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Entrance Examination, 2011
M.Sc. (Statistics-OR)

Hall Ticket No.								

Time : 2 hours
Max. Marks. 75

Part A : 25 marks
Part B : 50 marks

Instructions

1. Write your Booklet Code and Hall Ticket Number on the OMR Answer Sheet given to you. Also write the Hall Ticket Number in the space provided above.
2. There is negative marking.
3. Answers are to be marked on the OMR answer sheet .
4. Please read the instructions carefully before marking your answers on the OMR answer sheet.
5. Hand over the question paper booklet and the OMR answer sheet at the end of the examination.
6. No additional sheets will be provided. Rough work can be done in the question paper itself/space provided at the end of the booklet.
7. Calculators are not allowed.
8. There are a total of 50 questions in Part A and Part B together.
9. The appropriate answer should be coloured in either a blue or black ball point or sketch pen. DO NOT USE A PENCIL.

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PART A

- Find the correct answer and mark it on the **OMR** sheet.
- A right answer gets **1 mark** and wrong answer gets $-$**0.33 mark**

1. The number of 4 digit numbers greater than 1000 in which all the numerals are distinct is

   [A] $9^2 \times 8 \times 7$.

   [B] $10 \times 9 \times 8 \times 7$.

   [C] $10^2 \times 9 \times 8$.

   [D] $9 \times 8 \times 7 \times 6$.

2. Eight points are chosen on the circumference of a circle. How many different chords can be drawn by joining any two points?

   [A] 28.

   [B] 8.

   [C] 16.

   [D] 64.

3. Let A and B be two events with $P(A \cup B) = 0.6$ and $P(A \cap B) = 0.3$, then $P(A^c) + P(B^c)$ is

   [A] 0.8.

   [B] 1.1.

   [C] 1.2.

   [D] 1.

4. A and B_1 are independent events, so are A and B_2, then

   [A] A and $B_1 \cap B_2$ are always independent.

   [B] if B_1 and B_2 are independent, then A and $B_1 \cap B_2$ are always independent.

   [C] A and $B_1 \cup B_2$ are always independent.

   [D] none of the above is true.

5. Three numbers are selected from the set $\{1, 2, ..., 10\}$, the probability that the selected numbers are such that the minimum is at most 5 lies in the interval

   [A] $(\frac{1}{3}, \frac{1}{2}]$.

   [B] $(\frac{1}{2}, \frac{2}{3}]$.

   [C] $(\frac{2}{3}, \frac{3}{4}]$.

   [D] $(\frac{3}{4}, 1)$.

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6. If $p = 0.1$ and $n = 5$, then the corresponding binomial distribution is

   [A] right skewed.

   [B] left skewed.

   [C] symmetric.

   [D] bimodal.

7. A population is distributed according to the four standard blood types as follows $A : 42\%, \quad O : 33\%, \quad B : 18\%, \quad AB : 7\%$

   Assuming that the people chose their partners independently of the blood types, what is the probability that a randomly selected couple from this population will have same blood type?

   [A] $(0.42)^2$.

   [B] $(0.33)^2$.

   [C] $(0.18)^2$.

   [D] $(0.42)^2 + (0.33)^2 + (0.18)^2 + (0.07)^2$.

8. From a box containing N_1 white balls (all alike) and N_2 blue balls (all alike), n balls are drawn randomly with replacement, the expected number of white balls in the sample is

   [A] $\frac{nN_1}{N_2}$.

   [B] $\frac{nN_2}{N_1}$.

   [C] $\frac{nN_1}{N_1+N_2}$.

   [D] $\frac{N_1}{n(N_1+N_2)}$.

9. The expected values of two random variables X_1 and X_2 are equal to μ, whereas $Var(X_1) > Var(X_2)$, one can say that

   [A] X_1 and X_2 are identically distributed.

   [B] X_1 is more likely to be near μ than X_2.

   [C] X_2 is more likely to be near μ than X_1.

   [D] $P(X_2 > X_1) = 1$.

10. Let X denote the number of times a gambler has to throw two distinct fair dice till the sum of the numbers that show up is 8, then the random variable X is distributed as

   [A] Binomial (100,5/36).

   [B] Geometric (5/36).

   [C] Binomial (100,5/36).

   [D] Geometric (1/36).

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11. 10% of shirts produced in a work shop are defective, the expected number of defective shirts in a randomly selected sample of 100 shirts from this work shop is

[A] 1.

[B] 5.

[C] 10.

[D] 20.

12. If X has Poisson distribution with mean 10, then it is true that

[A] $P(X = 7) > P(X = 8)$.

[B] $P(X = 8) > P(X = 10)$.

[C] $P(X = 10) > P(X = 6)$.

[D] $P(X = 14) > P(X = 13)$.

13. Let X be a random variable having Poisson distribution with mean 2, also let $Y = 2X - 1$, then the variance of Y is

[A] 3.

[B] 4.

[C] 7.

[D] 8.

14. A number is randomly chosen from the interval $[0, 1]$, the probability that its second decimal digit is 1 is

[A] 0.1.

[B] 0.2.

[C] 0.5.

[D] 0.9.

15. The height of men is normally distributed with mean $\mu = 167$ cm and standard deviation $\sigma = 3$ cm. What is the percentage of the population of men that have height greater than 167 cm?

[A] 30%.

[B] 50%.

[C] 70%.

[D] 100%.

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16. The probability density function of a random variable X is

$$f_X(x) = \frac{e^{-|x|}}{2}, \quad -\infty < x < \infty.$$

Then the moment generating functions $M_X(t)$ and $M_Y(t)$ of X and $Y = -X$ respectively satisfy

[A] $M_Y(t) = -M_X(t) \quad \forall t \in \mathcal{R}$.

[B] $M_Y(t) = M_X(t) \quad \forall t \in \mathcal{R}$.

[C] $M_Y(t) = -M_X(t)$ only when $t \geq 0$.

[D] $M_Y(t) = -M_X(t)$ only when $t < 0$.

17. The median and standard deviation of 25 numbers are $\mathbf{m}$ and $\mathbf{s}$ respectively, 5 is added to the largest and 5 is subtracted from the smallest numbers. For this new set of observations, the median $\mathbf{m_1}$ and standard deviation $\mathbf{s_1}$ satisfy

[A] $\mathbf{m_1} > \mathbf{m}$ and $\mathbf{s_1} = \mathbf{s}$.

[B] $\mathbf{m_1} = \mathbf{m}$ and $\mathbf{s_1} > \mathbf{s}$.

[C] $\mathbf{m_1} = \mathbf{m}$ and $\mathbf{s_1} < \mathbf{s}$.

[D] $\mathbf{m_1} > \mathbf{m}$ and $\mathbf{s_1} > \mathbf{s}$.

18. The differences between every pair of real numbers $x_1, x_2, ...x_n$, $n \geq 2$ are known, however $x_1, x_2, ..., x_n$ are not known, if $\mathbf{m}$ and $\mathbf{s}$ are the mean and standard deviation respectively of these numbers, it is true that

[A] both $\mathbf{m}$ and $\mathbf{s}$ can be determined.

[B] neither $\mathbf{m}$ nor $\mathbf{s}$ can be determined.

[C] $\mathbf{s}$ can be determined but not $\mathbf{m}$.

[D] $\mathbf{m}$ can be determined but not $\mathbf{s}$.

19. The correlation coefficient $\rho_{X,Y}$ between two random variables X and Y is $-\frac{1}{2}$, so $\rho_{-X,Y}$ is

[A] $-\frac{1}{2}$.

[B] $\frac{1}{2}$.

[C] 0.

[D] 1.

20. The maximum likelihood estimator of any parameter θ

[A] is always unbiased for θ.

[B] is always a sufficient estimator for θ.

[C] has the smallest mean square error always.

[D] none of the above is correct.

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21. Four random numbers where selected from the interval $(0, \theta)$, they were 0.6, 0.49, 032, 0.83, the maximum likelihood estimate of θ is

[A] 0.83.

[B] 0.56.

[C] 0.715.

[D] 0.575.

22. Let $A = \{1, 2, 3, 4\}$ and $B = \{6, 7\}$. The number of different onto functions from A to B is

[A] 16.

[B] 2.

[C] 14.

[D] 8.

23. The sequence $\{a_n\}$ is defined as follows

$$a_n = \sum_{j=0}^{n} j \binom{n}{j} \frac{1}{2^n}, \quad n = 1, 2, ...$$

Identify the correct answer

[A] The sequence $\{a_n\}$ is not monotonic.

[B] $\lim_{n \to \infty} a_n = \infty$.

[C] $\lim_{n \to \infty} a_n = 1$.

[D] $\lim_{n \to \infty} a_n = 0$.

24. A and B are two $n \times n$ real matrices, if Rank $(B) = n - 3$ and $AB = 0$, then for any $n \geq 7$

[A] Rank (A)== Rank (B).

[B] Rank (A)$\leq$3.

[C] Rank (A)>n-3.

[D] Rank (A)> 3.

25. For any set of real numbers $x_1, x_2, ..., x_n$, where $n \geq 2$, let $\bar{E} = \frac{1}{n} \sum_{i=1}^{n} e^{x_i}$, then

[A] $e^{\bar{x}} > \bar{E}$.

[B] $e^{\bar{x}} = \bar{E}$.

[C] $e^{-\bar{x}} = \bar{E}$.

[D] $e^{\bar{x}} \leq \bar{E}$.

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PART B

- Find the correct answer and mark it on the **OMR** sheet.
- A right answer gets **2 marks** and wrong answer gets – 0.66 mark

26. The probability that at least one of the events A, B occurs is 0.8 and the probability that at most one of them occurs is 0.5, the probability that exactly one of them occurs

    [A] is 0.3.

    [B] is 0.2.

    [C] is 0.

    [D] cannot be determined from the data given.

27. A fair coin is tossed three times, given that at least one of the first two tosses showed tails, the probability that exactly one head showed up

    [A] is $\frac{1}{2}$.

    [B] is $\frac{1}{4}$.

    [C] is $\frac{1}{8}$.

    [D] cannot be determined.

28. The probability that among seven people no two were born in the same day of the week is

    [A] $\frac{1}{7^7}$.

    [B] $\frac{2}{7^7}$.

    [C] $\frac{5!}{7^7}$.

    [D] $\frac{7!}{7^7}$.

29. The coefficient of x in $P(x) = 2x^2 - bx + 3$ is selected from the set $\{-7, -6, ..., 0, 1, ..., 6, 7\}$ with probability $\frac{1}{15}$ each. The probability that $P(x)$ has real roots is

    [A] $\frac{1}{15}$.

    [B] $\frac{1}{5}$.

    [C] $\frac{2}{5}$.

    [D] $\frac{3}{5}$.

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30. The probability mass function of a random variable X is given by

$$P(X=x)=\frac{1}{3}\left(\frac{2}{3}\right)^{j-1} \quad j=1,2,...$$

$P(X=101|X>100)$ is

[A] $\frac{1}{3}$.

[B] $\frac{2}{3}$.

[C] $\left(\frac{1}{3}\right)^{100}$

[D] $\left(\frac{2}{3}\right)^{100}$

31. The random variable X takes values in the set $\{1,2,3,4,5\}$ and it is given that $P(X\geq 2)+P(X\geq 3)+P(X\geq 4)=\frac{7}{3}$ and $E(X)=\frac{11}{3}$ so $P(X=5)$

[A] cannot be determined from the data given.

[B] is $\frac{1}{2}$.

[C] is $\frac{1}{3}$.

[D] is $\frac{1}{4}$.

32. The amount of bread that a bakery sells in a day is a random variable with density function

$$f(x)=\begin{cases} cx & 0\leq x<3 \\ c(6-x) & 3\leq x<6 \\ 0 & \text{otherwise} \end{cases}$$

what is c?

[A] $\frac{1}{9}$.

[B] $\frac{2}{9}$.

[C] $\frac{1}{3}$.

[D] $\frac{4}{9}$.

33. The cubic equation $(x-3)(x-5)(x-8)+1$ has

[A] some complex roots.

[B] all real roots and the largest is in the interval $(5,6]$.

[C] all real roots and the smallest is in the interval $(1,2]$.

[D] all real roots and the largest is in the interval $(7,8]$.

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34. Let X be a random variable having distribution function

$$F(x) = \begin{cases} 0 & \text{if} \quad x < 0 \\ x + \frac{1}{8} & \text{if} \quad 0 \leq x < 0.5 \\ 1 & \text{if} \quad x \geq 0.5 \end{cases}$$

Then $P(0 < X \leq 0.5)$

[A] $\frac{1}{2}$.

[B] $\frac{5}{8}$.

[C] $\frac{3}{4}$.

[D] $\frac{7}{8}$.

35. From a bag containing 5 white balls (all alike), take out at least one ball, that is you may take out 1,2,3,4 or 5 balls with equal probabilities, paint the selected balls blue and replace them, now take out 3 balls without replacement, the probability that one of them is blue is

[A] 0.9.

[B] 0.7.

[C] 0.6.

[D] 0.3.

36. The mean marks of a class of 60 students is $\mathbf{m}$ and variance is $\mathbf{s^2}$, the mean marks of another class of 40 students is $\mathbf{m+5}$ and variance is $\mathbf{s^2}$. Identify the correct statement regarding the mean marks $\mathbf{M}$ and variance $\mathbf{S^2}$ of all the 100 students

[A] $\mathbf{M = m}$ and $\mathbf{S^2 > s^2}$.

[B] $\mathbf{M = m}$ and $\mathbf{S^2 = s^2}$.

[C] $\mathbf{M > m}$ and $\mathbf{S^2 = s^2}$.

[D] $\mathbf{M > m}$ and $\mathbf{S^2 > s^2}$.

37. For the two sets of numbers given below

$$\begin{aligned} A_1 &= \{7, 7, 9, 12, 14, 16, 18\} \quad \text{and} \\ A_2 &= \{8, 8, 8, 8, 8, 8, 19\} \end{aligned}$$

Let R_i, M_i and S_i, $i = 1, 2$, denote the range, median and standard deviation of the numbers of the sets A_i, $i = 1, 2$, respectively, it can be seen that

[A] $R_1 = R_2$, $M_1 = M_2$ and $S_1 = S_2$.

[B] $R_1 = R_2$, $M_1 > M_2$ and $S_1 > S_2$.

[C] $R_1 = R_2$, $M_1 > M_2$ and $S_1 < S_2$.

[D] $R_1 = R_2$, $M_1 > M_2$ and $S_1 = S_2$.

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38. The probability distribution of a random variable X is as follows

X	0	1	2	3
$P(X=x)$	p_1	p_1	p_2	p_2

where $0 < p_1 < 1/2$. From the above distribution we have the following random sample of size 5 $\{0, 2, 2, 3, 1\}$. The maximum likelihood estimate of p_1 is

[A] 0.1

[B] 0.2

[C] 0.3

[D] 0.4

39. Let $\lfloor x \rfloor$ denote the largest integer $\leq x$, $\forall x \in \mathcal{R}$, eg. $\lfloor 2.66 \rfloor = 2$, $\lfloor -3.5 \rfloor = -4$. The function $f : \mathcal{R} \to \mathcal{R}$ defined as

$$f(x) = x\lfloor x \rfloor$$

[A] is not continuous at any $x \in \mathcal{Z} \setminus \{0\}$.

[B] is not continuous at any $x \in \mathcal{Z}$.

[C] is continuous at every $x \in \mathcal{R}$ but not differentiable at $x \in \mathcal{Z}$.

[D] is continuous and differentiable at every $x \in \mathcal{R}$.

40. Suppose a man has invested in the share market which earns 10% in the first year and 50% in the second year, what is his average rate of return?

[A] 40.

[B] 10.

[C] 28.45.

[D] 30.

41. X_1 and X_2 are independent and identically distributed random variable with finite mean μ and finite variance σ^2, define $U_1 = \frac{2X_1+X_2}{3}$ and $U_2 = \frac{X_1+X_2}{2}$, then

[A] $E(U_1) = E(U_2)$ and $Var(U_1) = Var(U_2)$.

[B] $E(U_1) = E(U_2)$ and $Var(U_1) > Var(U_2)$.

[C] $E(U_1) > E(U_2)$ and $Var(U_1) = Var(U_2)$.

[D] $E(U_1) = E(U_2)$ and $Var(U_1) < Var(U_2)$.

42. Suppose $X_1, X_2, ...X_n$ is a random sample from the $Exp(1/\lambda)$ population. Denote $U = \frac{1}{n}\sum_{k=1}^{n} X_k$ and $V = \frac{1}{n}\sum_{k=1}^{n} X_k^2$, then an unbiased estimator for λ^2 is

[A] $U^2/2$.

[B] $2/U^2$.

[C] $V/2$.

[D] $2/V$.

43. $X_1, X_2, ..., X_n$ is a random sample from the $N(\mu, \mu^2)$ population, define U and V as in Question No: 42, the maximum likelihood estimator of μ is

[A] U.

[B] U^2.

[C] V.

[D] none of the above 3.

44. A die is rolled 60 times with resulting frequency distribution as given below

Faces	1	2	3	4	5	6
Frequency	11	10	9	10	12	8

To test the hypothesis that the die is fair, the value of the χ^2 goodness-of-fit test statistic is

[A] 0.

[B] 1.

[C] 2.

[D] 3.

45. Suppose $\bar{X}$ is the sample mean based on a random sample from a density $\theta x^{\theta-1}$, for $0 < x < 1$, and zero otherwise, where $\theta > 0$, then the moment estimator of θ is

[A] $\bar{X}$.

[B] $1/\bar{X}$.

[C] $(1-\bar{X})/\bar{X}$.

[D] $\bar{X}/(1-\bar{X})$

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46. Let X be sample of size 1 from discrete uniform distribution over the set $\{1, 2, \theta\}$, where $\theta \in \{3, 4\}$. For testing the hypothesis $H_0 : \theta = 3$ against $H_1 : \theta = 4$, the size and the power of the test which rejects H_0 if $X = 4$, are respectively.

[A] 0 and $\frac{1}{3}$.

[B] 0 and $\frac{1}{4}$.

[C] $\frac{1}{4}$ and $\frac{1}{3}$.

[D] $\frac{1}{4}$ and $\frac{1}{4}$.

47. C_i is the α_i, $i = 1, 2$, level critical region to test the null hypothesis H_0 against the alternative H_1, which of the following is true

[A] if $\alpha_1 < \alpha_2$ then $C_1 \subset C_2$.

[B] if $\alpha_1 < \alpha_2$ then $C_2 \subset C_1$.

[C] if $\alpha_1 < \alpha_2$ then $C_1 = C_2$.

[D] if $\alpha_1 \neq \alpha_2$ then C_1 and C_2 may be disjoint.

48. Let $f(x) = x^{1/x}$. Then the maximum value of f is

[A] $1/e$.

[B] e.

[C] $e^{1/e}$.

[D] 1.

49. Consider the linear programming problem

$$\begin{array}{lll} \text{minimise} & u_1 & +pu_2 \\ \text{subject to} & u_1 & \geq 0, u_2 \geq 0 \\ & 3u_1 & +u_2 \geq 3 \\ & u_1 & +2u_2 \geq 4 \\ & u_1 & +6u_2 \geq 6. \end{array}$$

For which value or values of p is there is no solution to this problem?

[A] only 2.

[B] only 0.

[C] any value less than 0.

[D] any value greater than 0.

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50. How many points with integer coordinates lie in the feasible region defined by

$$3x + 4y \leq 12, \quad x \geq 0, \quad y \geq 1.$$

[**A**] 3.

[**B**] 6.

[**C**] 8.

[**D**] 4.