(6 pages)

MAY 2011 **P/ID 37454/PMAD**

Time : Three hours Maximum : 100 marks

SECTION A — ($10 \times 2 = 20$ marks)

Answer ALL the questions.

Each question carries 2 marks.

1. A coin is tossed three times. What is the probability that heads appear twice?

2. Define the moment of order k.

3. The random variable X can take on the values $x_1 = -1$ and $x_2 = +1$ with probabilities $P(X = -1) = P(X = +1) = 0.5$. Determine the characteristic function of this random variable.

4. Define two-point distribution.

5. State Poisson law of large numbers.

6. State Borel-Cantelli Lemma.

7. Define parametric hypothesis and parametric test.

8. Define Pearson's χ^2 statistic.

9. Define null hypothesis and alternate hypothesis.

10. State fundamental lemma.

SECTION B — (5 × 6 = 30 marks)

Answer ALL questions.

Each question carries 6 marks.

11. (a) Let X be a random variable with the distribution function *F(x,y)*. Find the distribution of $Y = X^2$.

Or

(b) State and prove Chebyshev's inequality.

12. (a) The joint distribution of the random variable *(X, Y)* is given by the density.

$$f(x, y) = \begin{cases} \frac{1}{4}\left[1 + xy\,(x^2 - y^2)\right] & \text{, for } |x| \leq \text{ and } |y| \leq 1 \\ 0 & \text{for all other points} \end{cases}$$

Show that X and Y are not independent.

Or

(b) The random variable X has the distribution $N(1;2)$. Find the probability that X is greater than 3 in absolute value.

13. (a) State and prove the Bernoulli law of large numbers.

Or

(b) The random variables X_k $(k = 1, 2, \ldots\ldots, 16)$ are independent and have the same density $f(x) = \frac{1}{2\sqrt{2\pi}} \exp\left[-\frac{1}{2} \cdot \frac{(x-1)^2}{4}\right]$.

Find the density of

$$\overline{X} = \frac{1}{16} \sum_{k=1}^{16} X_k .$$

Also find $P\,(0 \le \overline{X} \le 2)$.

14. (a) We have good and defective items in a lot and the proportion p of the defective items is unknown. From this lot, draw a simple sample of size n = 30. Assign the number one to the appearance of a defective item and the number zero to the appearance of a good item. Suppose that there are four defective items in the sample. Test the hypothesis $H_0 (p = 0.10)$. Should we reject H_0 at the significance level $\alpha = 0.05$?

Or

(b) Consider a population in which the characteristic X has an arbitrary distribution whose first moment exists. Estimate the unknown expected value $E(X) = m$.

15. (a) The expected value m of a population, where the characteristic X has the normal distribution $N(m;\ 1)$, is unknown. Test the hypothesis $H_0(m = m_0)$ against the alternate hypothesis $H_1(m = m_1)$, where $m_1 > m_0$, by applying a most powerful test based on a simple sample for a given α and β. What value should n have?

Or

(b) Derive the OC function $L(Q)$ of the sequential probability ratio test.

SECTION C — (5 × 10 = 50 marks)

Answer ALL the questions.

Each question carries 10 marks.

16. (a) Prove that the equality $\rho^2 = 1$ is a necessary and sufficient condition for the relation $P(Y = aX + b) = 1$ to hold.

Or

(b) Let $\{A_n\}, n = 1, 2, \ldots\ldots,$ be a non increasing sequence of events and let A be their product. Prove that $P(A) = \lim_{n\to\infty} P(A_n)$.

17. (a) If the l th moment m_l of a random variable exists, prove that it is expressed by

$$m_l = \frac{\phi^l(0)}{i^l},$$

where $\phi^l(0)$ is the l th derivative of the characteristic function $\phi(t)$ of this random variable at $t = 0$.

Or

(b) Prove by means of an example that the values of the characteristic function in a finite interval do not uniquely determine the distribution.

18. (a) State and prove de Moivre-Laplace theorem.

Or

(b) State and prove Lapunov theorem.

19. (a) State and prove Smirnov theorem.

Or

(b) State and prove Rao-Cramer inequality.

20. (a) Explain uniformly most powerful test.

Or

(b) State and prove Wald's fundamental identity.