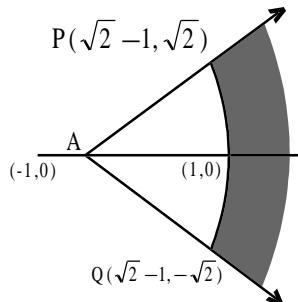


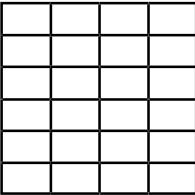
IIT - JEE 2005 Maths Question Paper (Screening)

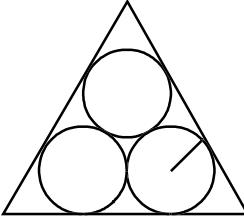
1. If $f(x)$ is a continuous and differentiable function and $f\left(\frac{1}{n}\right)=0 \forall n \geq 1$ and $n \in \mathbb{N}$, then
- (a) $f(x) = 0, x \in (0,1]$ (b) $f(0) = 0, f'(0) = 0$
 (c) $f'(0) = 0 = f''(0), x \in (0,1]$ (d) $f(0) = 0$ and $f'(0)$ need not be zero
2. A variable plane at a distance of 1 unit from the origin cuts the co-ordinate axes at A, B and C. If the centroid D (x, y, z) of triangle ABC satisfies the relation $\frac{1}{x^2} + \frac{1}{y^2} + \frac{1}{z^2} = k$, then the value of k is
- (a) 3 (b) 1 (c) $1/3$ (d) 9
3. If a, b, c are integers not all equal and ω is a cube root of unity ($\omega \neq 1$), then the minimum value of $|a + b\omega + c\omega^2|$ is
- (a) 0 (b) 1 (c) $\frac{\sqrt{3}}{2}$ (d) $\frac{1}{2}$
4. If $P(x)$ is a polynomial of degree less than or equal to 2 and S is the set of all such polynomials so that $P(1) = 1$. $P(0) = 0$ and $P'(x) > 0 \forall x \in [0, 1]$, then
- (a) $S = \emptyset$ (b) $S = \{(1 - a)x^2 + ax \mid 0 < a < 2\}$
 (c) $S = \{(1 - a)x^2 + ax \mid a \in (0, \infty)\}$ (d) $S = \{(1 - a)x^2 + ax \mid 0 < a < 1\}$
5. A circle is given by $x^2 + (y - 1)^2 = 1$, another circle C touches it externally and also the x-axis, then the locus of its centre is
- (a) $\{(x, y) : x^2 = 4y\} \cup \{(x, y) : y \leq 0\}$ (b) $\{(x, y) : x^2 + (y - 1)^2 = 4\} \cup \{(x, y) : y \leq 0\}$
 (c) $\{(x, y) : x^2 = y\} \cup \{(0, y) : y \leq 0\}$ (d) $\{(x, y) : x^2 = 4y\} \cup \{(0, y) : y \leq 0\}$
6. The locus of z which lies in shaded region is best represented by
- (a) $z : |z + 1| > 2, |\arg(z + 1)| < \pi/4$ (b) $z : |z - 1| > 2, |\arg(z - 1)| < \pi/4$
 (c) $z : |z + 1| < 2, |\arg(z + 1)| < \pi/2$ (d) $z : |z - 1| < 2, |\arg(z - 1)| < \pi/2$



7. $\cos(\alpha - \beta)$ and $\cos(\alpha + \beta) = 1/e$, where $\alpha, \beta \in [-\pi, \pi]$. Pairs of α, β which satisfy both the equations is/are
- (a) 0 (b) 1 (c) 2 (d) 4
8. In $\triangle ABC$, a, b, c are the lengths of its sides and A, B, C are the angles of triangle ABC. The correct relation is given by

- (a) $(b - c) \sin\left(\frac{B - C}{2}\right) = a \cos\frac{A}{2}$ (b) $(b - c) \cos\frac{A}{2} = a \sin\left(\frac{B - C}{2}\right)$
 (c) $(b + c) \sin\left(\frac{B + C}{2}\right) = a \cos\frac{A}{2}$ (d) $(b - c) \cos\left(\frac{A}{2}\right) = 2a \sin\left(\frac{B + C}{2}\right)$.

9. If $\int_{\sin x}^1 t^2 (f(t)) dt = (1 - \sin x)$, then $f\left(\frac{1}{\sqrt{3}}\right)$ is
- (a) $1/3$ (b) $1/\sqrt{3}$ (c) 3 (d) $\sqrt{3}$
10. The value of $\binom{30}{0}\binom{30}{10} - \binom{30}{1}\binom{30}{11} + \binom{30}{2}\binom{30}{12} - \dots + \binom{30}{20}\binom{30}{30}$ is, where $\binom{n}{r} = {}^n C_r$
- (a) $\binom{30}{10}$ (b) $\binom{30}{15}$ (c) $\binom{60}{30}$ (d) $\binom{31}{10}$
11. A rectangle with sides $2m - 1$ and $2n - 1$ is divided into squares of unit length by drawing parallel lines as shown in the diagram, then the number of rectangles possible with odd side lengths is
- 
- (a) $(m + n + 1)^2$ (b) 4^{m+n-1} (c) $m^2 n^2$ (d) $mn(m + 1)(n + 1)$
12. If $f(x)$ is a twice differentiable function and given that $f(1) = 1$, $f(2) = 4$, $f(3) = 9$, then
- (a) $f''(x) = 2$, for $\forall x \in (1, 3)$ (b) $f''(x) = f'(x) = 5$ for some $x \in (2, 3)$
 (c) $f''(x) = 3$, $\forall x \in (2, 3)$ (d) $f''(x) = 2$, for some $x \in (1, 3)$
13. The solution of primitive integral equation $(x^2 + y^2) dy = xy dx$, is $y = y(x)$. If $y(1) = 1$ and $y(x_0) = e$, then x_0 is
- (a) $\sqrt{2(e^2 - 1)}$ (b) $\sqrt{2(e^2 + 1)}$ (c) $\sqrt{3e}$ (d) $\sqrt{\frac{e^2 + 1}{2}}$
14. $\int_{-2}^0 (x^3 + 3x^2 + 3x + 3 + (x+1)\cos(x+1)) dx$ is equal to
- (a) -4 (b) 0 (c) 4 (d) 6
15. In the quadratic equation $ax^2 + bx + c = 0$, if $\Delta = b^2 - 4ac$ and $\alpha + \beta, \alpha^2 + \beta^2, \alpha^3 + \beta^3$ are in G.P. where α, β are the roots of $ax^2 + bx + c = 0$, then
- (a) $\Delta \neq 0$ (b) $b\Delta = 0$ (c) $c\Delta = 0$ (d) $\Delta = 0$
16. If the functions $f(x)$ and $g(x)$ are defined on $R \rightarrow R$ such that
- $$f(x) = \begin{cases} 0, & x \in \text{rational} \\ x, & x \in \text{irrational} \end{cases}, \quad g(x) = \begin{cases} 0, & x \in \text{irrational} \\ x, & x \in \text{rational} \end{cases}, \text{ then } (f - g)(x) \text{ is}$$
- (a) one-one and onto (b) neither one-one nor onto
 (c) one-one but not onto (d) onto but not one-one
17. The function given by $y = | |x| - 1 |$ is differentiable for all real numbers except the points
- (a) $\{0, 1, -1\}$ (b) ± 1 (c) 1 (d) -1
18. If $y = y(x)$ and it follows the relation $x \cos y + y \cos x = \pi$, then $y''(0)$
- (a) 1 (b) -1 (c) π (d) $-\pi$
19. X and Y are two sets and $f : X \rightarrow Y$. If $\{f(c) = y; c \subset X, y \rightarrow Y\}$ and $\{f^{-1}(d) = x; d \subset Y, x \subset X\}$, then the true statement is
- (a) $f(f^{-1}(b)) = b$ (b) $f^{-1}(f(a)) = a$ (c) $f(f^{-1}(b)) = b, b \subset y$ (d) $f^{-1}(f(a)) = a, a \subset x$

20. $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & -2 & 4 \end{bmatrix}$, $I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ and $A^{-1} = \left[\frac{1}{6}(A^2 + cA + dI) \right]$, then the value of c and d are
 (a) -6, -11 (b) 6, 11 (c) -6, 11 (d) 6, -11
21. The area bounded by the parabolas $y = (x + 1)^2$ and $y = (x - 1)^2$ and the line $y = 1/4$ is
 (a) 4 sq. units (b) 1/6 sq. units (c) 4/3 sq. units (d) 1/3 sq. units
22. Tangent to the curve $y = x^2 + 6$ at a point P(1, 7) touches the circle $x^2 + y^2 + 16x + 12y + c = 0$ at a point Q.
 Then the coordinates of Q are
 (a) (-6, -11) (b) (-9, -13) (c) (-10, -15) (d) (-6, -7)
23. If $P = \begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$, $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ and $Q = PAP^T$ and $x = P^T Q^{2005} P$, then x is equal to
 (a) $\begin{bmatrix} 1 & 2005 \\ 0 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} 4+2005\sqrt{3} & 6015 \\ 2005 & 4-2005\sqrt{3} \end{bmatrix}$ (c) $\frac{1}{4} \begin{bmatrix} 2+\sqrt{3} & 1 \\ -1 & 2-\sqrt{3} \end{bmatrix}$ (d) $\frac{1}{4} \begin{bmatrix} 2005 & 2-\sqrt{3} \\ 2+\sqrt{3} & 2005 \end{bmatrix}$
24. For the primitive integral equation $ydx + y^2dy = x dy$; $x \in R$, $y > 0$, $y = y(x)$, $y(1)=1$, then $y(-3)$ is
 (a) 3 (b) 2 (c) 1 (d) 5
25. In an equilateral triangle, 3 coins of radii 1 unit each are kept so that they touch each other and also the sides of the triangle. Area of the triangle is
- 
- (a) $4+2\sqrt{3}$ (b) $6+4\sqrt{3}$ (c) $12+\frac{7\sqrt{3}}{4}$ (d) $3+\frac{7\sqrt{3}}{4}$
26. The minimum area of triangle formed by the tangent to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ and coordinate axes is
 (a) ab sq. units (b) $\frac{a^2 + b^2}{2}$ sq. units (c) $\frac{(a+b)^2}{2}$ sq. units (d) $\frac{a^2 + ab + b^2}{3}$ sq. units
27. If $\vec{a}, \vec{b}, \vec{c}$ are three non-zero, non-coplanar vectors are $\vec{b}_1 = \vec{b} - \frac{\vec{b} \cdot \vec{a}}{|\vec{a}|^2} \vec{a}$, $\vec{b}_2 = \vec{b} + \frac{\vec{b} \cdot \vec{a}}{|\vec{a}|^2} \vec{a}$,
 $\vec{c}_1 = \vec{c} - \frac{\vec{c} \cdot \vec{a}}{|\vec{a}|^2} \vec{a} + \frac{\vec{b} \cdot \vec{c}}{|\vec{c}|^2} \vec{b}_1$, $\vec{c}_2 = \vec{c} - \frac{\vec{c} \cdot \vec{a}}{|\vec{a}|^2} \vec{a} - \frac{\vec{b}_1 \cdot \vec{c}}{|\vec{b}_1|^2} \vec{b}_1$, $\vec{c}_3 = \vec{c} - \frac{\vec{c} \cdot \vec{a}}{|\vec{c}|^2} \vec{a} + \frac{\vec{b} \cdot \vec{c}}{|\vec{c}|^2} \vec{b}_1$, $\vec{c}_4 = \vec{c} - \frac{\vec{c} \cdot \vec{a}}{|\vec{c}|^2} \vec{a} - \frac{\vec{b} \cdot \vec{c}}{|\vec{b}|^2} \vec{b}_1$, then the set of orthogonal vector is
 (a) $(\vec{a}, \vec{b}_1, \vec{c}_3)$ (b) $(\vec{a}, \vec{b}_1, \vec{c}_2)$ (c) $(\vec{a}, \vec{b}_1, \vec{c}_1)$ (d) $(\vec{a}, \vec{b}_2, \vec{c}_2)$
28. A six faced fair dice is thrown until 1 comes, then the probability that 1 comes in even no.of trials is
 (a) 5/11 (b) 5/6 (c) 6/11 (d) 1/6