Date : 11-04-2010 **Duration : 3 Hours** **Max. Marks : 252**

PAPER - 1

Please read the instructions carefully. You are allotted 5 minutes specifically for this purpose.

INSTRUCTIONS

A. General :

1. This Question Paper contains 32 pages having 84 questions.
2. The question paper CODE is printed on the right hand top corner of this sheet and also on the back page of this booklet.
3. No additional sheets will be provided for rough work.
4. Blank paper, clipboard, log tables, slide rules, calculators, cellular phones, pagers and electronic gadgets in any form are not allowed.
5. The answer sheet, a machine-gradable **Objective Response Sheet (ORS)**, is provided separately.
6. Do not Tamper / mutilate the **ORS** or this booklet.
7. Do not break the seals of the question-paper booklet before instructed to do so by the invigilators.

B. Filling the bottom-half of the ORS :

8. The ORS has **CODE** printed on its lower and upper Parts.
9. Make sure the **CODE** on the ORS is the same as that on this booklet. **If the Codes do not match, ask for a change of the Booklet.**
10. Write your Registration No., Name and Name of centre and sig with pen in appropriate boxes. **Do not write these anywhere else.**
11. Darken the appropriate bubbles below your registration number with HB **Pencil**.

C. Question paper format and Marking scheme :

12. The question paper consists of 3 parts (Chemistry, Mathematics and Physics). Each part consists of four Sections.
13. For each question is **Section–I,** you will be **awarded 3 marks** if you have darkened only the bubble corresponding to the correct answer and **zero mark** if no bubbles are darkened. In all other cases, **minus one (–1)** will be awarded.
14. For each question is **Section–II,** you will be **awarded 3 marks** if you have darken only the bubble corresponding to the correct answer and **zero mark** if no bubbles are darkened. Partial marks will be answered for partially correct answers. No negative marks will be awarded in this Section.
15. For each question is **Section–III,** you will be **awarded 3 marks** if you have darken only the bubble corresponding to the correct answer and **zero mark** if no bubbles are darkened. In all other cases, **minus one (–1)** will be awarded.
16. For each question is **Section–IV,** you will be **awarded 3 marks** if you darken the bubble corresponding to the correct answer and **zero mark** if no bubbles is darkened. **No negative marks** will be awarded for in this Section.

Useful Data :

Atomic Numbers : Be 4; N 7; O 8; Al 13 ; Si 14; Cr 24 ; Fe 26; Fe 26; Zn 30; Br 35.

1 amu = 1.66×10^{-27} kg

$h = 6.626 \times 10^{-34}$ J s

$m_e = 9.1 \times 10^{-31}$ kg

$c = 3.0 \times 10^{8}$ m s^{-1}

$R_H = 2.18 \times 10^{-18}$ J

R = 0.082 L-atm K^{-1} mol^{-1}

$N_A = 6.022 \times 10^{23}$

$e = 1.6 \times 10^{-19}$ C

F = 96500 C mol^{-1}

$4\pi\varepsilon_0 = 1.11 \times 10^{-10}$ J^{-1} C^2 m^{-1}

PART- I

CHEMISTRY

SECTION - I

Single Correct Choice Type

This section contains 8 multiple choice questions. Each question has four choices (A), (B), (C) and (D) out of which ONLY ONE is correct.

1. The species which by definition has **ZERO** standard molar enthalpy of formation at 298 K is :

(A) $Br_2(g)$ (B) $Cl_2(g)$

(C) $H_2O(g)$ (D) $CH_4(g)$

Ans. (B)

Sol. ΔH_f° $(Cl_2, g) = 0$, As ΔH_f° of elements in their standard state is taken to be zero.

2. The bond energy (in **kcal mol^{-1}**) of a C–C single bond is approximately :

(A) 1 (B) 10

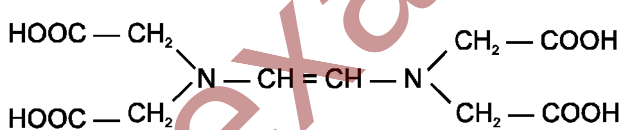
(C) 100 (D) 1000

Ans. (C)

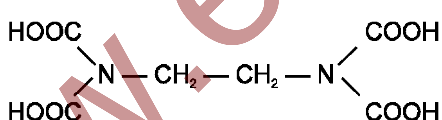
Sol. $E_{C-C} \cong 100$ KCal/mole.

3. The correct structure of ethylenediaminetetraacetic acid (EDTA) is :

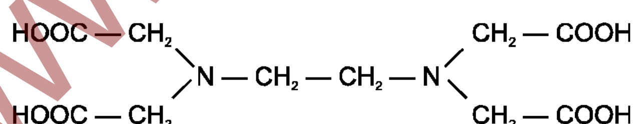
(A)



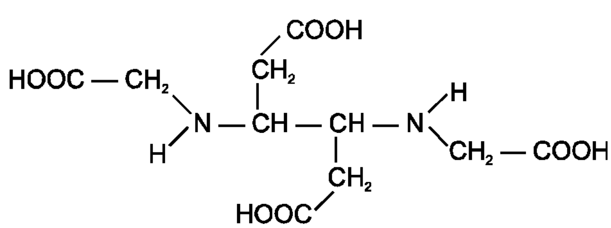
(B)



(C)

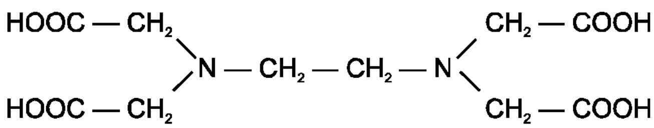


(D)



Ans. (C)

Sol. Structure of EDTA is :



4. The ionization isomer of $[Cr(H_2O)_4Cl(NO_2)]Cl$ is :

(A) $[Cr(H_2O)_4(O_2N)]Cl_2$ (B) $[Cr(H_2O)_4Cl_2](NO_2)$

(C) $[Cr(H_2O)_4Cl(ONO)]Cl$ (D) $[Cr(H_2O)_4Cl_2(NO_2)].H_2O$

Ans. **(B)**

Sol. The ionisation isomer for the given compound will be obtained by exchanging ligand with counter ion as : $[Co(H_2O)_4Cl_2](NO_2)$.

5. The synthesis of 3-octyne is achieved by adding a bromoalkane into a mixture of sodium amide and an alkyne. The bromoalkane and alkyne respectively are :

(A) $BrCH_2CH_2CH_2CH_2CH_3$ and $CH_3CH_2C\equiv CH$ (B) $BrCH_2CH_2CH_3$ and $CH_3CH_2CH_2C\equiv CH$

(C) $BrCH_2CH_2CH_2CH_2CH_3$ and $CH_3C\equiv CH$ (D) $BrCH_2CH_2CH_2CH_3$ and $CH_3CH_2C\equiv CH$

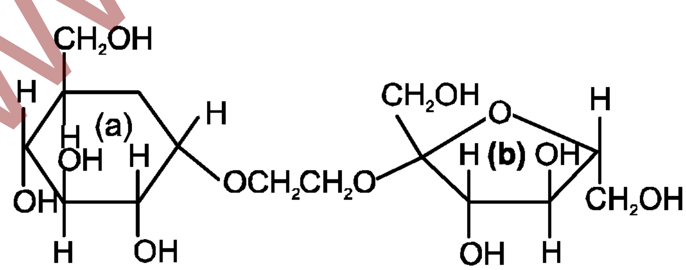
Ans. **(D)**

Sol. $CH_3–CH_2–C\equiv C–H \xrightarrow{NaNH_2} CH_3–CH_2–C\equiv C^{\ominus} \xrightarrow{CH_3–CH_2–CH_2–CH_2–Br}$

$CH_3–CH_2–C\equiv C–CH_2–CH_2–CH_2–CH_3$

3–octyne

6. The correct statement about the following disaccharide is :

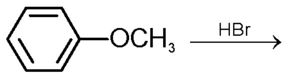


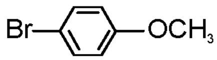
(A) Ring **(a)** is pyranose with α-glycosidic link (B) Ring **(a)** is furanose with α-glycosidic link

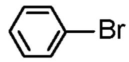
(C) Ring **(b)** is furanose with α-glycosidic link (D) Ring **(b)** is pyranose with β-glycosidic link

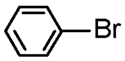
Ans. **(A)**

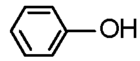
Sol. Ring (a) is pyranose with α-glycosidic linkage and ring (b) is furanose with β-glycosidic linkage.

7. In the reaction  the products are :

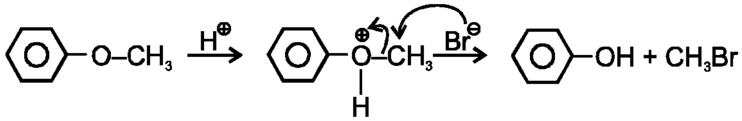
(A)  and H_2

(B)  and CH_3Br

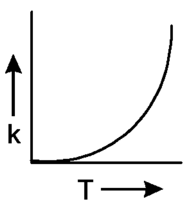
(C)  and CH_3OH

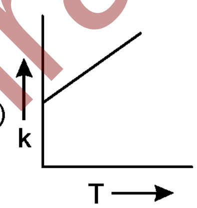
(D)  and CH_3Br

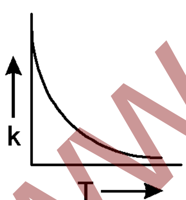
Ans. **(D)**

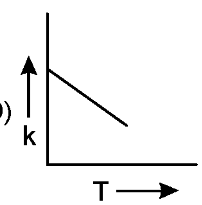
Sol. 

8. Plots showing the variation of the rate constant (k) with temperature (T) are given below. The plot that follows Arrhenius equation is :

(A) 

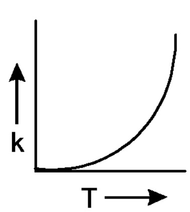
(B) 

(C) 

(D) 

Ans. **(A)**

Sol. $k = Ae^{-E_a/RT}$

So, variation will be 

SECTION - II

Multiple Correct Correct Type

This section contains 5 multiple choice questions. Each question has four choices (A), (B), (C) and (D) out of which ONE OR MORE may be correct.

9. Aqueous solutions of HNO_3, KOH, CH_3COOH, and CH_3COONa of identical concentrations are provided. The pair (s) of solutions which form a buffer upon mixing is (are) :

(A) HNO_3 and CH_3COOH (B) KOH and CH_3COONa

(C) HNO_3 and CH_3COONa (D) CH_3COOH and CH_3COONa

Ans. (C,D)

Sol. (C) $HNO_3 + CH_3COONa$ mixture can act as buffer solution if volume of HNO_3 solution taken is lesser than volume of CH_3COONa solution because of following reaction :

$CH_3COONa + HNO_3 \longrightarrow CH_3COOH + NaNO_3$

(D) $CH_3COOH + CH_3COONa$ - mixture will act as buffer.

10. Among the following, the intensive property is (properties are) :

(A) molar conductivity (B) electromotive force

(C) resistance (D) heat capacity

Ans. (A,B)

Sol. Molar conductivity and electromotive force (emf) are intensive properties as these are size independent.

11. The reagent (s) used for softening the temporary hardness of water is (are) :

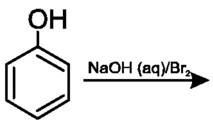
(A) $Ca_3(PO_4)_2$ (B) $Ca(OH)_2$

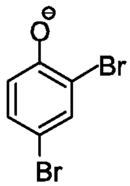
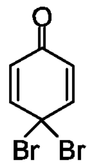
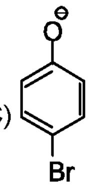
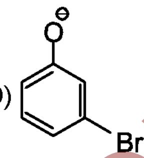
(C) Na_2CO_3 (D) NaOCl

Ans. (B,C)

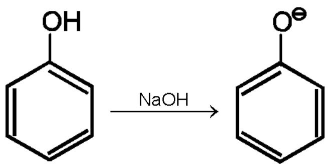
Sol. (B) $HCO_3^- + OH^- \longrightarrow H_2O + CO_3^{2-}$ and $CaCO_3$ will get precipitated.

(C) $CO_3^{2-} + Ca^{2+} \longrightarrow CaCO_3$, $CaCO_3$ (white) precipitated.

12. In the reaction  the intermediate (s) is (are) :

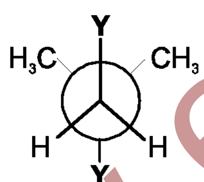
(A)  (B)  (C)  (D) 

Ans. (A,C)

Sol. 

It is ortho-para directing.

13. In the Newman projection for 2, 2-dimethylbutane

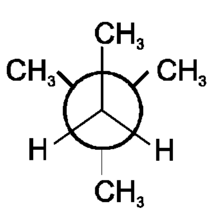
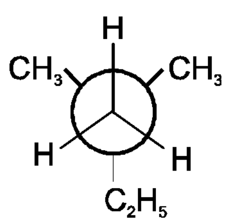


X and **Y** can respectively be :

(A) H and H (B) H and C_2H_5

(C) C_2H_5 and H (D) CH_3 and CH_3

Ans. (B,D)

Sol.  $\equiv CH_3-\underset{CH_3}{\overset{CH_3}{C}}-CH_2-CH_3$,  $\equiv CH_3-\underset{CH_3}{\overset{CH_3}{C}}-C_2H_5$

SECTION - III

Comprehension Type

This section contains 2 Paragraphs. Based upon the first paragraph 3 multiple choice questions and based upon the second paragraph 3 multiple choice questions have to be answered. Each of these questions has four choices (A), (B), (C) and (D) out of which ONLY ONE is correct.

Paragraph for Question Nos. 14 to 16

Copper is the most noble of the first row transition metals and occurs in small deposits in several countries, Ores of copper include chalcanthite ($CuSO_4.5H_2O$), atacamite ($Cu_2Cl(OH)_3$), cuprite (Cu_2O), copper glance (Cu_2S) and malachite ($Cu_2(OH)_2CO_3$). However, 80% of the world copper production comes from the ore chalcopyrite ($CuFeS_2$). The extraction of copper from chalcopyrite involves partial roasting, removal of iron and self-reduction.

14. Partial roasting of Chalcopyrite produces :

(A) Cu_2S and FeO
(B) Cu_2O and FeO
(C) CuS and Fe_2O_2
(D) Cu_2O and Fe_2O_2

Ans. **(A)**

Sol. $2\,CuFeS_2 + 4O_2 \longrightarrow Cu_2S + 2FeO + 3SO_2$

15. Iron is removed from chalcopyrite as :

(A) FeO
(B) FeS
(C) Fe_2O_3
(D) $FeSiO_3$

Ans. **(D)**

Sol. Iron is removed in the form of slag of $FeSiO_3$

$FeO + SiO_2 \longrightarrow FeSiO_3$

16. In self-reduction, the reduction species is :

(A) S
(B) O^{2-}
(C) S^{2-}
(D) SO_2

Ans. **(C)**

Sol. S^{2-} acts as reducing species in self reduction reaction

$2Cu_2O + Cu_2S \longrightarrow 6Cu + SO_2$

Paragraph for Question Nos. 17 to 18

The concentration of potassium ions inside a biological cell is at least twenty times higher than the outside. The resulting potential difference across the cell is important in several processes such as transmission of nerve impulses and maintaining the ion balance. A simple model for such a concentration cell involving a metal M is :

$M(s) \mid M^+ (aq ; 0.05 \text{ molar}) \parallel M^+ (aq ; 1 \text{ molar}) \mid M(s)$

For the above electrolytic cell the magnitude of the cell potential $|E_{cell}| = 70$ mV.

17. For the above cell :

(A) $E_{cell} < 0$; $\Delta G > 0$
(B) $E_{cell} > 0$; $\Delta G < 0$
(C) $E_{cell} < 0$; $\Delta G^o > 0$
(D) $E_{cell} > 0$; $\Delta G^o < 0$

Ans. **(B)**

Sol. $M (s) \mid M^+ (aq, 0.05\ M) \parallel M^+ (aq, 1\ M) \mid M(s)$

Anode : $M (s) \longrightarrow M^+ (aq) + e^-$

Cathode : $M^+ (aq) + e^- \longrightarrow M (s)$

$$M^+ (aq)|_c \rightleftharpoons M^+ (aq)|_a$$

$$E_{cell} = E^\circ_{cell} - \frac{0.0591}{1} \log \frac{M^+ (aq)|_a}{M^+ (aq)|_c}$$

$$= 0 - \frac{0.0591}{1} \log \left\{\frac{0.05}{1}\right\}$$

$= +$ ve $= 70$ mV and hence $\Delta G = -nFE_{cell} = -$ ve.

18. If the 0.05 molar solution of M^+ is replaced by a 0.0025 molar M^+ solution, then the magnitude of the cell potential would be :

(A) 35 mV
(B) 70 mV
(C) 140 mV
(D) 700 mV

Ans. **(C)**

Sol. $$E_{cell} = \frac{-0.0591}{1} \log \left\{\frac{0.0025}{1}\right\} = -\frac{0.0591}{1} \log \left\{\frac{0.05}{20}\right\}$$

$$= 70 \text{ mV} + \frac{0.0591}{1} \log 20 = 140 \text{ mV}.$$

SECTION - IV

(Integer Type)

This section contains TEN questions. The asnwer to each question is a single-digit integer, ranging from 0 to 9. The correct digit below the question number in the ORS is to be bubbled.

19. The concentration of R in the reaction $R \rightarrow P$ was measured as a function of time and the following data is obtained :

[R](molar)	1.0	0.75	0.40	0.10
t(min.)	0.0	0.05	0.12	0.18

The order of the reaction is :

Ans. 0

Sol. $K = \dfrac{C_0 - C}{t} = \dfrac{1 - 0.75}{0.05} = \dfrac{0.25}{0.05} = 5$

$K = \dfrac{0.75 - 0.40}{0.07} = \dfrac{0.35}{0.07} = 5$

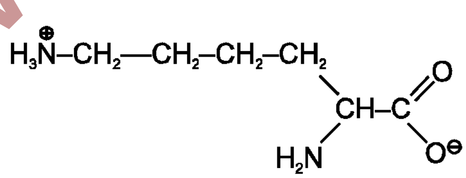
So, reaction must be of zero order.

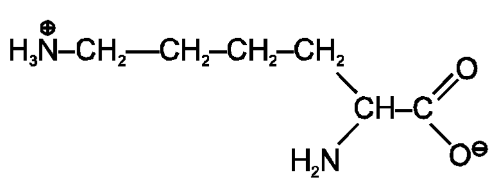
20. The number of neutrons emitted when ${}^{235}_{92}U$ undergoes controlled nuclear fission to ${}^{142}_{54}Xe$ and ${}^{90}_{38}Sr$ is :

Ans. 3

Sol. ${}^{235}_{92}U \longrightarrow {}^{142}_{54}Xe + {}^{90}_{38}Sr + 3\,{}^{1}_{0}n$

21. The total number of basic groups in the following form of lysine is :



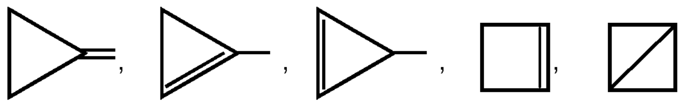


Ans. 2

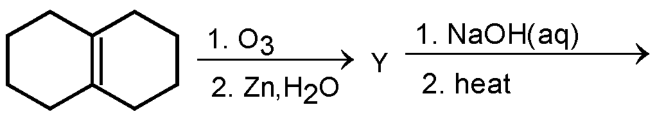
Sol. $-NH_2$ and $-COO^{\ominus}$ are two basic groups.

22. The total number of cyclic isomers possible for a hydrocarbon with the molecular formula C_4H_6 is :

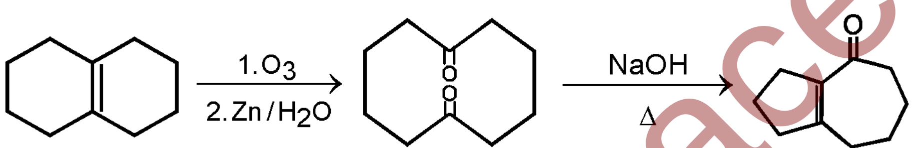
Ans. 5

Sol. 

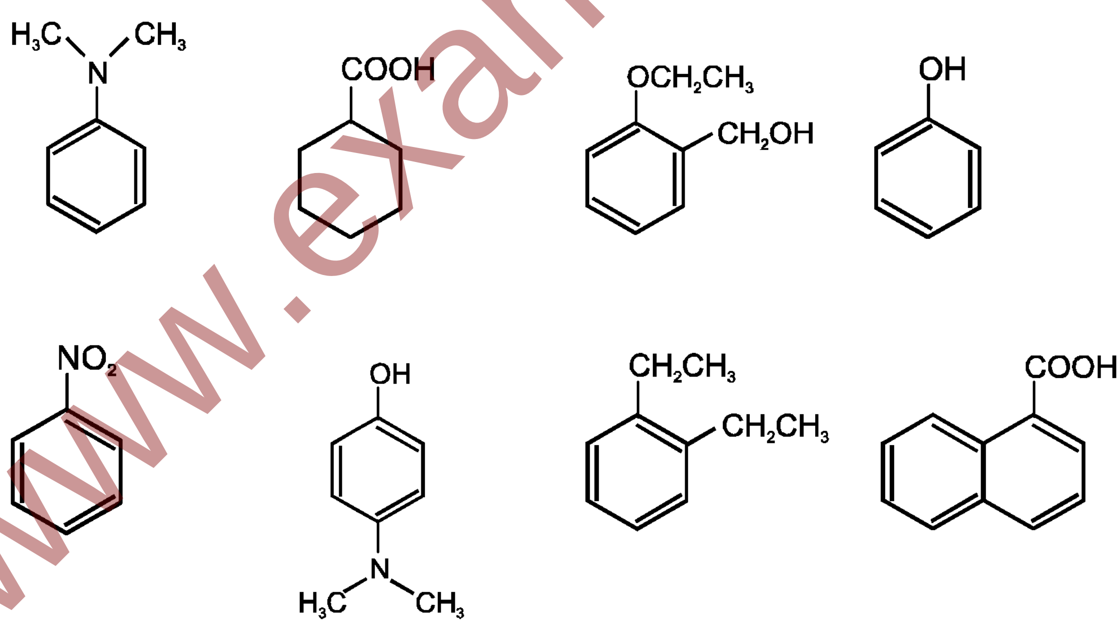
23. In the scheme given below, the total number of intramolecular aldol condensation products formed from 'Y' is:



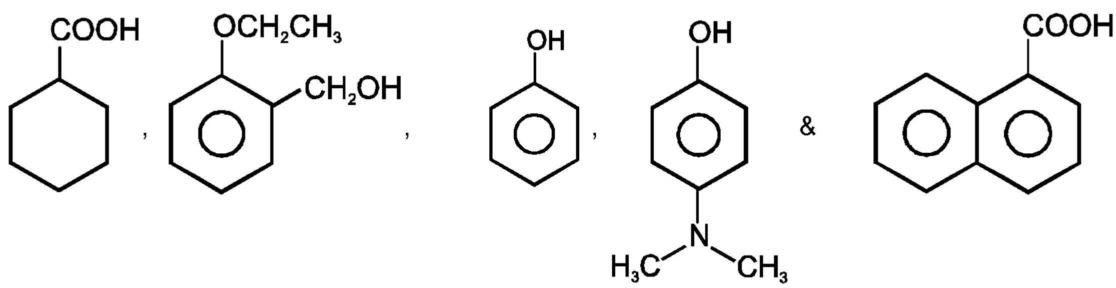
Ans. 1

Sol. 

24. Amongst the following, the total number of compounds soluble in aqueous NaOH is :



Ans. 5

Sol. 

are soluble in aqueous NaOH.

25. Amongst the following, the total number of compounds whose aqueous solution turns red litmus paper blue is :

KCN K_2SO_4 $(NH_4)_2C_2O_4$ $NaCl$ $Zn(NO_3)_2$

$FeCl_3$ K_2CO_3 NH_4NO_3 $LiCN$

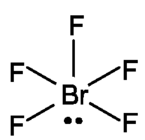
Ans. **3**

Sol. Basic solutions will convert red litmus blue.

$\left.\begin{array}{l} KCN \\ K_2CO_3 \\ LiCN \end{array}\right\}$, their aqueous solution will be basic due to anionic hydrolysis.

26. Based on VSEPR theory, the number of 90 degree F–Br–F angles is BrF_5 is :

Ans. **8**

Sol. 

27. The value of n in the molecular formula $Be_nAl_2Si_6O_{18}$ is :

Ans. **3**

Sol. $Be_nAl_2Si_6O_{18}$. The value of n = 3 by charge balancing.

$(Be^{2+})_3\ (Al^{3+})_2\ (Si_6O_{18})^{12-}$ $[2 \times n + 2\ (+3) + (-12) = 0\ \therefore\ n = 3]$.

28. A student performs a titration with different burettes and finds titre values of 25.2 mL, 25.25 mL, and 25.0 mL. The number of significant figures in the average titre value is :

Ans. **3**

Sol. Average titre value = $\dfrac{25.2 + 25.25 + 25.0}{3} = \dfrac{75.45}{3}$ = 25.15 = 25.2 mL

number of significant figures will be 3.

PART-II

SECTION - I

Single Correct Choice Type

This section contains 8 multiple choice questions. Each question has four choices (A), (B), (C) and (D) out of which ONLY ONE is correct.

29. Let p and q be real numbers such that $p \neq 0$, $p^3 \neq q$ and $p^3 \neq -q$. If α and β are nonzero complex numbers satisfying $\alpha + \beta = -p$ and $\alpha^3 + \beta^3 = q$, then a quadratic equation having $\frac{\alpha}{\beta}$ and $\frac{\beta}{\alpha}$ as its roots is

(A) $(p^3 + q)x^2 - (p^3 + 2q)x + (p^3 + q) = 0$
(B) $(p^3 + q)x^2 - (p^3 - 2q)x + (p^3 + q) = 0$
(C) $(p^3 - q)x^2 - (5p^3 - 2q)x + (p^3 - q) = 0$
(D) $(p^3 - q)x^2 - (5p^3 + 2q)x + (p^3 - q) = 0$

Ans. (B)

Sol. Product = 1

Sum = $\dfrac{\alpha^2 + \beta^2}{\alpha\beta} = \dfrac{(\alpha+\beta)^2 - 2\alpha\beta}{\alpha\beta}$

Since $\alpha^3 + \beta^3 = q \quad \Rightarrow \quad -p(\alpha^2 + \beta^2 - \alpha\beta) = q$

$((\alpha + \beta)^2 - 3\alpha\beta) = -\dfrac{q}{p} \quad \Rightarrow \quad p^2 + \dfrac{q}{p} = 3\alpha\beta$

Hence sum = $\dfrac{\left\{p^2 - \frac{2}{3}\left(\frac{p^3+q}{p}\right)\right\}3p}{(p^3+q)} = \dfrac{p^3 - 2q}{p^3 + q}$

so the equation is $x^2 + \left(\dfrac{p^3 - 2q}{p^3 + q}\right)x + 1 = 0$

$\Rightarrow \quad (p^3 + q)x^2 - (p^3 - 2q)x + (p^3 + q) = 0$

30. Let f, g and h be real-valued functions defined on the interval [0, 1] by $f(x) = e^{x^2} + e^{-x^2}$, $g(x) = xe^{x^2} + e^{-x^2}$ and $h(x) = x^2e^{x^2} + e^{-x^2}$. If a, b and c denote, respectively, the absolute maximum of f, g and h on [0, 1], then

(A) $a = b$ and $c \neq b$
(B) $a = c$ and $a \neq b$
(C) $a \neq b$ and $c \neq d$
(D) $a = b = c$

Ans. (D)

Sol. Clearly $f(x) = e^{x^2} + e^{-x^2}$

$f'(x) = 2x\,(e^{x^2} - e^{-x^2}) \geq 0$ increasing $\Rightarrow$ $f_{max} = f(1) = e + \frac{1}{e}$

$g(x) = x\,e^{x^2} + e^{-x^2}$ $\Rightarrow$ $g(x) = e^{x^2} + 2x^2 e^{x^2} - 2x\,e^{-x^2} > 0$ increasing $\Rightarrow$ $g_{max} = g(1) = e + \frac{1}{e}$

$h(x) = x^2\,e^{x^2} + e^{-x^2}$ $\Rightarrow$ $h'(x) = 2x\,e^{x^2} + 2x^3 e^{x^2} - 2x\,e^{-x^2} = 2x\left(e^{x^2} + x^2 e^{x^2} - e^{-x^2}\right) > 0$

$\Rightarrow$ $h_{max} = h(1) = e + \frac{1}{e}$,

so a = b = c

31. If the angle A, B and C of a triangle are in arithmetic progression and if a, b and c denote the lengths of the sides opposite to A, B and C respectively, then the value of the expression $\frac{a}{c}\sin 2C + \frac{c}{a}\sin 2A$ is

(A) $\frac{1}{2}$ (B) $\frac{\sqrt{3}}{2}$ (C) 1 (D) $\sqrt{3}$

Ans. (D)

Sol. $\frac{a}{c}\sin 2C + \frac{c}{a}\sin 2A = \frac{2}{2R}(a\cos C + c\cos A) = \frac{b}{R} = 2\sin B = 2\sin 60^\circ = \sqrt{3}$

32. Equation of the plane containing the straight line $\frac{x}{2} = \frac{y}{3} = \frac{z}{4}$ and perpendicular to the plane containing the straight line $\frac{x}{3} = \frac{y}{4} = \frac{z}{2}$ and $\frac{x}{4} = \frac{y}{2} = \frac{z}{3}$ is

(A) $x + 2y - 2z = 0$ (B) $3x + 2y - 2z = 0$ (C) $x - 2y + z = 0$ (D) $5x + 2y - 4z = 0$

Ans. (C)

Sol. Direction ratio of normal to plane containing the straight line

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 4 & 2 \\ 4 & 2 & 3 \end{vmatrix} = 8\hat{i} - \hat{j} - 10\hat{k}$$

Required plane $\begin{vmatrix} x-0 & y-0 & z-0 \\ 2 & 3 & 4 \\ 8 & -1 & -10 \end{vmatrix} = 0$ $\Rightarrow$ $-26x + 52y - 26z = 0$ $\Rightarrow$ $x - 2y + z = 0$

33. Let ω be a complex cube root of unity with $\omega \neq 1$. A fair die is thrown three times. If r_1, r_2 and r_3 are the numbers obtained on the die, then the probability that $\omega^{r_1} + \omega^{r_2} + \omega^{r_3} = 0$ is

(A) $\frac{1}{18}$ (B) $\frac{1}{9}$ (C) $\frac{2}{9}$ (D) $\frac{1}{36}$

Ans. (C)

Sol. $\omega^{r_1} + \omega^{r_2} + \omega^{r_3} = 0$; r_1, r_2, r_3 are to be selected form {1, 2, 3, 4, 5, 6}

As we know that $1 + \omega + \omega^2 = 0$

$\therefore$ form r_1, r_2, r_3, one has remainder 1, other has remainder 2 and third has remainder 0 when divided by 3.

$\therefore$ we have to select r_1, r_2, r_3 form (1, 4) or (2, 5) or (3, 6) which can be done in ${}^2C_1 \times {}^2C_1 \times {}^2C_1$ ways

value of r_1, r_2, r_3 can be interchanged in 3! ways.

$\therefore$ required probability = $\frac{({}^2C_1 \times {}^2C_1 \times {}^2C_1) \times 3!}{6 \times 6 \times 6} = \frac{2}{9}$

34. Let P, Q, R and S be the points on the plane with position vectors $-2\hat{i} - \hat{j}$, $4\hat{i}$, $3\hat{i} + 3\hat{j}$ and $-3\hat{i} + 2\hat{j}$ respectively. The quadrilateral PQRS must be a

(A) parallelogram, which is neither a rhombus nor a rectangle

(B) square

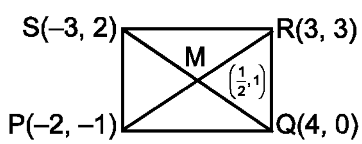
(C) rectangle, but not a square

(D) rhombus, but not a square

Ans. (A)

Sol. PQ = $\sqrt{36+1} = \sqrt{37}$ = RS, PQ $\neq$ PS

PS = $\sqrt{1+9} = \sqrt{10}$ = QR



slope of PQ = $\frac{1}{6}$, slope of PS = – 3

PQ is not $\perp$ to PS

So it is parallelogram, which is neither a rhombus nor a rectangle

35. The number of 3 × 3 matrices A whose entries are either 0 or 1 and for which the system $A\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ has exactly two distinct solutions, is

(A) 0 (B) $2^9 - 1$ (C) 168 (D) 2

Ans. (A)

Sol. $a_1x + b_1y + c_1z = 1$

$a_2x + b_2y + c_2z = 0$

$a_3x + b_3y + c_3z = 0$

No three planes can meet at two distinct points. So number of matrices is 0

36. The value of $\lim\limits_{x\to 0} \dfrac{1}{x^3}\displaystyle\int_0^x \dfrac{t\ell n(1+t)}{t^4+4}\,dt$ is

(A) 0 (B) $\dfrac{1}{12}$ (C) $\dfrac{1}{24}$ (D) $\dfrac{1}{64}$

Ans. (B)

Sol. $\lim\limits_{x\to 0} \dfrac{x\ell n(1+x)}{(x^4+4)\times 3x^2} = \lim\limits_{x\to 0} \dfrac{1}{4}\times\dfrac{1}{3} = \dfrac{1}{12}$

SECTION - II

Multiple Correct ChoiceType

This section contains 5 multiple choice questions. Each question has four choices (A), (B), (C) and (D) out of which ONE OR MORE may be correct.

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37. The value(s) of $\displaystyle\int_0^1 \dfrac{x^4(1-x)^4}{1+x^2}\,dx$ is (are)

(A) $\dfrac{22}{7}-\pi$ (B) $\dfrac{2}{105}$ (C) 0 (D) $\dfrac{71}{15}-\dfrac{3\pi}{2}$

Ans. (A)

Sol. $\displaystyle\int_0^1 \dfrac{x^4(1-x)^4}{1+x^2} = \int_0^1 \dfrac{x^4[(1+x^2)-2x]^2}{1+x^2} = \int_0^1 \dfrac{x^4[(1+x^2)^2-4x(1+x^2)+4x^2]}{1+x^2}$

$= \displaystyle\int_0^1 x^4\left[(1+x^2)-4x+\dfrac{4x^2}{1+x^2}\right]dx = \int\left[x^6+x^4-4x^5+\dfrac{4x^6}{1+x^2}\right]dx$

Now on polynomial division of x^6 by $1 + x^2$, we obtain

$= \displaystyle\int\left[x^6+x^4-4x^5+4\left[(x^4-x^2+1)-\dfrac{1}{1+x^2}\right]\right]dx = \int\left[\left(x^6-4x^5+5x^4-4x^2+4\right)-\dfrac{4}{1+x^2}\right]dx$

$$= \left[\frac{x^7}{7} - \frac{4x^6}{6} + \frac{5 \cdot x^5}{5} - \frac{4x^3}{3} + 4x\right]_0^1 - 4\left[\tan^{-x} x\right]_0^1 = \left(\frac{1}{7} - \frac{4}{6} + 1 - \frac{4}{3} + 4\right) - 4\left(\frac{\pi}{4}\right) = \left[\frac{1}{7} - \frac{12}{6} + 5\right] - \pi$$

$$= \left(\frac{1}{7} + 3\right) - \pi = \frac{22}{7} - \pi$$

38. Let f be a real-valued function defined on the interval $(0, \infty)$ by $f(x) = \ell n\, x + \int_0^x \sqrt{1+\sin t}\, dt$. Then which of the following statement(s) is (are) true?

(A) $f''(x)$ exists for all $x \in (0, \infty)$

(B) $f'(x)$ exists for all $x \in (0, \infty)$ and f' is continuous on $(0, \infty)$, but not differentiable on $(0, \infty)$

(C) there exists $\alpha > 1$ such that $|f'(x)| < |f(x)|$ for all $x \in (\alpha, \infty)$

(D) there exists $\beta > 1$ such that $|f(x)| + |f'(x)| \le \beta$ for all $x \in (0, \infty)$

Ans. (B, C)

Sol. $f(x) = \ell n\, x + \int_0^2 \sqrt{1+\sin t}\, dt$

$$f'(x) = \frac{1}{x} + \sqrt{1+\sin x}$$

$$f''(x) = -\frac{1}{x^2} + \frac{\cos x}{2\sqrt{1+\sin x}}$$

(A) f'' is not defined for $x = -\frac{\pi}{2} + 2n\pi$, $n \in I$

so (A) is wrong

(B) $f'(x)$ always exist for $x > 0$

(C) $|f'| < |f|$

Since $f' > 0$ and $f > 0$

$f' < f$

$$\frac{1}{x} + \sqrt{1+\sin x} < \ell n\, x + \int_0^x \sqrt{1+\sin x}\, dx$$

LHS is bounded RHS is Increasing with range ∞

So there exist some α beyond which RHS is greater than LHS

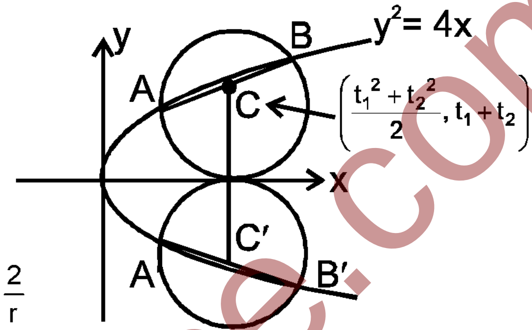
(D) $|f| + |f'| \le b$ is wrong as f is MI & its range is not bound while β is finite

39. Let A and B be two distinct points on the parabola $y^2 = 4x$. If the axis of the parabola touches a circle of radius r having AB as its diameter, then the slope of the line joining A and B can be

(A) $-\frac{1}{r}$ (B) $\frac{1}{r}$ (C) $\frac{2}{r}$ (D) $-\frac{2}{r}$

Ans. (C, D)

Sol. $A(t_1^2, 2t_1)$, $B(t_2^2, 2t_2)$



centre of circle $\left(\frac{t_1^2+t_2^2}{2}, t_1+t_2\right)$

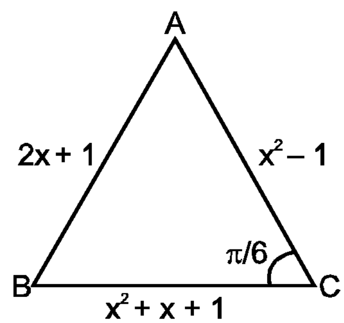
$\Rightarrow$ $|t_1 + t_2| = r$, slope of AB $= \frac{2}{t_1+t_2} = \pm\frac{2}{r}$

40. Let ABC be a rectangle such that $\angle ACB = \frac{\pi}{6}$ and let a, b and c denote the lengths of the sides opposite to A, B and C respectively. The value(s) of x for which $a = x^2 + x + 1$, $b = x^2 - 1$ and $c = 2x + 1$ is (are)

(A) $-(2+\sqrt{3})$ (B) $1 + \sqrt{3}$ (C) $2 + \sqrt{3}$ (D) $4\sqrt{3}$

Ans. (A, B)

Sol. $\cos\frac{\pi}{6} = \frac{(x^2-1)^2+(x^2+x+1)^2-(2x+1)^2}{2(x^2+x+1)(x^2-1)}$



$\frac{\sqrt{3}}{2} = \frac{(x^2-1)^2+(x^2+3x+2)(x^2-x)}{2(x^2+x+1)(x^2-1)}$

$\frac{\sqrt{3}}{2} = \frac{(x^2-1)^2+(x+1)(x+2)x(x-1)}{2(x^2+x+1)(x^2-1)}$

$\Rightarrow$ $\sqrt{3} = \frac{x^2-1+x(x+2)}{x^2+x+1}$ $\Rightarrow$ $\sqrt{3}(x^2 + x + 1) = 2x^2 + 2x - 1$

$\Rightarrow$ $(\sqrt{3} - 2)x^2 + (\sqrt{3} - 2)x + (\sqrt{3} + 1) = 0$

on solving

$\Rightarrow$ $x^2 + x - (3\sqrt{3}+5) = 0$

$x = \sqrt{3} + 1, -(2+\sqrt{3})$

41. Let z_1 and z_2 be two distinct complex numbers and let $z = (1 - t)\, z_1 + tz_2$ for some real number t with $0 < t < 1$. If Arg(w) denotes the principal argument of a nonzero complex number w, then

(A) $|z - z_1| + |z - z_2| = |z_1 - z_2|$

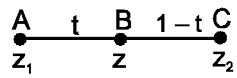
(B) $\text{Arg}\,(z - z_1) = \text{Arg}\,(z - z_2)$

(C) $\begin{vmatrix} z - z_1 & \bar{z} - \bar{z}_1 \\ z_2 - z_1 & \bar{z}_2 - \bar{z}_1 \end{vmatrix} = 0$

(D) $\text{Arg}\,(z - z_1) = \text{Arg}\,(z_2 - z_1)$

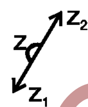
Ans. (A, C, D)

Sol. (A) $|z - z_1| + |z - z_2| = |z_1 - z_2|$

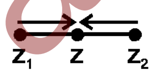


AB + BC = AC

(B) $\text{Arg}\,(z - z_1) - \text{Arg}\,(z - z_2) = \pi$



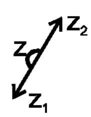
(C) $\begin{vmatrix} z & \bar{z} & 1 \\ z_1 & \bar{z}_1 & 1 \\ z_2 & \bar{z}_2 & 1 \end{vmatrix} = 0$



$\begin{vmatrix} z - z_1 & \bar{z} - \bar{z}_1 & 0 \\ z_1 - z_2 & \bar{z}_1 - \bar{z}_2 & 0 \\ z_2 & \bar{z}_2 & 1 \end{vmatrix} = 0$

$\begin{vmatrix} z - z_1 & \bar{z} - \bar{z}_1 \\ z_1 - z_2 & \bar{z}_1 - \bar{z}_2 \end{vmatrix} = 0$

(D) $\text{Arg}\,(z - z_1) = \text{Arg}\,(z_2 - z_1)$



SECTION - III
Comprehension Type

This section contains 2 Paragraphs. Based upon the first paragraph 3 multiple choice questions and based upon the second paragraph 3 multiple choice questions have to be answered. Each of these questions has four choices (A), (B), (C) and (D) out of which ONLY ONE is correct.

Paragraph for Question Nos. 42 to 44

Let p be an odd prime number and T_p be the following set of 2 × 2 matrices :

$$T_p = \left\{ A = \begin{bmatrix} a & b \\ c & a \end{bmatrix} : a, b, c \in \{0, 1, 2, \ldots\ldots, p-1\} \right\}$$

42. The number of A in T_p such that A is either symmetric or skew-symmetric or both, and det (A) divisible by p is

(A) $(p-1)^2$ (B) $2(p-1)$ (C) $(p-1)^2 + 1$ (D) $2p-1$

Ans. (D)

Sol. $A = \begin{bmatrix} a & b \\ c & a \end{bmatrix}$ where a, b, c $\in \{0,1,2,\ldots.p-1\}$

Case- I A is symmetric matrix $\Rightarrow$ b = c

$\Rightarrow$ det(A) = $a^2 - b^2$ is divisible by p

$\Rightarrow$ (a – b) (a + b) is divisible by p

(a) a – b is divisible by p if a = b, then 'p' cases are possible

(b) a + b is divisible by p if a + b = p, then $\frac{(p-1)}{2} \times 2 = (p-1)$ cases are possible

Case - II

A is skew symmetric matrix

if a = 0, b + c = 0, then det (A) = b^2

$\Rightarrow b^2$ can never be divisible by p. So No case is possible

Total number of A is possible = 2p – 1

43. The number of A in T_p such that the trace of A is not divisible by p but det (A) is divisible by p is

[Note : The trace of matrix is the sum of its diagonal entries.]

(A) $(p-1)(p^2-p+1)$ (B) $p^3 - (p-1)^2$

(C) $(p-1)^2$ (D) $(p-1)(p^2-2)$

Ans. (C)

Sol. $a^2 - bc \div p$

a can be chosen in p – 1 ways ($a \neq 0$)

Let a be 4 & p = 5

so a^2 = 16 and hence bc should be chosen such that $a^2 - bc \div p$

Now b can be chosen in p – 1 ways and c in only 1 (one be is chosen)

Explanation : If $b = 1 \Rightarrow c = 1$
$b = 2 \Rightarrow c = 3$
$b = 3 \Rightarrow c = 2$
$b = 4 \Rightarrow c = 4$

Hence a can be chosen in $p - 1$ ways
and then b can be chosen in $p - 1$ ways
c can be chosen in 1 ways
so $(p - 1)^2$

44. The number of A in T_p such that det (A) is not divisible by p is
(A) $2p^2$ (B) $p^3 - 5p$ (C) $p^3 - 3p$ (D) $p^3 - p^2$

Ans. (D)

Sol. As = Total cases – ($a \neq 0$ and $|A|$ is divisible by p) – ($a = 0$ and $|A|$ is divisible by p)
$= p^3 - (p - 1)^2 - (2p - 1) = p^3 - p^2$
– bc $\div$ p since b & c both we coprime to p
$\Rightarrow$ one of them must be zero.
If $b = 0 \Rightarrow$ c can be chosen in {0,1,....p – 1}
If $c = 0 \Rightarrow$ b can be chosen in {0,1,....p – 1}

Paragraph for Question Nos. 45 to 46

The circle $x^2 + y^2 - 8x = 0$ and hyperbola $\frac{x^2}{9} - \frac{y^2}{4} = 1$ intersect at the points A and B.

45. Equation of a common tangent with positive slope to the circle as well as to the hyperbola is
(A) $2x - \sqrt{5}\,y - 20 = 0$ (B) $2x - \sqrt{5}\,y + 4 = 0$
(C) $3x - 4y + 8 = 0$ (D) $4x - 3y + 4 = 0$

Ans. (B)

Sol. Let equation of tangent to ellipse

$$\frac{\sec\theta}{3}x - \frac{\tan\theta}{2}y = 1$$

$2\sec\theta\, x - 3\tan\theta\, y = 6$
It is also tangent to circle $x^2 + y^2 - 8x = 0$

$$\Rightarrow \frac{|8\sec\theta - 6|}{\sqrt{4\sec^2\theta + 9\tan^2\theta}} = 4$$

$(8\sec\theta - 6)^2 = 16\,(13\sec^2\theta - 9)$
$\Rightarrow 12\sec^2\theta + 8\sec\theta - 15 = 0$

$\Rightarrow \quad \sec\theta = \frac{5}{6}$ and $-\frac{3}{2}$ but $\sec \neq \frac{5}{6}$

$\Rightarrow \quad \sec\theta = -\frac{3}{2} \qquad \Rightarrow \quad \tan\theta = \frac{\sqrt{5}}{2} \qquad \therefore$ slope is positive

Equation of tangent = $2x - \sqrt{5}\,y + 4 = 0$

46. Equation of the circle with AB as its diameter is

(A) $x^2 + y^2 - 12x + 24 = 0$

(B) $x^2 + y^2 + 12x + 24 = 0$

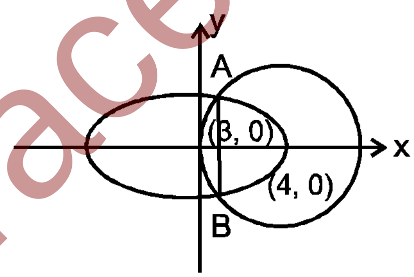
(C) $x^2 + y^2 + 24x - 12 = 0$

(D) $x^2 + y^2 - 24x - 12 = 0$

Ans. (D)

Sol. $x^2 + y^2 - 8x = 0$

$\frac{x^2}{9} - \frac{y^2}{4} = 1 \quad \Rightarrow \quad 4x^2 - 9y^2 = 36$



$\Rightarrow \quad 4x^2 - 9(8x - x^2) = 36$

$13x^2 - 72x - 36 = 0$

$13x^2 - 78x + 6x - 36 = 0$

$(13x + 6)(x - 6) = 0$

$\Rightarrow \quad x = -\frac{6}{13}$ and $x = 6$

But $x > 0 \quad \Rightarrow \quad x = 6$

$\Rightarrow \quad A(6, \sqrt{2})$ and $B(6, -\sqrt{2})$

$\Rightarrow$ Equation of circle with AB as a diameter $x^2 + y^2 - 12x + 24 = 0$

SECTION - IV
(Integer Type)

This section contains TEN questions. The asnwer to each question is a single-digit integer, ranging from 0 to 9. The correct digit below the question number in the ORS is to be bubbled.

47. The number of all possible values of θ, where $0 < \theta < \pi$, for which the system of equations

$$(y + z) \cos 3\theta = (xyz) \sin 3\theta$$

$$x \sin 3\theta = \frac{2\cos 3\theta}{y} + \frac{2\sin 3\theta}{z}$$

$$(xyz) \sin 3\theta = (y + 2z) \cos 3\theta + y \sin 3\theta$$

have a solution (x_0, y_0, z_0) with $y_0 z_0 \neq 0$, is

$$(y + z) \cos 3\theta = (xyz) \sin 3\theta$$

$$x \sin 3\theta = \frac{2\cos 3\theta}{y} + \frac{2\sin 3\theta}{z}$$

$$(xyz) \sin 3\theta = (y + 2z) \cos 3\theta + y \sin 3\theta$$

Ans. 3

Sol. Let $xyz = t$

$t \sin 3\theta - y \cos 3\theta - z \cos 3\theta = 0$ ....(1)

$t \sin 3\theta - 2y \sin 3\theta - 2z \cos 3\theta = 0$ ....(2)

$t \sin 3\theta - y (\cos 3\theta + \sin 3\theta) - 2z \cos 3\theta = 0$ ....(3)

$y_0 . z_0 \neq 0$ hence homogeneous equation has non-trivial solution

$$D = \begin{vmatrix} \sin 3\theta & -\cos 3\theta & -\cos 3\theta \\ \sin 3\theta & -2\cos 3\theta & -2\cos 3\theta \\ \sin 3\theta & -(\cos 3\theta + \sin 3\theta) & -2\cos 3\theta \end{vmatrix} = 0$$

$\Rightarrow \sin 3\theta \cos 3\theta (\sin 3\theta - \cos 3\theta) = 0$

$\Rightarrow \sin 3\theta = 0$ or $\cos 3\theta = 0$ or $\tan 3\theta = 1$

Case - I $\sin 3\theta = 0$

From equation (2)

$z = 0$ not possible

Case - II $\cos 3\theta = 0$, $\sin 3\theta \neq 0$

$t. \sin 3\theta = 0 \quad \Rightarrow \quad t = 0 \quad \Rightarrow \quad x = 0$

From equation (2)

$y = 0$ not possible

Case- III $\tan 3\theta = 1$

$\Rightarrow \quad 3\theta = n\pi + \frac{\pi}{4}, n \in I$

$\Rightarrow \quad x. y. z \sin 3\theta = 0 \quad \Rightarrow \theta = \frac{n\pi}{3} + \frac{\pi}{12}, n \in I$

$\Rightarrow \quad x = 0$, $\sin 3\theta \neq 0 \quad \Rightarrow \theta = \frac{\pi}{12}, \frac{5\pi}{12}, \frac{9\pi}{12}$

Hence 3 solutions

48. Let f be a real-valued differentiable function on **R** (the set of all real numbers) such that f(1) = 1. If the y-intercept of the tangent at any point P(x, y) on the curve y = f(x) is equal to the cube of the abscissa of P, then the value of f(–3) is equal to

Ans. 9

Sol. $Y - y = m(X - x)$

y-intercept ($x = 0$)

$y = y - mx$

Given that $y - mx = x^3$ $\Rightarrow$ $x\dfrac{dy}{dx} - y = -x^3$

$\Rightarrow \dfrac{dy}{dx} - \dfrac{y}{x} = -x^2$

Intergrating factor $e^{-\int \frac{1}{x}dx} = \dfrac{1}{x}$

solution $y \cdot \dfrac{1}{x} = \int \dfrac{1}{x}\cdot(-x^2)\,dx$ $\Rightarrow$ $f(x) = y = -\dfrac{x^3}{2} + cx$

Given f(1) = 1 $\Rightarrow$ $c = \dfrac{3}{2}$

$\therefore$ $f(x) = -\dfrac{x^3}{2} + \dfrac{3x}{2}$ $\Rightarrow$ $f(-3) = 9$

49. The number of values of θ in the interval $\left(-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right)$ such that $\theta \neq \dfrac{n\pi}{5}$ for n = 0, ±1, ± 2 and $\tan\theta = \cot 5\theta$ as well as $\sin 2\theta = \cos 4\theta$ is

Ans. 3

Sol. $\tan\theta = \cot 5\theta$

$\Rightarrow \dfrac{\sin\theta}{\cos\theta} = \dfrac{\cos 5\theta}{\sin\theta}$ $\Rightarrow$ $\cos 6\theta = 0$

$\Rightarrow 6\theta = (2n + 1)\dfrac{\pi}{2}$

$\Rightarrow \theta = (2n + 1)\dfrac{\pi}{12};\ n \in I$

$\Rightarrow\ \theta = -\frac{5\pi}{12}, -\frac{\pi}{4}, -\frac{\pi}{12}, \frac{\pi}{12}, \frac{\pi}{4}, \frac{5\pi}{12}$ ....(1)

$\sin 2\theta = \cos 4\theta$

$\Rightarrow\quad \sin 2\theta = 1 - 2\sin^2 2\theta$

$\Rightarrow\quad 2\sin^2 2\theta + \sin 2\theta - 1 = 0$

$\Rightarrow\quad \sin 2\theta = -1, \frac{1}{2}$

$\Rightarrow\quad 2\theta = (4m-1)\frac{\pi}{2}, p\pi + (-1)^p \frac{\pi}{6}$

$\Rightarrow\quad q = (4m-1)\frac{\pi}{4}, \frac{p\pi}{2} + (-1)^p \frac{\pi}{12}; m, \ p \in I$

$\Rightarrow\quad \theta = -\frac{\pi}{4}, \frac{\pi}{12}, \frac{5\pi}{12}$ ....(2)

From (1) & (2)

$\theta \in \left\{-\frac{\pi}{4}, \frac{\pi}{12}, \frac{5\pi}{12}\right\}$

Number of solution

50. The maximum value of the expression $\frac{1}{\sin^2\theta + 3\sin\theta\cos\theta + 5\cos^2\theta}$ is

Ans. 2

Sol. $f(\theta) = \frac{1}{\sin^2\theta + 3\sin\theta\cos\theta + 5\cos^2\theta} = \frac{1}{\frac{1-\cos 2\theta}{2} + \frac{3}{2}\sin 2\theta + \frac{5(1+\cos 2\theta)}{2}} = \frac{2}{6 + 3\sin 2\theta + 4\cos 2\theta}$

$\therefore\quad f(\theta)_{max} = \frac{2}{6-5} = 2$

51. If $\vec{a}$ and $\vec{b}$ are vectors in space given by $\vec{a} = \frac{\hat{i} - 2\hat{j}}{\sqrt{5}}$ and $\vec{b} = \frac{2\hat{i} + \hat{j} + 3\hat{k}}{\sqrt{14}}$, then the value of

$(2\vec{a} + \vec{b}) \cdot \left[(\vec{a} \times \vec{b}) \times (\vec{a} - 2\vec{b})\right]$ is

Ans. 5

Sol. $\vec{a} = \frac{\hat{i}-2\hat{j}}{\sqrt{5}}$, $\vec{b} = \frac{2\hat{i}+\hat{j}+3\hat{k}}{\sqrt{14}}$

$|\vec{a}| = 1$, $|\vec{b}| = 1$

$\vec{a}.\vec{b} = 0$

$(2\vec{a}+\vec{b}) . \{(\vec{a}\times\vec{b})\times(\vec{a}-2\vec{b})\}$

$= (2\vec{a}+\vec{b}) . \{\vec{a}.(\vec{a}-2\vec{b})\vec{b}-(\vec{b}.(\vec{a}-2\vec{b})\vec{a}\}$

$= (2\vec{a}+\vec{b}) . \{(\vec{a}.\vec{a}-2\vec{a}.\vec{b})\vec{b}-(\vec{b}.\vec{a}-2\vec{b}.\vec{b})\vec{a}\}$

$= (2\vec{a}+\vec{b}) . \{(1-0)\vec{b}-(0+2)\vec{a}\}$

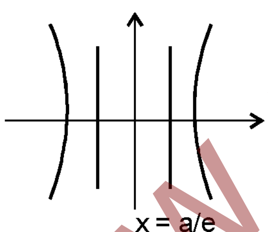
$= (2\vec{a}+\vec{b}) . \{\vec{b}+2\vec{a}\}$

$= 2(\vec{a}.\vec{b})+4(\vec{a}.\vec{a})+\vec{b}.\vec{b}+2(\vec{b}.\vec{a}) = 0 + 4 + 1 + 0 = 5$

52. The line 2x + y = 1 is tangent to the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$. If this line passes through the point of intersection of the nearest directrix and the x-axis, then the eccentricity of the hyperbola is

Ans. 2

Sol.



$y = -2x + 1$

$0 = - -\frac{2a}{e} + 1$

$\Rightarrow \frac{a}{e} = \frac{1}{2}$

$e = 2a$

$e^2 = 1 + \frac{b^2}{a^2}$

$= 1 + \frac{(4a^2-1)}{a^2}$

$e^2 = 1 + 4 - \frac{1}{a^2}$

$e^2 = 5 - \frac{4}{e^2}$

$c^2 = a^2 m^2 - b^2$

$\Rightarrow\ 1 = 4a^2 - b^2$

$\Rightarrow\ 1 + b^2 - 4a^2 = 0$

$\Rightarrow\ e^4 - 5e^2 + 4 = 0$

$\Rightarrow\ (e^2 - 1)(e^2 - 4) = 0$

$e^2 - 1 \neq 0 \qquad e = 2$

53. If the distance between the plane $Ax - 2y + z = d$ and the plane containing the lines

$\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x-2}{3} = \frac{y-3}{4} = \frac{z-4}{5}$ is $\sqrt{6}$, then |d| is

Ans. 6

Sol. Equation of plane is $\begin{vmatrix} x-1 & y-2 & z-3 \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{vmatrix} = 0$

$x - 2y + z = 0$ ....(1)

$Ax - 2y + z = d$ ....(2)

Compare $\frac{A}{1} = \frac{-2}{-2} = \frac{1}{1} \Rightarrow A = 1$

Distance between planes is $\left|\frac{d}{\sqrt{1+1+4}}\right| = \sqrt{6}$

$\Rightarrow\ |d| = 6$

54. For any real number, let [x] denote the largest integer less than or equal to x. Let f be a real valued function defined on the interval [–10, 10] by

$$f(x) = \begin{cases} x - [x] & \text{if } [x] \text{ is odd,} \\ 1 + [x] - x & \text{if } [x] \text{ is even} \end{cases}$$

Then the value of $\frac{\pi^2}{10}\int_{-10}^{10} f(x)\cos \pi x\, dx$ is

Ans. 4

Sol. $f(x) = \begin{cases} \{x\} & , \quad 2n-1 \le x < 2n \\ 1-\{x\} & , \quad 2n \le x < 2n+1 \end{cases}$

Clearly f(x) is a periodic function with period = 2

Hence f(x) . cos πx is also periodic with period = 2

$$\frac{\pi^2}{10}\int_{-10}^{10} f(x)\cos(\pi x)\,dx = \pi^2\int_0^2 f(x)\cos(\pi x)\,dx = \pi^2\int_0^1 ((1-\{x\})+\{-x\})\cos(\pi x)\,dx$$

$$= \quad 2\pi^2\int_0^1 (-x\cos\pi x)\,dx = -2\pi^2\left[\frac{x\sin\pi x}{\pi}+\frac{\cos\pi x}{\pi^2}\right]_0^1 = -2\pi^2\left(-\frac{2}{\pi^2}\right) = 4$$

55. Let ω be the complex number $\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}$. Then the number of distinct complex numbers

z satisfying $\begin{vmatrix} z+1 & \omega & \omega^2 \\ \omega & z+\omega^2 & 1 \\ \omega^2 & 1 & z+\omega \end{vmatrix} = 0$ is equal to

Ans. 1

Sol. $\omega = \cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}$

$R_1 \to R_1 + R_2 + R_3$

$$\begin{vmatrix} z & \omega & \omega^2 \\ z & z+\omega^2 & 1 \\ z & 1 & z+\omega \end{vmatrix} = 0 \Rightarrow z\begin{vmatrix} 1 & \omega & \omega^2 \\ 1 & z+\omega^2 & 1 \\ 1 & 1 & z+\omega \end{vmatrix} = 0$$

z = 0

$$\begin{vmatrix} 1 & \omega & \omega^2 \\ 0 & z+\omega^2-\omega & 1-\omega^2 \\ 0 & 1-\omega & z+\omega-\omega^2 \end{vmatrix} = 0$$

$(z + \omega^2 - \omega)(z + \omega - \omega^2) - (1 - \omega)(1 - \omega^2) = 0$

$z^2 = 0$

only one solution

56. Let S_k, k = 1, 2,...., 100, denote the sum of the infinite geometric series whose first term is $\frac{k-1}{k!}$ and the common ratio is $\frac{1}{k}$. Then the value of $\frac{100^2}{100!} + \sum_{k=1}^{100} \left|(k^2 - 3k + 1) S_k\right|$ is

Ans. 2

Sol.

$$\sum_{k=1}^{100} \left|(k^2 - 3k + 1) S_k\right|$$

$$\sum_{k=1}^{100} \left(\frac{k-1}{(k-2)!} - \frac{k-1+1}{(k-1)!}\right)$$

$$\sum_{k=1}^{100} \frac{1}{(k-3)!} + \frac{1}{(k-2)!} - \frac{1}{(k-2)!} - \frac{1}{(k-1)!}$$

$$\sum_{k=1}^{100} \left(\frac{1}{(k-3)!} - \frac{1}{(k-1)!}\right)$$

$$S = \left(1 - \frac{1}{2!}\right) + \left(\frac{1}{1!} - \frac{1}{3!}\right) + \left(\frac{1}{2!} - \frac{1}{4!}\right) + \left(\frac{1}{3!} - \frac{1}{5!}\right) + \left(\frac{1}{4!} - \frac{1}{6!}\right) + \ldots \left(\frac{1}{94!} - \frac{1}{96!}\right)$$

$$+ \left(\frac{1}{95!} - \frac{1}{97!}\right) + \left(\frac{1}{96!} - \frac{1}{98!}\right) + \left(\frac{1}{97!} - \frac{1}{99!}\right) = 2 - \frac{1}{98!} - \frac{1}{99!}$$

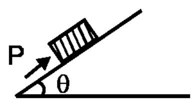
$$\therefore \quad E = \frac{100^2}{100!} + 2 - \frac{1}{98!} - \frac{1}{99.98!}$$

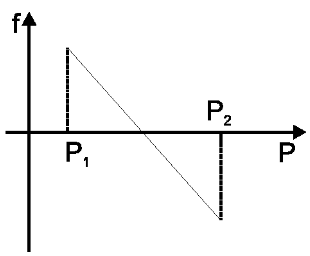
$$= \frac{100^2}{100!} + 2 - \frac{100}{99!} = \frac{100^2}{100.99!} + 2 - \frac{100}{99!} = 2$$

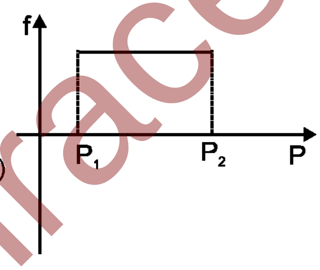
PART-III
SECTION - I
Single Correct Choice Type

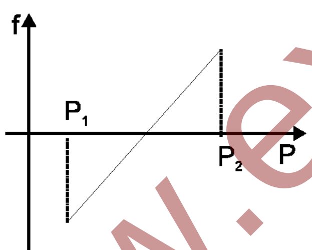
This section contains **8 multiple choice questions**. Each question has four choices (A), (B), (C) and (D) four choices (A), (B), (C) and (D) out of which **ONLY ONE** is correct.

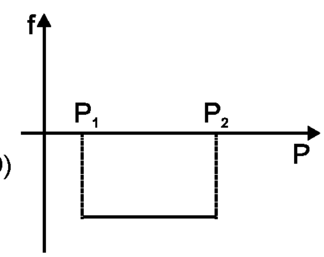
57. A block of mass m is on inclined plane of angle θ. The coefficient of friction between the block and the plane is μ and $\tan\theta > \mu$. The block is held stationary by applying a force P parallel to the plane. The direction of force pointing up the plane is taken to be positive. As P is varied from $P_1 = mg(\sin\theta - \mu\cos\theta)$ to $P_2 = mg(\sin\theta + \mu\cos\theta)$, the frictional force f versus P graph will look like :



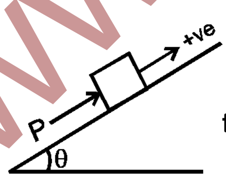
(A)


(B)


(C)


(D)


Ans. (A)

Sol.


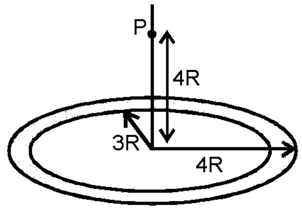
$\tan\theta > u$

$P_1 = mg\sin\theta - \mu mg\cos\theta$

$P_2 = mg\sin\theta + \mu mg\cos\theta$

Initially block has tendency to slide down and as $\tan\theta > \mu$, maximum friction $\mu mg\cos\theta$ will act in positive direction. When magnitude P is increased from P_1 to P_2, friction reverse its direction from positive to negative and becomes maximum i.e.$\mu mg\cos\theta$ in opposite direction.

58. A thin uniform annular disc (see figure) of mass M has outer radius 4R and inner radius 3R. The work required to take a unit mass from point P on its axis to infinity is :



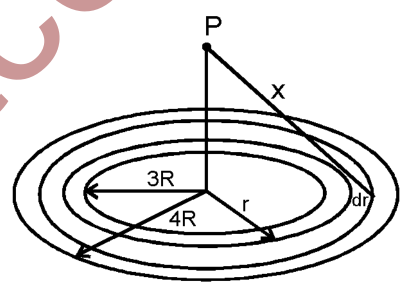
(A) $\frac{2GM}{7R}\left(4\sqrt{2}-5\right)$ (B) $-\frac{2GM}{7R}\left(4\sqrt{2}-5\right)$ (C) $\frac{GM}{4R}$ (D) $\frac{2GM}{5R}\left(\sqrt{2}-1\right)$

Ans. (A)

Sol. $W_{ext} = U_{\infty} - U_P$

$$W_{ext} = 0 - \int -\left(-\frac{Gdm}{x}.1\right)$$

$$W_{ext} = G\int \frac{M}{\pi \times 7R^2}\frac{2\pi r dr}{\sqrt{16R^2+r^2}} = \frac{2GM}{7R^2}\int \frac{rdr}{\sqrt{16R^2+r^2}}$$



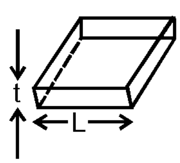
$$= \frac{2GM}{7R^2}\int \frac{zdz}{z} = \frac{2GM}{7R^2}[Z]$$

$$W_{ext} = \frac{2GM}{7R^2}\left[\sqrt{16R^2+r^2}\right]_{3R}^{4R}$$

$$W_{ext} = \frac{2GM}{7R^2}\left[4\sqrt{2}R - 5R\right]$$

$$W_{ext} = \frac{2GM}{7R^2}\left[4\sqrt{2} - 5\right].$$

59. Consider a thin square sheet of side L and thickness t, made of a material of resitivity ρ. The resistance between two opposite faces, shown by the shaded areas in the figure is :



(A) directly proportional to L (B) directly proportional to t
(C) independent of L (D) independent of t

Ans. (C)

Sol. $R = \frac{\rho l}{A}$

$R = \frac{\rho L}{tL} = \frac{\rho}{t}$

Independent of L.

60. A real gas behaves like an ideal gas if its

(A) pressure and temperature are both high
(B) pressure and temperature are both low
(C) pressure is high and temperature is low
(D) pressure is low and temperature is high

Ans. (D)

Sol. At low pressure and high temperature inter molecular forces become ineffective. So a real gas behaves like an ideal gas.

61. Incandescent bulbs are designed by keeping in mind that the resistance of their filament increases with increase in temperature. If at room temperature, 100 W, 60 W and 40 W bulbs have filament resistances R_{100}, R_{60} and R_{40}, respectively, the relation between these resistance is :

(A) $\frac{1}{R_{100}} = \frac{1}{R_{40}} + \frac{1}{R_{60}}$
(B) $R_{100} = R_{40} + R_{60}$
(C) $R_{100} > R_{60} > R_{40}$
(D) $\frac{1}{R_{100}} > \frac{1}{R_{60}} > \frac{1}{R_{40}}$

Ans. (D)

Sol. $100 = \frac{V^2}{R'_{100}} \Rightarrow \frac{1}{R'_{100}} = \frac{100}{V^2}$

where R'_{100} is resistance at any temperature corresponds to 100 W

$60 = \frac{V^2}{R'_{60}} \Rightarrow \frac{1}{R'_{60}} = \frac{60}{V^2}$

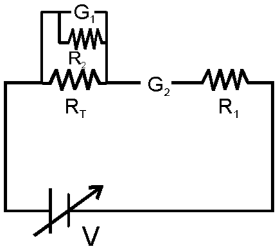
$40 = \frac{V^2}{R'_{40}} \Rightarrow \frac{1}{R'_{40}} = \frac{40}{V^2}$

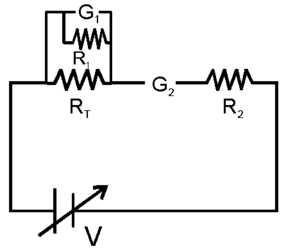
From above equations we can say

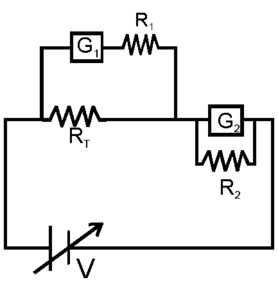
$$\frac{1}{R'_{100}} > \frac{1}{R'_{60}} > \frac{1}{R'_{40}}.$$

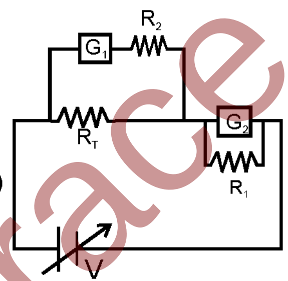
So, most appropriate answer is option (D).

62. To verify Ohm's law, a student is provided with a test resistor R_T, a high resistance R_1, a small resistance R_2, two identical galvanometers G_1 and G_2, and a variable voltage source V. The correct circuit to carry out the experiment is :

(A)


(B)


(C)


(D)


Ans. (C)

Sol. To verify Ohm's law one galvaometer is used as ammeter and other galvanometer is used as voltameter. Voltameter should have high resistance and ammeter should have low resistance as voltameter is used in parallel and ammeter in series that is in option (C).

63. An AC voltage source of variable angular frequency ω and fixed amplitude V connected in series with a capacitance C and an electric bulb of resistance R (inductance zero). When ω is increased :

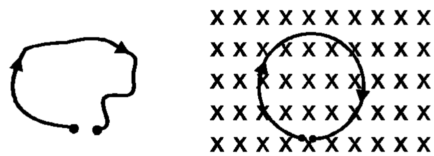
(A) the bulb glows dimmer
(B) the bulb glows brighter
(C) total impedence of the circuit is unchanged
(D) total impedence of the circuit increases

Ans. (B)

Sol. $i_{rms} = \dfrac{V_{rms}}{\sqrt{R^2 + \dfrac{1}{\omega^2 c^2}}}$

when ω increases, i_{rms} increases so the bulb glows brighter

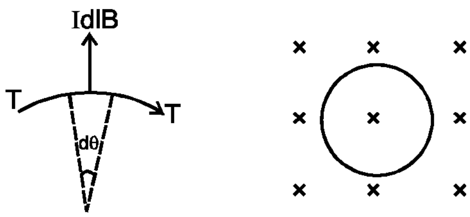
64. A thin flexible wire of length L is connected to two adjacent fixed points carries a current I in the clockwise direction, as shown in the figure. When system is put in a uniform magnetic field of strength B going into the plane of paper, the wire takes the shape of a circle. The tension in the wire is :



(A) IBL (B) $\frac{IBL}{\pi}$ (C) $\frac{IBL}{2\pi}$ (D) $\frac{IBL}{4\pi}$

Ans. (C)

Sol.



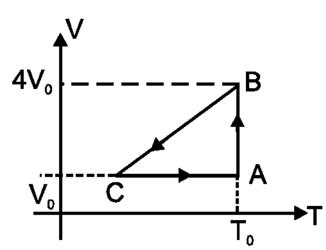
$2T\sin\left(\frac{d\theta}{2}\right) = IdlB \quad \Rightarrow \quad 2T\frac{d\theta}{2} = IRd\theta B \quad \Rightarrow \quad T = BIR = \frac{BIL}{2\pi}.$

SECTION - II

Multiple Correct Choice Type

This section contains **5 multiple choice questions**. Each question has four choices (A), (B), (C) and (D) out of which **ONE OR MORE** may be correct.

65. One mole of an ideal gas in initial state A undergoes a cyclic process ABCA, as shown in the figure. Its pressure at A is P_0. Choose the correct option(s) from the following :



(A) Internal energies at A and B are the same

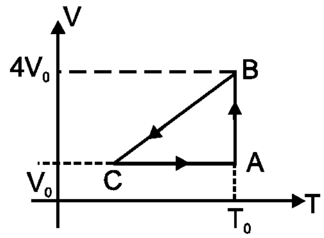
(B) Work done by the gas in process AB is $P_0V_0 \,\ell n\, 4$

(C) Pressure at C is $\frac{P_0}{4}$

(D) Temperature at C is $\frac{T_0}{4}$

Ans. (A, B)

Sol.



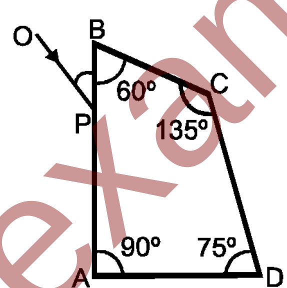
$\because \quad U = \frac{f}{2} nRT,$ where f,n,R are constants. Also temperature T is same at A & B.

$\therefore \quad U_A = U_B$

Also, $\Delta W_{AB} = nRT_0 \, \ell n\left(\frac{V_f}{V_i}\right) = nRT_0 \, \ell n \frac{4V_0}{V_0} = nRT_0 \, \ell n4 = P_0V_0 \, \ell n4$

So, answers are (A) & (B).

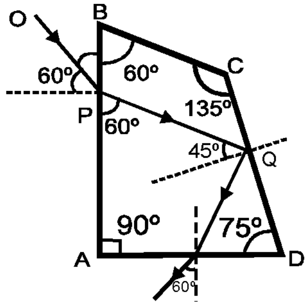
66. A ray OP of monochromatic light is incident on the face AB of prism ABCD near vertex B at an incident angle of 60° (see figure). If the refractive index of the material of the prism is $\sqrt{3}$, which of the following is (are) correct ?



(A) The ray gets totally internally reflected at face CD
(B) The ray comes out through face AD
(C) The angle between the incident ray and the emergent ray is 90°
(D) The angle between the incident ray and the emergent ray is 120°

Ans. (A, B, C)

Sol.



By refraction at face AB :

$$1.\sin 60° = \sqrt{3}\ .\ \sin r_1$$

So, $r_1 = 30°$

This shows that the refracted ray is parallel to side BC of prism. For side 'CD' angle of incidence will be 45°, which can be calculated from quadrilateral PBCQ.

By refraction at face CD :

$$\sqrt{3}\sin 45° = 1\ \sin r_2$$

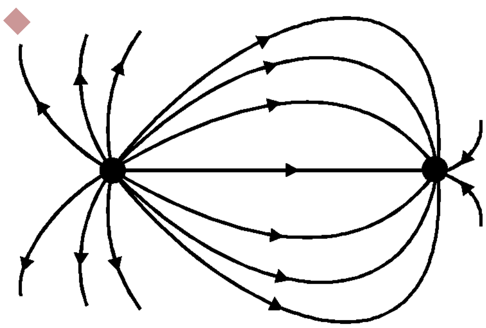
So, $\sin r_2 = \frac{\sqrt{3}}{\sqrt{2}}$

which is impossible. So, there will be T.I.R. at face CD.

Now, by geometry angle of incidence at AD will be 30°. So, angle of emergence will be 60°.

Hence, angle between incident and emergent beams is 90°

67. A few electric field lines for a system of two charges Q_1 and Q_2 fixed at two different points on the x-axis are shown in the figure. These lines suggest that :



(A) $|Q_1| > |Q_2|$

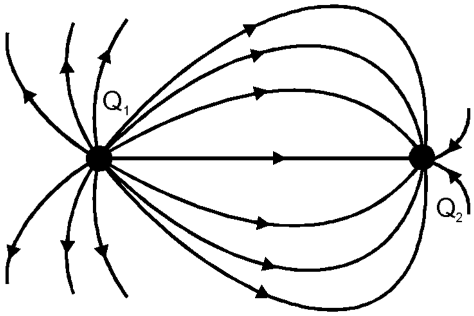
(B) $|Q_1| < |Q_2|$

(C) at a finite distance to the left of Q_1 the electric field is zero

(D) at a finite distance to the right of Q_2 the electric field is zero

Ans. (A, D)

Sol.



From the diagram, it can be observed that Q_1 is positive, Q_2 is negative.

No. of lines on Q_1 is greater and number of lines is directly proportional to magnitude of charge.

So, $|Q_1| > |Q_2|$

Electric field will be zero to the right of Q_2 as it has small magnitude & opposite sign to that of Q_1 .

68. A student uses a simple pendulum of exactly 1m length to determine g, the acceleration due to gravity. He uses a stop watch with the least count of 1sec for this and records 40 seconds for 20 oscillations. For this observation, which of the following statement(s) is (are) true ?

(A) Error ΔT in measuring T, the time period, is 0.05 seconds

(B) Error ΔT in measuring T, the time period, is 1 second

(C) Percentage error in the determination of g is 5%

(D) Percentage error in the determination of g is 2.5%

Ans. (A, C)

Sol. Since, $t = nT$. So, $T = \frac{t}{n} = \frac{40}{20}$ or $T = 2$ sec.

Now, $\Delta t = n\,\Delta T$

and $\frac{\Delta t}{t} = \frac{\Delta T}{T}$

So, $\frac{1}{40} = \frac{\Delta T}{2} \Rightarrow \Delta T = 0.05$

Time period, $T = 2\pi\sqrt{\frac{\ell}{g}}$

So, $\frac{\Delta T}{T} = -\frac{1}{2}\frac{\Delta g}{g}$ or $-\frac{\Delta g}{g} = 2\frac{\Delta T}{T}$

So, percentage error in g = $\frac{\Delta g}{g} \times 100$

$= -2\frac{\Delta T}{T} \times 100 = -\,2 \times \frac{0.05}{2} \times 100$

= 5%.

69. A point mass of 1kg collides elastically with a stationary point mass of 5 kg. After their collision, the 1 kg mass reverses its direction and moves with a speed of 2 ms^{-1}. Which of the following statement(s) is (are) correct for the system of these two masses ?

(A) Total momentum of the system is 3 kg ms^{-1}

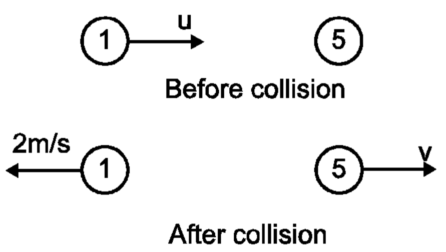
(B) Momentum of 5 kg mass after collision is 4 kg ms^{-1}

(C) Kinetic energy of the centre of mass is 0.75 J

(D) Total kinetic energy of the system is 4 J

Ans. (A, C)

Sol.



Since collision is elastic, so e = 1

Velocity of approach = velocity of separation

So, $u = v + 2$ ....(i)

By momentum conservation :

$1 \times u = 5v - 1 \times 2$

$u = 5v - 2$

$v + 2 = 5v - 2$

So, $v = 1$ m/s

and $u = 3$ m/s

Momentum of system = $1 \times 3 = 3$ kgm/s

Momentum of 5kg after collision = $5 \times 1 = 5$ kgm/s

So, kinetic energy of centre of mass = $\frac{1}{2}(m_1 + m_2)\left(\frac{m_1 u}{m_1 + m_2}\right)^2$

$= \frac{1}{2}(1 + 5)\left(\frac{1 \times 3}{6}\right)^2$

$= 0.75$ J

Total kinetic energy = $\frac{1}{2} \times 1 \times 3^2$

$= 4.5$ J.

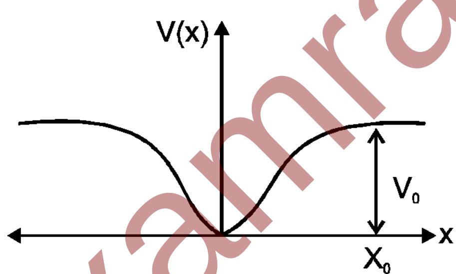
SECTION–III

Paragraph Type

This section contains **2 paragraphs**. Based upon the first paragraph **3 multiple choice questions** and based upon the second paragraph **2 multiple choice questions** have to be answered. Each of these questions has four choices (A) ,(B),(C) and (D) out of which **ONLY ONE** is correct.

Paragraph for Question Nos. 70 to 72

When a particle of mass m moves on the x-axis in a potential of the form $V(x) = kx^2$, it performs simple harmonic motion. The corresponding time period is proportional to $\sqrt{\frac{m}{k}}$, as can be seen easily using dimensional analysis. However, the motion of a particle can be periodic even when its potential energy increases on both sides of x = 0 in a way different from kx^2 and its total energy is such that the particle does not escape to infinity. Consider a particle of mass m moving on the x-axis. Its potential energy is $V(x) = \alpha x^4$ ($\alpha > 0$) for |x| near the origin and becomes a constant equal to V_0 for $|x| \geq X_0$ (see figure)



70. If the total energy of the particle is E, it will perform periodic motion only if :

(A) E < 0 (B) E > 0 (C) $V_0 > E > 0$ (D) $E > V_0$

Ans. (C)

Sol. When $0 < E < V_0$ there will be acting a restoring force to perform oscillation because in this case particle will be in the region $|x| \leq x_0$.

71. For periodic motion of small amplitude A, the time period T of this particle is proportional to :

(A) $A\sqrt{\frac{m}{\alpha}}$ (B) $\frac{1}{A}\sqrt{\frac{m}{\alpha}}$ (C) $A\sqrt{\frac{\alpha}{m}}$ (D) $\frac{1}{A}\sqrt{\frac{\alpha}{m}}$

Ans. (B)

Sol. $V = \alpha x^4$

T.E. = $\frac{1}{2} m\omega^2 A^2 = \alpha A^4$ (not strictly applicable just for dimension matching it is used)

$$\omega^2 = \frac{2\alpha A^2}{m} \quad \Rightarrow \quad T \propto \frac{1}{A}\sqrt{\frac{m}{\alpha}}$$

72. The acceleration of this particle for $|x| > X_0$ is :

(A) proportional to V_0

(B) proportional to $\frac{V_0}{mX_0}$

(C) proportional to $\sqrt{\frac{V_0}{mX_0}}$

(D) zero

Ans. (D)

Sol. $F = -\frac{dU}{dx}$

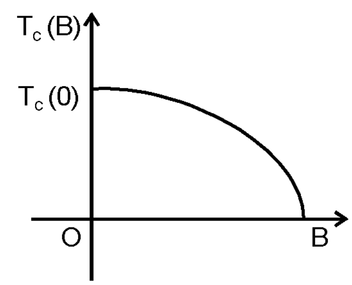
as for $|x| > x_0$ $\quad V = V_0 = \text{constant}$

$\Rightarrow \quad \frac{dU}{dx} = 0$

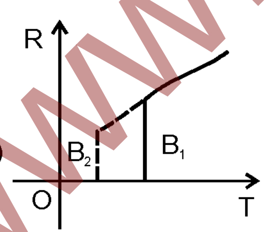
$\Rightarrow \quad F = 0.$

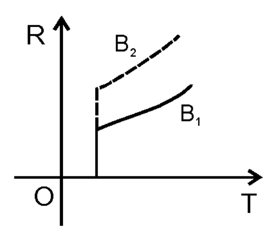
Paragraph for Question Nos. 73 to 74

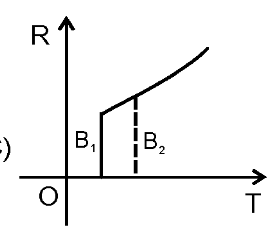
Electrical resistance of certain materials, known as superconductors, changes abruptly from a nonzero value to zero as their temperature is lowered below a critical temperature $T_C(0)$. An interesting property of superconductors is that their critical temperature becomes smaller than $T_C(0)$ if they are placed in a magnetic field, i.e., the critical temperature $T_C(B)$ is a function of the magnetic field strength B. The dependence of $T_C(B)$ on B is shown in the figure.

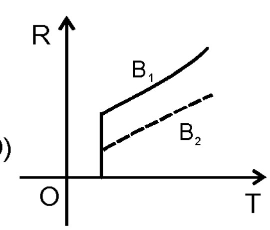


73. In the graphs below, the resistance R of a superconductor is shown as a function of its temperature T for two different magnetic fields B_1 (solid line) and B_2 (dashed line). If B_2 is larger than B_1, which of the following graphs shows the correct variation of R with T in these fields?

(A)


(B)


(C)


(D)


Ans. (A)

Sol. As the magnetic field is greater, the critical temperature is lower and as B_2 is larger than B_1. Graph 'A' is correct.

74. A superconductor has T_C (0) = 100 K. When a magnetic field of 7.5 Tesla is applied, its T_C decreases to 75 K. For this material one can definitely say that when :

(A) B = 5 Tesla, T_C (B) = 80 K
(B) B = 5 Tesla, 75 K < T_C (B) < 100 K
(C) B = 10 Tesla, 75 K < T_C (B) < 100 K
(D) B = 10 Tesla, T_C (B) = 70 K

Ans. (B)

Sol.

For	B = 0	T_C = 100 k
	B = 7.5 T	T_C = 75 k
For	B = 5T	$75 < T_C < 100$

SECTION–IV

Integer Type

This Section contains **10 questions.** The answer to each question is a **single-digit integer,** ranging from 0 to 9. The correct digit below the question number in the ORS is to be bubbled.

75. A piece of ice (heat capacity = 2100 J kg^{-1} °C^{-1} and latent heat = 3.36×10^5 J kg^{-1}) of mass m grams is at –5 °C at atmospheric pressure. It is given 420 J of heat so that the ice starts melting. Finally when the ice-water mixture is in equilibrium, it is found that 1 gm of ice has melted. Assuming there is no other heat exchange in the process, the value of m is :

Ans. **8 gm**

Sol. S = 2100 J kg^{-1} °C^{-1}

L = 3.36×10^5 J kg^{-1}

$420 = m\,S\,\Delta Q + (1) \times 10^{-3} \times L$

$420 = m\,s(5) + 3.36 \times 10^2$

$420 - 336 = m(2100) \times 5$

$m = \frac{1}{125} \times 1000 = 8$ gm.

76. A stationary source is emitting sound at a fixed frequency f_0, which is reflected by two cars approaching the source. The difference between the frequencies of sound reflected from the cars is 1.2% of f_0. What is the difference in the speeds of the cars (in km per hour) to the nearest integer ? The cars are moving at constant speeds much smaller than the speed of sound which is 330 ms^{-1}.

Ans. **7**

Sol. Let speed of cars are V_1 and V_2

frequency received by car $f_1 = \left(\frac{v}{v - v_1}\right) f_0$

frequency reflected by car $f_2 = \left(\frac{v+v_1}{v}\right)\left(\frac{v}{v-v_1}\right) f_0$

$$\Delta f = f_2' - f_2 = \left(\frac{v+v_2}{v-v_2} - \frac{v+v_1}{v-v_1}\right) f_0$$

$$\Delta f = \left(\frac{(v+v_2)(v-v_1)-(v+v_1)(v-v_2)}{(v-v_1)(v-v_2)}\right) f_0$$

$$\Delta f = \frac{2v(v_2-v_1)\,f_0}{(v-v_1)(v-v_2)} \approx \frac{2(v_2-v_1)\,f_0}{v}$$

Given $\frac{2(v_2-v_1)\,f_0}{v} = \frac{1.2}{100} f_0$

$\Rightarrow \quad v_2 - v_1 = 7.126$

Answer in nearest integer is 7.

77. The focal length of a thin biconvex lens is 20cm. When an object is moved from a distance of 25cm in front of it to 50cm, the magnification of its image changes from m_{25} to m_{50}. The ratio $\frac{m_{25}}{m_{50}}$ is :

Ans. **6**

Sol. When object distance is 25.

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

$$\frac{1}{v} - \frac{1}{(-25)} = \frac{1}{20} \quad \Rightarrow \quad v = 100 \text{ cm.}$$

$$m_{25} = \frac{v}{u} = \frac{100}{-25} = -4.$$

When object distance is 50.

$$\frac{1}{v} - \frac{1}{(-50)} = \frac{1}{20} \quad \Rightarrow \quad u = \frac{100}{3} \text{ cm}$$

$$m_{50} = \frac{\frac{100}{3}}{-50} = -\frac{2}{3} \qquad \frac{m_{25}}{m_{50}} = \frac{-4}{-\frac{2}{3}} = 6.$$

Alternate :

$$\frac{m_{25}}{m_{50}} = \frac{\frac{f}{20-25}}{\frac{f}{20-50}} = \frac{-30}{-5} = 6$$

78. An α-particle and a proton are accelerated from rest by a potential difference of 100V. After this, their de-Broglie wavelength are λ_α and λ_p respectively. The ratio $\frac{\lambda_p}{\lambda_\alpha}$, to the nearest integer, is :

Ans. **3**

Sol. $P_1 = \sqrt{2m(100\,eV)}$

$$\lambda_P = \frac{h}{\sqrt{2m(100\,eV)}} \quad \Rightarrow \quad \lambda_\alpha = \frac{h}{\sqrt{2(4m)(100\,eV)}}$$

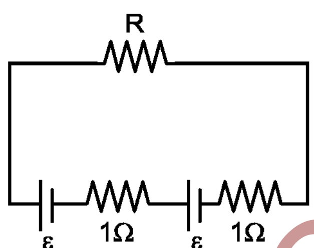
$$\frac{\lambda_P}{\lambda_\alpha} = \sqrt{8}$$

$\Rightarrow$ The ratio $\frac{\lambda_p}{\lambda_\alpha}$, to the nearest integer, is equal to 3.

79. When two identical batteries of internal resistance 1Ω each are connected in series across a resistor R, the rate of heat produced in R is J_1. When the same batteries are connected in parallel across R, the rate is J_2. If $J_1 = 2.25\ J_2$ the value of R in Ω is :

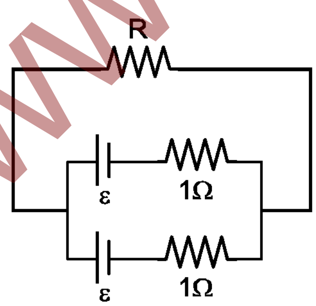
Ans. **4**

Sol.



$$i = \frac{2\varepsilon}{2+R}$$

$$J_1 = \left(\frac{2\varepsilon}{2+R}\right)^2 R$$



$$\varepsilon_{eq} = \frac{\frac{\varepsilon}{1}+\frac{\varepsilon}{1}}{\frac{1}{1}+\frac{1}{1}} = \varepsilon$$

$$r_{eq} = \frac{1}{2} \qquad \Rightarrow \qquad i = \frac{\varepsilon}{\frac{1}{2}+R} = \frac{2\varepsilon}{2R+1}$$

$$J_2 = \left(\frac{2\varepsilon}{1+2R}\right)^2 R$$

Given $J_1 = \frac{9}{4} J_2$

$$\Rightarrow \quad \left(\frac{2\varepsilon}{2+R}\right)^2 R = \frac{9}{4}\left(\frac{2\varepsilon}{1+2R}\right)^2 R$$

$$\Rightarrow \quad \frac{2}{2+R} = \frac{3}{1+2R}$$

$\Rightarrow \quad 2 + 4R = 6 + 3R$

$\Rightarrow \quad R = 4\Omega.$

80. Two spherical bodies A (radius 6 cm) and B (radius 18 cm) are at temperature T_1 and T_2 respectively. The maximum intensity in the emission spectrum of A is at 500 nm and in that of B is at 1500 nm. Considering them to be black bodies, what will be the ratio of the rate of total energy radiated by A to that of B ?

Ans. **9**

Sol. $(\lambda_m)_B = 3(\lambda_m)_A$

$\Rightarrow \quad T_A = 3T_B$

$E_1 = 4\pi\,(6)^2\,\sigma T_A^{\,4} = 4\pi(6)^2\,(3T_B)^4$

$E_2 = 4\pi\,(18)^2\,\sigma\,T_B^{\,4}$

$\frac{E_1}{E_2} = 9.$

81. When two progressive waves $y_1 = 4\sin(2x - 6t)$ and $y_2 = 3\sin\left(2x - 6t - \frac{\pi}{2}\right)$ are superimposed, the amplitude of the resultant wave is :

Ans. **5**

Sol. $A_{eq} = \sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos\phi}$

$A_{eq} = \sqrt{4^2 + 3^2 + 2(4)(3)\cos\frac{\pi}{2}}$

$A_{eq} = 5.$

82. A 0.1 kg mass is suspended from a wire of negligible mass. The length of the wire is 1m and its cross-sectional area is 4.9×10^{-7} m^2. If the mass is pulled a little in the vertically downward direction and released, it performs simple harmonic motion of angular frequency 140 rad s^{-1}. If the Young's modulus of the material of the wire is $n \times 10^9$ Nm^{-2}, the value of n is :

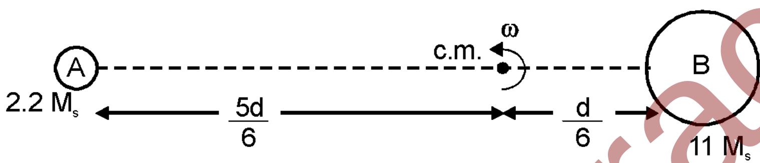
Ans. 4

Sol. $\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{yA/\ell}{m}} = \sqrt{\frac{yA}{\ell m}} \quad \Rightarrow \quad \sqrt{\frac{(n \times 10^9) \times (4.9 \times 10^{-7})}{1 \times 0.1}} = 140 \quad \Rightarrow \quad n = 4.$

83. A binary star consists of two stars A (mass 2.2 M_S) and B (mass 11 M_S) is the same mass of the sun. They are separated by distance d and are rotating about their centre of mass, which is stationary. The ratio of the total angular momentum of hte binary star to the angular momentum of star B about the centre of mass is :

Ans. 6

Sol.



$$\frac{\text{Total angular momentum about c.m.}}{\text{Angular momentum of B about c.m.}} = \frac{(2.2M_s)\left(\omega\frac{5d}{6}\right)\left(\frac{5d}{6}\right) + (11M_s)\left(\omega\frac{d}{6}\right)\left(\frac{d}{6}\right)}{(11M_s)\left(\omega\frac{d}{6}\right)\left(\frac{d}{6}\right)} = 6.$$

84. Gravitational acceleration on the surface of a planet is $\frac{\sqrt{6}}{11}g$, where g is the gravitational acceleration on the surface of the earth. The average mass density of the planet is $\frac{2}{3}$ times that of the earth. If the escape speed on the surface of the earth is taken to be 11 kms^{-1}, the escape speed on the surface of the planet in kms^{-1} will be :

Ans. 3

Sol. $g = \frac{GM}{R^2} = \frac{(G)\rho\left(\frac{4}{3}\pi R^3\right)}{R^2}$; $g \alpha \rho R$

$\frac{g'}{g} = \left(\frac{\rho'}{\rho}\right)\left(\frac{R'}{R}\right) = \left(\frac{2}{3}\right)\left(\frac{R'}{R}\right) = \frac{\sqrt{6}}{11}$ Given, $\frac{R'}{R} = \frac{3\sqrt{6}}{22}$ $\quad V_e = \sqrt{\frac{GM}{R}} = \sqrt{\frac{(G)\left((\rho)\right)\left(\frac{4}{3}\pi R^3\right)}{R}}$

$\Rightarrow \quad V_e \alpha R\sqrt{\rho}$; $V_e = 3$ km/hr.