2011

MATHEMATICS

Paper : 1.4

(Differential Equation)

Full Marks : 80

Time : 3 hours

The figures in the margin indicate full marks for the questions

PART—A

(Objective-type questions)

(*Marks* : 32)

All questions are compulsory. Each question carries four keys *(a)*, *(b)*, *(c)* and *(d)* of which one gives the correct answer. The key giving the correct answer is to be mentioned :

1. In general solution of the differential equation, the arbitrary constants are

(a) dependent

(b) independent

(c) dependent variables

(d) None of the above 2

2. The differential equation associated with the primitive $y = Ax^2 + Bx + C$ is given by

(a) $\frac{d^3y}{dx^3} = 0$

(b) $\frac{d^2y}{dx^2} = 0$

(c) $\frac{dy}{dx} - 2Ax - B = 0$

(d) $\frac{dy}{dx} + 2Ax + B = 0$ 3

3. In the differential equation

$$xy'' - 2(x+1)y' + (x+2)y = (x-2)e^{2x}$$

where $x > 0$, $y = e^x$ is a part of CF if

(a) $1 - P + Q = 0$

(b) $1 + P + Q = 0$

(c) $P + Qx + 1 = 0$

(d) $Px + Q + 1 = 0$ 3

4. If the functions $1, x, x^2$ are linearly independent, then the differential equation having them as its independent solutions is

(a) $y'' = 0$

(b) $y'' + 1 = 0$

(c) $y''' = 0$

(d) $y''' + 1 = 0$ 3

5. If both the functions $P(x)$ and $Q(x)$ are analytic at $x = x_0$, then the point $x = x_0$ in the equation $y'' + P(x)y' + Q(x) = 0$ is called

(a) an ordinary point

(b) a singular point

(c) a regular singular point

(d) an irregular point 2

6. The differential equation of the type $(1 - x^2)y'' - 2xy' + n(n+1)y = 0$ is called

(a) Bessel's equation

(b) Gauss's equation

(c) Legendre's equation

(d) Laguerre's equation 2

7. The solutions of the simultaneous equations

$$\frac{dx}{yz} = \frac{dy}{zx} = \frac{dz}{xy}$$

are given by

(a) $x - y = C_1$, $x - z = C_2$

(b) $x + y = C_1$, $x - z = C_2$

(c) $x^2 - y^2 = C_1,\ x^2 - z^2 = C_2$

(d) $x^2 + y^2 = C_1,\ x^2 - z^2 = C_2$ 3

8. The order of the partial differential equation

$$\frac{\partial^2 v}{\partial t^2} + k\left(\frac{\partial^3 v}{\partial x^3}\right)^2 = 0$$

where k is constant, is

(a) 1 (b) 2

(c) 5 (d) 6 2

9. In partial differential equations, if

$$p = \frac{\partial z}{\partial x},\ q = \frac{\partial z}{\partial y}$$

then the following notations are adopted

(a) $r = \frac{\partial^2 z}{\partial x^2},\ s = \frac{\partial^2 z}{\partial x \partial y},\ t = \frac{\partial^2 z}{\partial y^2}$

(b) $r = \frac{\partial^2 z}{\partial y^2},\ s = \frac{\partial^2 z}{\partial x \partial y},\ t = \frac{\partial^2 z}{\partial x^2}$

(c) $r = \frac{\partial^2 z}{\partial y^2},\ s = \frac{\partial^2 z}{\partial x^2},\ t = \frac{\partial^2 z}{\partial x \partial y}$

(d) $r = \frac{\partial^2 z}{\partial x \partial y},\ s = \frac{\partial^2 z}{\partial x^2},\ t = \frac{\partial^2 z}{\partial y^2}$ 2

10. The complete integral of the partial differential equation

$$z = px + qy + C\sqrt{1 + p^2 + q^2}$$

is

(a) $z = ax + by + C\sqrt{1 + p^2 + q^2}$

(b) $z = x + y + C\sqrt{1 + a^2 + b^2}$

(c) $z = ax + by + C\sqrt{1 + a^2 + b^2}$

(d) $z = a + b + C\sqrt{1 + a^2 + b^2}$ 2

11. The non-linear PDE of order one but of any degree can be solved by using

(a) Lagrange's method

(b) Charpit's method

(c) Cauchy's method

(d) Bernoulli's method 2

12. The equations $f(x, y, p, q) = 0$ and $g(x, y, p, q) = 0$ are compatible if

(a) $\dfrac{\partial(f, g)}{\partial(x, p)} + \dfrac{\partial(f, g)}{\partial(y, q)} = 0$

(b) $\dfrac{\partial(f, g)}{\partial(x, p)} - \dfrac{\partial(f, g)}{\partial(y, q)} = 0$

(c) $\dfrac{\partial(f, g)}{\partial(x, y)} + \dfrac{\partial(f, g)}{\partial(p, q)} = 0$

(d) $\dfrac{\partial(f, g)}{\partial(x, y)} - \dfrac{\partial(f, g)}{\partial(p, q)} = 0$ 3

13. The general solution of the PDE

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$$

is of the form

(a) $u = f(x + iy) + g(x - iy)$

(b) $u = f(x + iy) + g(x + iy)$

(c) $u = f(x + iy) - g(x + iy)$

(d) $u = f(x - iy) - g(x - iy)$ 3

PART—B

(Subjective-type questions)

(*Marks* : 48)

14. Answer any *two* parts : 6×2=12

(a) Solve :

$$\frac{d^2 y}{dx^2} - \frac{3}{x}\frac{dy}{dx} + \frac{3}{x^2} y = 2x - 1$$

(b) Solve :

$$\frac{d^2y}{dx^2}+4y=4\tan 2x$$

by using the method of variation of parameter.

(c) Use Wronskian to show that the functions x, x^2, x^3 are independent. Determine the differential equation with these as independent solutions.

15. Answer any *two* parts : 6×2=12

(a) Find the solution in series of

$$\frac{d^2y}{dx^2}+x\frac{dy}{dx}+x^2y=0$$

about $x=0$.

(b) Solve the simultaneous equations :

$$\frac{a\,dx}{(b-c)yz}=\frac{b\,dy}{(c-a)zx}=\frac{c\,dz}{(a-b)xy}$$

(c) Solve :

$$(yz+2x)\,dx+(zx-2z)\,dy+(xy-2y)\,dz=0$$

16. Answer any *two* parts : 6×2=12

(a) Find a partial differential equation by eliminating the constants a, b, c from

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$$

(b) Solve by Lagrange's method :

$$x(y-z)p+y(z-x)q=z(x-y)$$

(c) Find the integral surface of the partial differential equation

$$(x-y)p+(y-x-z)q=z$$

through the circle $z=1,\ x^2+y^2=1$.

17. Answer any *two* parts : 6×2=12

(a) Using Charpit's method, find a complete integral of the partial differential equation $px+qy=pq$.

(b) Prove that the complete integral of the equation $(px+qy-z)^2=1+p^2+q^2$ is $(ax+by+cz)=(a^2+b^2+c^2)^{1/2}$.

(c) Find a complete integral of the partial differential equation $z^2=pqxy$.

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