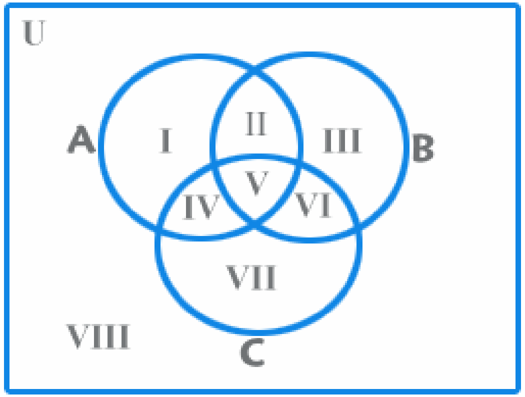
Set A is said to be a **subset** of Set B if each and every element of Set A is also contained in Set B. Set A is said to be a **proper subset** of Set B if Set B has at least one element that is not contained in Set A. A set with 'n' elements will have 2^n subsets ($2^n - 1$ proper subsets)

The **Universal set** is defined as the set of all possible objects under consideration.

Funda: Any set is a subset of itself, but not a proper subset. The empty set, denoted by ∅, is also a subset of any given set *X*. The empty set is always a proper subset, except of itself. Every other set is then a subset of the universal set.

Union of two sets is represented as A ∪ B and consists of elements that are present in either Set A or Set B or both. **Intersection of two sets** is represented as A ∩ B and consists of elements that are present in both Set A and Set B. **n(A∪B) = n(A) + n(B) — n(A∩B)**

Venn Diagram: A venn diagram is used to visually represent the relationship between various sets. What do each of the areas in the figure represent?



I – only A; II – A and B but not C; III – Only B; IV – A and C but not B; V – A and B and C; VI – B and C but not A; VII – Only C

n(A∪B∪C) = n(A) + n(B) + n(C) — n(A∩B) — n(A∩C) - n(B∩C) + n(A∩B∩C)

Binomial Theorem

For some basic values:

$$(a+b)^0 = 1$$
$$(a+b)^1 = a+b$$
$$(a+b)^2 = a^2+2ab+b^2$$
$$(a+b)^3 = a^3+3a^2b+3ab^2+b^3$$
$$(a+b)^4 = a^4+4a^3b+6a^2b^2+4ab^3+b^4$$
$$(a+b)^5 = a^5+5a^4b+10a^3b^2+10a^2b^3+5ab^4+b^5$$

Theorem

$$\boldsymbol{(a+b)^n = {}^n_0C\, a^n b^0 + {}^n_1C\, a^{n-1} b^1 + {}^n_2C\, a^{n-2} b^2 \ldots + {}^n_nC\, a^0 b^n}$$

$$(x+1)^n = x^n + nx^{n-1} + {}^n_2C\, x^{n-2} \ldots + nx + 1$$

$${}^n_0C + {}^n_1C + {}^n_2C \ldots + {}^n_nC = 2^n$$

$${}^n_0C + {}^n_2C + {}^n_4C \ldots = {}^n_1C + {}^n_3C + {}^n_5C \ldots = \frac{2^n}{2} = 2^{n-1}$$

Some basic properties

Funda: There is one more term than the power of the exponent, n. That is, there are terms in the expansion of $(a + b)^n$.

Funda: In each term, the sum of the exponents is n, the power to which the binomial is raised.

Funda: The exponents of a start with n, the power of the binomial, and decrease to 0. The last term has no factor of a. The first term has no factor of b, so powers of b start with 0 and increase to n.

Funda: The coefficients start at 1 and increase through certain values about “half”-way and then decrease through these same values back to 1.

Funda: To find the remainder when $(x + y)^n$ is divided by x, find the remainder when y^n is divided by x.

Funda: $(1+x)^n \cong 1 + nx$, when x<<1

Permutation & Combination

When two tasks are performed in succession, i.e., they are connected by an **'AND'**, to find the total number of ways of performing the two tasks, you have to **MULTIPLY** the individual number of ways. When only one of the two tasks is performed, i.e. the tasks are connected by an '**OR**', to find the total number of ways of performing the two tasks you have to **ADD** the individual number of ways.

Eg: In a shop there are'd' doors and 'w' windows.
*Case1: If a thief wants to enter via a door or window, he can do it in – **(d+w) ways**.*
*Case2: If a thief enters via a door and leaves via a window, he can do it in – **(d x w) ways**.*

Linear arrangement of 'r' out of 'n' distinct items (^nP_r): The first item in the line can be selected in 'n' ways AND the second in (n — 1) ways AND the third in (n — 2) ways AND so on. So, the total number of ways of arranging 'r' items out of 'n' is

$$(n)(n-1)(n-2)\ldots(n-r+1) = \frac{n!}{(n-r)!}$$

Circular arrangement of 'n' distinct items: Fix the first item and then arrange all the other items linearly with respect to the first item. This can be done in **(n — 1)!** ways.

Funda: In a necklace, it can be done in $\frac{(n-1)!}{2}$ ways.

Selection of r items out of 'n' distinct items (^nCr): Arrange of r items out of n = Select r items out of n and then arrange those r items on r linear positions.

$$^nP_r = {}^nC_r \times r! \rightarrow {}^nC_r = \frac{^nP_r}{r!} = \frac{n!}{r!(n-r)!}$$

Continued >>

Dearrangement If 'n' things are arranged in a row, the number of ways in which they can, be deranged so that none of them occupies its original place is

$$n!\left(\frac{1}{0!}-\frac{1}{1!}+\frac{1}{2!}-\ \ldots+\frac{(-1)^n}{n!}\right)$$

Funda: Number of ways of arranging 'n' items out of which `p' are alike, 'q' are alike, 'r' are alike in a line is given by = $\frac{\boldsymbol{n}\,!}{\boldsymbol{p}!\boldsymbol{q}!\boldsymbol{r}!}$

Partitioning

'n' similar items in **'r' distinct** groups	No restrictions	$^{n+r-1}C_{r-1}$
	No group empty	$^{n-1}C_{r-1}$
'n' distinct items in **'r' distinct** groups	No restrictions	r^n
	Arrangement in a group important	$\frac{(\boldsymbol{n}+\boldsymbol{r}-\mathbf{1})!}{(\boldsymbol{r}-\mathbf{1})!}$
'n' similar items in **'r' similar** groups	List the cases and then find out in how many ways is each case possible	
'n' similar items in **'r' similar** groups	List the cases and then find out in how many ways is each case possible	

Probability

P(A) = $\frac{Number\ of\ favorable\ outcomes}{Total\ number\ of\ outcomes}$

For **Complimentary Events:** *P(A) + P(A') = 1*

For **Exhaustive Events:** *P(A) + P(B) +P(C)… = 1*

Addition Rule:

P (A U B) = P(A) + P(B) – P(A ∩ B)

For **Mutually Exclusive Events** P(A ∩ B) = 0

⇨ *P (A U B) = P(A) + P(B)*

Multiplication Rule:

P(A ∩ B) = P(A) P(B/A) = P(B) P(A/B)

For **Independent Events** P(A/B) = P(B) and P(B/A) = P(B)

⇨ *P(A ∩ B) = P(A).P(B)*
⇨ *P (A U B) = P(A) + P(B) – P(A).P(B)*

Funda: If the probability of an event occurring is P, then the probability of that event occurring 'r' times in 'n' trials is = ^nC_r x P^r x $(1-P)^{n-r}$

Odds

Odds in favor = $\frac{Number\ of\ favorable\ outcomes}{Number\ of\ unfavorable\ outcomes}$

Odds against = $\frac{Number\ of\ unfavorable\ outcomes}{Number\ of\ favorable\ outcomes}$

Sequence, Series & Progression

Arithmetic Progression

$$a_n = a_1 + (n-1)d$$

$$S_n = \frac{n}{2}(a_1 + a_n) = \frac{n}{2}[2a_1 + (n-1)d]$$

Funda: Number of terms $= \frac{a_n - a_1}{d} + 1$

Geometric Progression

$$a_n = ar^{n-1}$$

$$S_n = \frac{a(1-r^n)}{1-r}$$

Sum till infinite terms = $\frac{a}{1-r}$ (Valid only when r<1)

Sum of first n natural numbers

⇨ $1 + 2 + 3 \ldots + n = \frac{\boldsymbol{n(n+1)}}{\boldsymbol{2}}$

Sum of squares of first n natural numbers

⇨ $1^2 + 2^2 + 3^2 \ldots + n^2 = \frac{\boldsymbol{n(n+1)(2n+1)}}{\boldsymbol{6}}$

Sum of cubes of first n natural numbers

⇨ $1^3 + 2^3 + 3^3 \ldots + n^3 = \left[\frac{\boldsymbol{n(n+1)}}{\boldsymbol{2}}\right]^{\boldsymbol{2}}$

Funda: Sum of first n odd numbers

⇨ $1 + 3 + 5 \ldots + (2n-1) = n^2$

Funda: Sum of first n even numbers

⇨ $2 + 4 + 6 \ldots 2n = n(n+1)$

Funda: If you have to consider 3 terms in an AP, consider {a-d,a,a+d}. If you have to consider 4 terms, consider {a-3d,a-d,a+d,a+3d}

Funda: If all terms of an AP are multiplied with k or divided with k, the resultant series will also be an AP with the common difference dk or d/k respectively.

Functions

Domain: Set of real and finite values that the independent variable can take.

Range: Set of real and finite values that the dependent variable can have corresponding to the values of the independent variable

Co-Domain: Set of real and finite values that the dependent variable can have.

Funda: Range is a subset of Co-Domain. Co-domain may or may not have values which do not have a pre-image in the domain.

Funda: It is not a function if for some value in the domain, the relationship gives more than one value. Eg: f(x) = $\sqrt{x}$ (At x = 4, f(x) could be both +2 and -2)

Funda: Domain cannot have any extra value ie the values at which the function does not exist.

One to One: Every element in the Domain has one and only one image in the Co-Domain. Every element in Co-Domain has one and only one pre-image in the Domain.

Many to One: If at least two elements in Domain have the same image in the co-domain.

Onto Function: If for every element in the Co-Domain there is at least one pre-image in the Domain. In this case, Range = Co-Domain

Into Function: If there is at least one element in the Co-Domain which does not have a pre-image in the Domain. In this case, Range is a proper subset of Co-Domain.

Even Function: f(x) is even if and only if f(-x) = f(x) for all values of x. The graph of such a function is symmetric about the Y-Axis

Odd Function: f(x) is odd if and only if f(-x) = - f(x) for all values of x. The graph is symmetric about the origin

Funda: If f(x) is an odd function and f(0) exists

⇨ f(0) = 0

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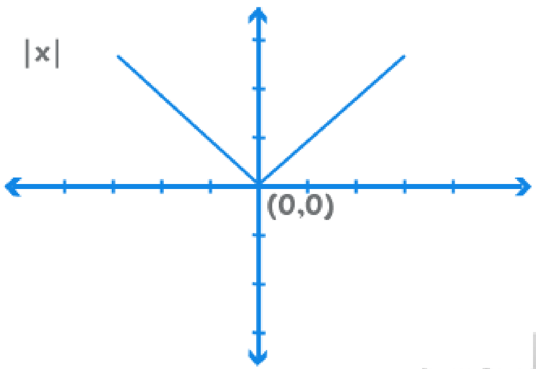
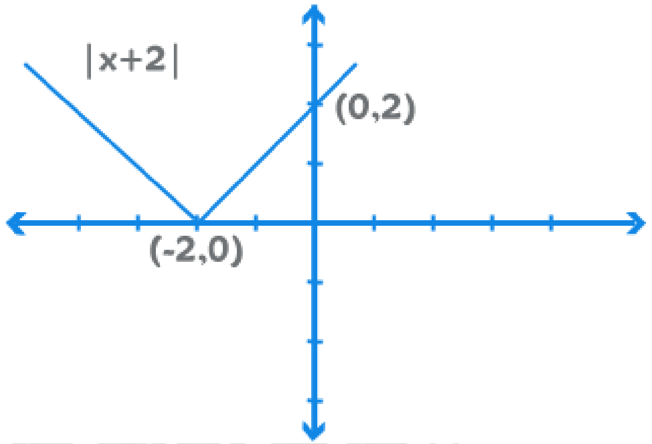
Graphs

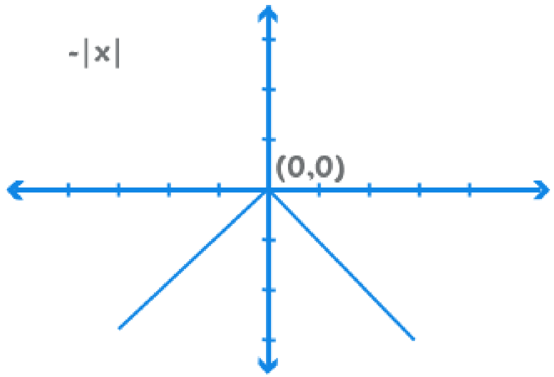
f(x) = |x|



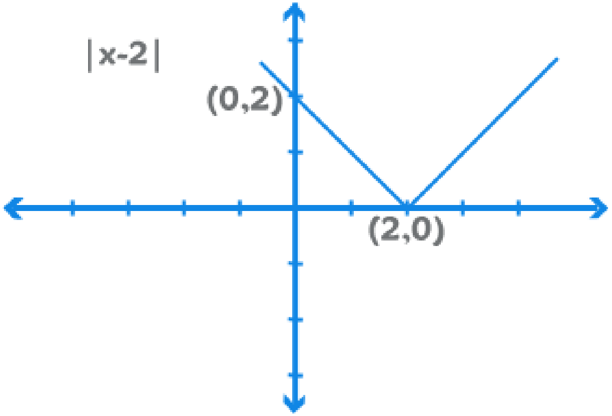
If we consider f(x+2), it shifts left by 2 units



If we consider –f(x), it gets mirrored in the X-Axis.



If we consider f(x-2), it shifts right by 2 units.



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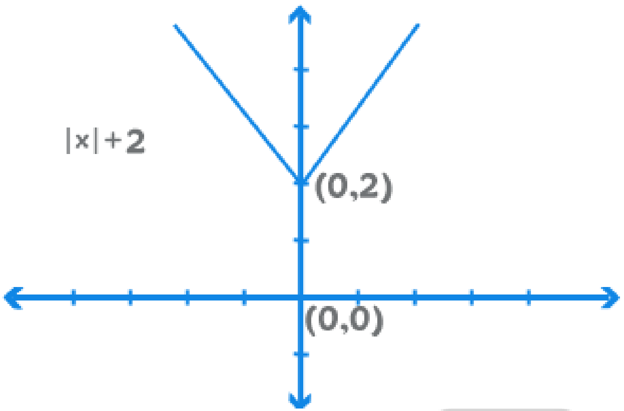
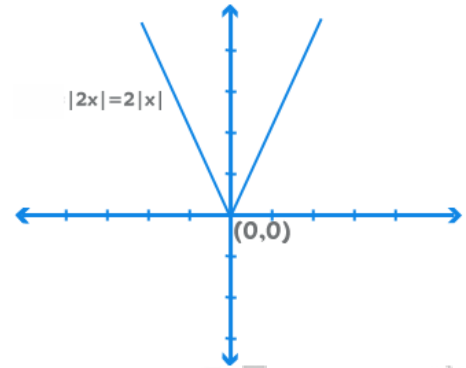
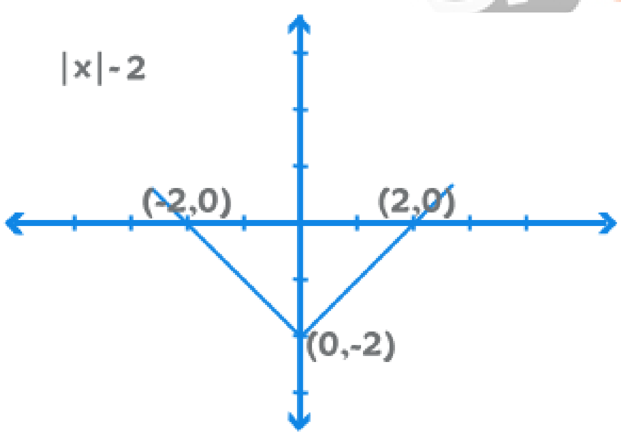
If we consider f(x) + 2, it shifts up by 2 units.



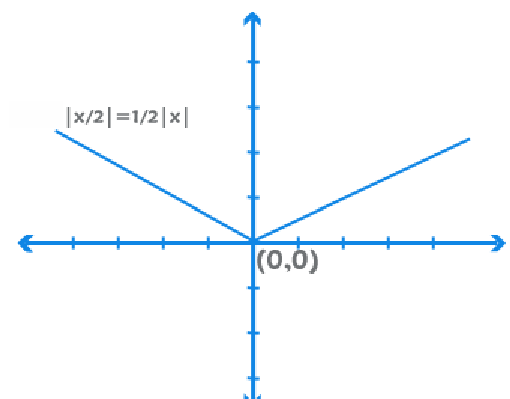
If we consider f(2x) or 2f(x) ,the slope doubles and the rise and fall become much sharper than earlier



If we consider f(x) – 2, it shifts down by 2 units.



If we consider f(x/2) or ½ f(x), the slope halves and the rise and fall become much flatter than earlier.



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