IITJEE 2009

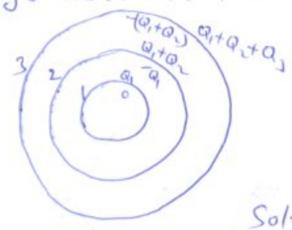
Paper 1 Code (0)

Physics

41. Three concentric metallic spherical shells of radii $R, 2R, 3R$, are given charges Q_1, Q_2, Q_3, respectively. It is found that the surface charge densities on the outer surfaces of the shells are equal. Then, the ratio of the charges given to the shells, $Q_1 : Q_2 : Q_3$, is

(A) $1:2:3$ (B) $1:3:5$ (C) $1:4:9$ (D) $1:8:18$

(41) charge distribution is as shown



$$\sigma_1 : \sigma_2 : \sigma_3 = \frac{Q_1}{4\pi\epsilon_0 R^2} : \frac{Q_1 + Q_2}{4\pi\epsilon_0 (2R)^2} : \frac{Q_1 + Q_2 + Q_3}{4\pi\epsilon_0 (3R)^2}$$

Solving we get: $\sigma_1 : \sigma_2 : \sigma_3 = 1:3:5$

(B) is correct

42. A block of base 10 cm × 10 cm and height 15 cm is kept on an inclined plane. The coefficient of friction between them is $\sqrt{3}$. The inclination θ of this inclined plane from the horizontal plane is gradually increased from 0°. Then

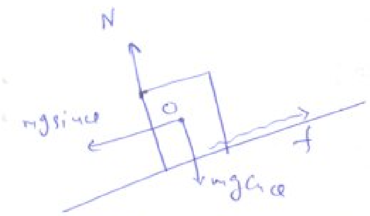
(A) at $\theta = 30°$, the block will start sliding down the plane

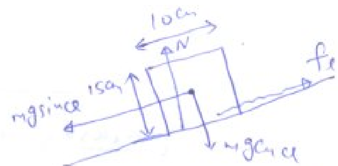
the block will remain at rest on the plane up to certain θ and then it will topple

(C) at $\theta = 60°$, the block will start sliding down the plane and continue to do so at higher angles

(D) at $\theta = 60°$, the block will start sliding down the plane and on further increasing θ, it will topple at certain θ

(42) Work shop question.





Sliding: $mg\sin\theta = f_\ell$

$\Rightarrow mg\sin\theta = \mu mg\cos\theta$

$\Rightarrow \tan\theta = \mu = \sqrt{3}$

Toppling (Torque about O)

$N \times 5 = f \times \frac{15}{2}$ — (1)

also $N = mg\cos\theta$ and $f = mg\sin\theta$ — (2)

(1) and (2) $\Rightarrow \tan\theta = \frac{2}{3}$

So $\theta_{toppling} < \theta_{sliding}$ (B) is correct

43. A ball is dropped from a height of 20 m above the surface of water in a lake. The refractive index of water is 4/3. A fish inside the lake, in the line of fall of the ball, is looking at the ball. At an instant, when the ball is 12.8 m above the water surface, the fish sees the speed of ball as [Take $g = 10$ m/s^2.]

(A) 9 m/s (B) 12 m/s (C) 16 m/s (D) 21.33 m/s

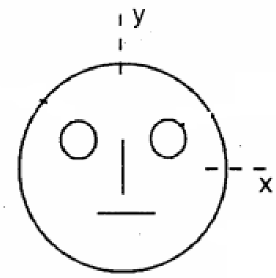
(43) Workshop question

$V = \sqrt{2gh} = \sqrt{2 \times 10(20 - 12.8)} = 12$ m/s

$V_{\text{observed by fish}} = \mu V = \frac{4}{3} \times 12 = 16$ m/s

(C)

44. Look at the drawing given in the figure which has been drawn with ink of uniform line-thickness. The mass of ink used to draw each of the two inner circles, and each of the two line segments is m. The mass of the ink used to draw the outer circle is $6m$. The coordinates of the centres of the different parts are: outer circle (0, 0), left inner circle $(-a, a)$, right inner circle (a, a), vertical line (0, 0) and horizontal line $(0, -a)$. The y-coordinate of the centre of mass of the ink in this drawing is

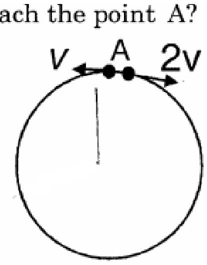


(A) $\frac{a}{10}$ (B) $\frac{a}{8}$ (C) $\frac{a}{12}$ (D) $\frac{a}{3}$

44. $y_c = \frac{ma + ma + m \times 0 + 6m \times 0 - ma}{10m} = \frac{a}{10}$

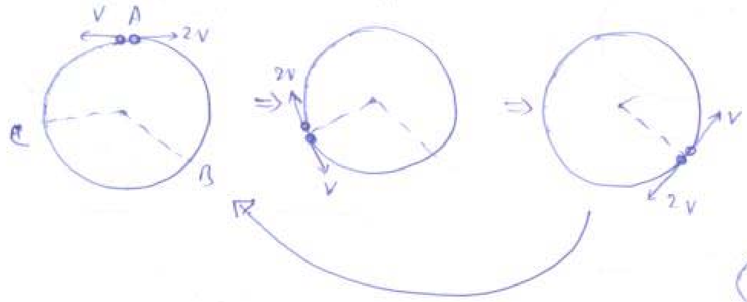
A

45. Two small particles of equal masses start moving in opposite directions from a point A in a horizontal circular orbit. Their tangential velocities are v and $2v$, respectively, as shown in the figure. Between collisions, the particles move with constant speeds. After making how many elastic collisions, other than that at A, these two particles will again reach the point A?



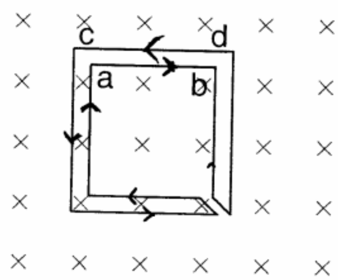
(A) 4 (B) 3 (C) 2 (D) 1

45 2 Collision as shown



C

46. The figure shows certain wire segments joined together to form a coplanar loop. The loop is placed in a perpendicular magnetic field in the direction going into the plane of the figure. The magnitude of the field increases with time. I_1 and I_2 are the currents in the segments **ab** and **cd**. Then,



(A) $I_1 > I_2$

(B) $I_1 < I_2$

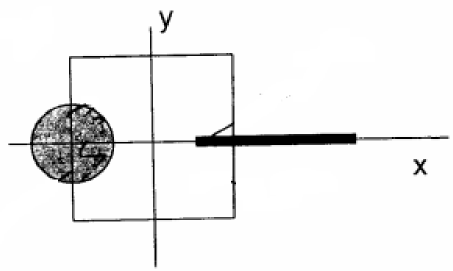
(C) I_1 is in the direction **ba** and I_2 is in the direction **cd**

(D) I_1 is in the direction **ab** and I_2 is in the direction **dc**

46 Lenz law (workshop question)

D is correct

47. A disk of radius $a/4$ having a uniformly distributed charge $6\,\text{C}$ is placed in the x-y plane with its centre at $(-a/2, 0, 0)$. A rod of length a carrying a uniformly distributed charge $8\,\text{C}$ is placed on the x-axis from $x = a/4$ to $x = 5a/4$. Two point charges $-7\,\text{C}$ and $3\,\text{C}$ are placed at $(a/4, -a/4, 0)$ and $(-3a/4, 3a/4, 0)$, respectively. Consider a cubical surface formed by six surfaces $x = \pm a/2$, $y = \pm a/2$, $z = \pm a/2$. The electric flux through this cubical surface is



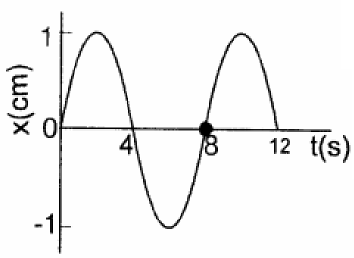
(A) $\dfrac{-2\,\text{C}}{\varepsilon_0}$ (B) $\dfrac{2\,\text{C}}{\varepsilon_0}$ (C) $\dfrac{10\,\text{C}}{\varepsilon_0}$ (D) $\dfrac{12\,\text{C}}{\varepsilon_0}$

47

$$\phi = \frac{q_{enclosed}}{\epsilon_0} = \frac{3+2-7}{\epsilon_0} = \frac{-2}{\epsilon_0}$$

A

48. The x-t graph of a particle undergoing simple harmonic motion is shown below. The acceleration of the particle at $t = 4/3$ s is



(A) $\dfrac{\sqrt{3}}{32}\pi^2\ \text{cm/s}^2$ (B) $\dfrac{-\pi^2}{32}\ \text{cm/s}^2$

(C) $\dfrac{\pi^2}{32}\ \text{cm/s}^2$ (D) $-\dfrac{\sqrt{3}}{32}\pi^2\ \text{cm/s}^2$

(48) $x = A\sin\omega t$

$\Rightarrow x = 1\sin\frac{\pi}{4}\cdot\frac{4}{3}$

$= \frac{\sqrt{3}}{2}$

$\Rightarrow a = -\omega^2 x = -\frac{\pi^2\sqrt{3}}{32}$

$\frac{2\pi}{\omega} = 8$

$\Rightarrow \omega = \frac{\pi}{4}$

(D)

49. If the resultant of all the external forces acting on a system of particles is zero, then from an inertial frame, one can surely say that

(A) linear momentum of the system does not change in time
(B) kinetic energy of the system does not change in time
(C) angular momentum of the system does not change in time
(D) potential energy of the system does not change in time

(49) (A)

50. A student performed the experiment of determination of focal length of a concave mirror by u-v method using an optical bench of length 1.5 meter. The focal length of the mirror used is 24 cm. The maximum error in the location of the image can be 0.2 cm. The 5 sets of (u, v) values recorded by the student (in cm) are: (42, 56), (48, 48), (60, 40), (66, 33), (78, 39). The data set(s) that **cannot** come from experiment and is (are) incorrectly recorded, is (are)

(A) (42, 56) (B) (48, 48) (C) (66, 33) (D) (78, 39)

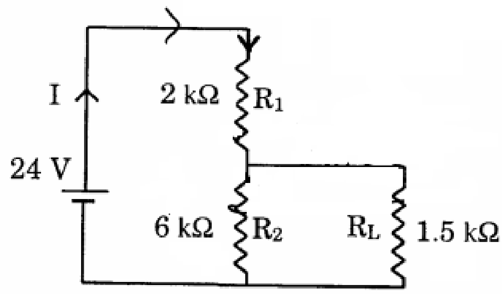
(50) (C) (D)

$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$

$\Rightarrow \frac{1}{v} = \frac{1}{f} - \frac{1}{u}$

$\Rightarrow v = \frac{uf}{u-f}$, check the values.

51. For the circuit shown in the figure



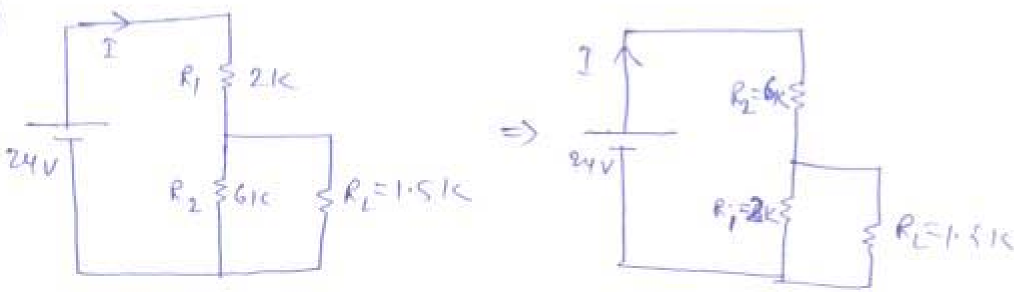
(A) the current I through the battery is 7.5 mA

(B) the potential difference across R_L is 18 V

(C) ratio of powers dissipated in R_1 and R_2 is 3

(D) if R_1 and R_2 are interchanged, magnitude of the power dissipated in R_L will decrease by a factor of 9

51



$R_{eq} = \frac{16}{5}$

$I = 7.5$

$V_{R_1} = 7.5 \times 2 = 15$

$V_{R_L} = 24 - 15 = 9$ V

$\frac{P_{R_1}}{P_{R_L}} = \frac{15^2/2}{9^2/6} = \frac{25}{3}$

$P_{R_L} = \frac{9^2}{1.5}$

if R_1 & R_2 interchanged

$I = \frac{7}{2}$ A

$V_L = 24 - \frac{7}{2} \times 6 = 3$ V

$P'_{R_L} = \frac{3^2}{1.5}$

~~Required ratio~~

$\frac{P'_{R_L}}{P_{R_L}} = \frac{1}{9}$

(A) (D)

52. C_v and C_p denote the molar specific heat capacities of a gas at constant volume and constant pressure, respectively. Then

(A) $C_p - C_v$ is larger for a diatomic ideal gas than for a monoatomic ideal gas

(B) $C_p + C_v$ is larger for a diatomic ideal gas than for a monoatomic ideal gas

(C) C_p / C_v is larger for a diatomic ideal gas than for a monoatomic ideal gas

(D) $C_p \cdot C_v$ is larger for a diatomic ideal gas than for a monoatomic ideal gas

52

$C_p - C_v = R$

$C_p/C_v = \gamma$

$C_p = \frac{\gamma R}{\gamma - 1}$

$C_v = \frac{R}{\gamma - 1}$

(B) & (D)

Paragraph for Question Nos. 53 to 55

Scientists are working hard to develop nuclear fusion reactor. Nuclei of heavy hydrogen, ${}^2_1H$, known as deuteron and denoted by D, can be thought of as a candidate for fusion reactor. The D-D reaction is ${}^2_1H + {}^2_1H \rightarrow {}^3_2He + n + \text{energy}$. In the core of fusion reactor, a gas of heavy hydrogen is fully ionized into deuteron nuclei and electrons. This collection of ${}^2_1H$ nuclei and electrons is known as plasma. The nuclei move randomly in the reactor core and occasionally come close enough for nuclear fusion to take place. Usually, the temperatures in the reactor core are too high and no material wall can be used to confine the plasma. Special techniques are used which confine the plasma for a time t_0 before the particles fly away from the core. If n is the density (number/volume) of deuterons, the product nt_0 is called Lawson number. In one of the criteria, a reactor is termed successful if Lawson number is greater than 5×10^{14} s/cm^3.

It may be helpful to use the following: Boltzmann constant $k = 8.6 \times 10^{-5}$ eV/K; $\frac{e^2}{4\pi\varepsilon_0} = 1.44 \times 10^{-9}$ eVm.

53. In the core of nuclear fusion reactor, the gas becomes plasma because of

(A) strong nuclear force acting between the deuterons

(B) Coulomb force acting between the deuterons

(C) Coulomb force acting between deuteron-electron pairs

(D) the high temperature maintained inside the reactor core

53 → D

54. Assume that two deuteron nuclei in the core of fusion reactor at temperature T are moving towards each other, each with kinetic energy $1.5\ kT$, when the separation between them is large enough to neglect Coulomb potential energy. Also neglect any interaction from other particles in the core. The minimum temperature T required for them to reach a separation of 4×10^{-15} m is in the range

(A) $1.0 \times 10^{9}\ \text{K} < T < 2.0 \times 10^{9}\ \text{K}$

(B) $2.0 \times 10^{9}\ \text{K} < T < 3.0 \times 10^{9}\ \text{K}$

(C) $3.0 \times 10^{9}\ \text{K} < T < 4.0 \times 10^{9}\ \text{K}$

(D) $4.0 \times 10^{9}\ \text{K} < T < 5.0 \times 10^{9}\ \text{K}$

(54) using conservation of energy

$2 \times KE = -\Delta U$

$\Rightarrow 2 \times 1.5\ kT = \dfrac{v^2}{4\pi\epsilon_0 r}$, solving $T \simeq 1.4 \times 10^{9}$ K (A)

55. Results of calculations for four different designs of a fusion reactor using D-D reaction are given below. Which of these is most promising based on Lawson criterion?

(A) deuteron density $= 2.0 \times 10^{12}\ \text{cm}^{-3}$, confinement time $= 5.0 \times 10^{-3}$ s

(B) deuteron density $= 8.0 \times 10^{14}\ \text{cm}^{-3}$, confinement time $= 9.0 \times 10^{-1}$ s

(C) deuteron density $= 4.0 \times 10^{23}\ \text{cm}^{-3}$, confinement time $= 1.0 \times 10^{-11}$ s

(D) deuteron density $= 1.0 \times 10^{24}\ \text{cm}^{-3}$, confinement time $= 4.0 \times 10^{-12}$ s

(55) $n t_0 > 5 \times 10^{14}$

for given values. (B) has greatest value

$= 7.2 \times 10^{14}$ (B)

Paragraph for Question Nos. 56 to 58

When a particle is restricted to move along x-axis between $x = 0$ and $x = a$, where a is of nanometer dimension, its energy can take only certain specific values. The allowed energies of the particle moving in such a restricted region, correspond to the formation of standing waves with nodes at its ends $x = 0$ and $x = a$. The wavelength of this standing wave is related to the linear momentum p of the particle according to the de Broglie relation. The energy of the particle of mass m is related to its linear momentum as $E = \frac{p^2}{2m}$. Thus, the energy of the particle can be denoted by a quantum number 'n' taking values 1, 2, 3, ... ($n = 1$, called the ground state) corresponding to the number of loops in the standing wave.

Use the model described above to answer the following three questions for a particle moving in the line $x = 0$ to $x = a$. Take $h = 6.6 \times 10^{-34}$ J s and $e = 1.6 \times 10^{-19}$ C.

56. The allowed energy for the particle for a particular value of n is proportional to

(A) a^{-2} (B) $a^{-3/2}$ (C) a^{-1} (D) a^2

56 $\lambda = \frac{h}{p}$

$E = \frac{p^2}{2m}$

and $a = n \cdot \frac{\lambda}{2}$ (standing wave)

56 → A

$\Rightarrow \quad E = \frac{n^2 h^2}{8 m a^2} \Rightarrow E \propto a^{-2}$

$E_1 = \frac{1^2 \times h^2}{8 m a^2} \simeq 8$ meV

also $E = \frac{1}{2} m v^2 \Rightarrow v^2 = \frac{2 n^2 h^2}{8 m^2 a^2} \Rightarrow v \propto n$

57. If the mass of the particle is $m = 1.0 \times 10^{-30}$ kg and $a = 6.6$ nm, the energy of the particle in its ground state is closest to

(A) 0.8 meV (B) 8 meV (C) 80 meV (D) 800 meV

(57) $\lambda = \frac{h}{p}$ (57) → B

$E = \frac{p^2}{2m}$

and $a = n \cdot \frac{\lambda}{2}$ (standing wave)

$\Rightarrow E = \frac{n^2 h^2}{8ma^2} \Rightarrow E \propto a^{-2}$

$E_1 = \frac{1^2 \times h^2}{8ma^2} = 8 \text{ meV}$

also $E = \frac{1}{2}mv^2 \Rightarrow v^2 = \frac{2n^2h^2}{8m^2a^2} \Rightarrow v \propto n$

58. The speed of the particle, that can take discrete values, is proportional to

(A) $n^{-3/2}$ (B) n^{-1} (C) $n^{1/2}$ (D) n

(58) $\lambda = \frac{h}{p}$ (58) → D

$E = \frac{p^2}{2m}$

and $a = n \cdot \frac{\lambda}{2}$ (standing wave)

$\Rightarrow E = \frac{n^2 h^2}{8ma^2} \Rightarrow E \propto a^{-2}$

$E_1 = \frac{1^2 \times h^2}{8ma^2} = 8 \text{ meV}$

also $E = \frac{1}{2}mv^2 \Rightarrow v^2 = \frac{2n^2h^2}{8m^2a^2} \Rightarrow v \propto n$

59. Six point charges, each of the same magnitude q, are arranged in different manners as shown in **Column II**. In each case, a point M and a line PQ passing through M are shown. Let E be the electric field and V be the electric potential at M (potential at infinity is zero) due to the given charge distribution when it is at rest. Now, the whole system is set into rotation with a constant angular velocity about the line PQ. Let B be the magnetic field at M and μ be the magnetic moment of the system in this condition. Assume each rotating charge to be equivalent to a steady current.

Column I

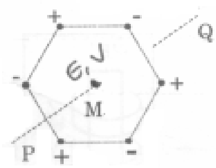
(A) $E = 0$

(B) $V \neq 0$

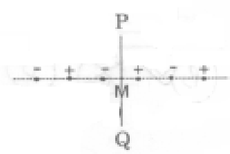
(C) $B = 0$

(D) $\mu \neq 0$

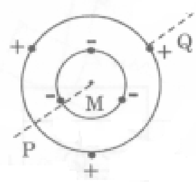
Column II

(p) 

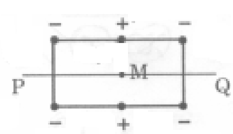
Charges are at the corners of a regular hexagon. M is at the centre of the hexagon. PQ is perpendicular to the plane of the hexagon.

(q) 

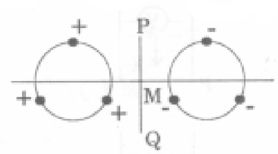
Charges are on a line perpendicular to PQ at equal intervals. M is the mid-point between the two innermost charges.

(r) 

Charges are placed on two coplanar insulating rings at equal intervals. M is the common centre of the rings. PQ is perpendicular to the plane of the rings.

(s) 

Charges are placed at the corners of a rectangle of sides a and $2a$ and at the mid points of the longer sides. M is at the centre of the rectangle. PQ is parallel to the longer sides.

(t) 

Charges are placed on two coplanar, identical insulating rings at equal intervals. M is the mid-point between the centres of the rings. PQ is perpendicular to the line joining the centres and coplanar to the rings.

59

A → P, R, S

B → R, S

C → P, Q, T

D → R, S

60. **Column II** shows five systems in which two objects are labelled as X and Y. Also in each case a point P is shown. **Column I** gives some statements about X and/or Y. Match these statements to the appropriate system(s) from **Column II**.

Column I

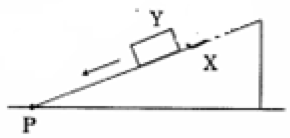
(A) The force exerted by X on Y has a magnitude Mg.

(B) The gravitational potential energy of X is continuously increasing.

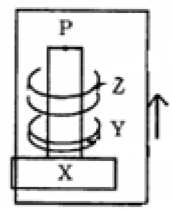
(C) Mechanical energy of the system X + Y is continuously decreasing.

(D) The torque of the weight of Y about point P is zero.

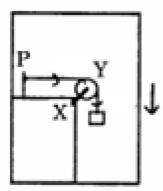
Column II

(p) 

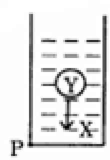
Block Y of mass M left on a fixed inclined plane X, slides on it with a constant velocity.

(q) 

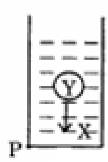
Two ring magnets Y and Z, each of mass M, are kept in frictionless vertical plastic stand so that they repel each other. Y rests on the base X and Z hangs in air in equilibrium. P is the topmost point of the stand on the common axis of the two rings. The whole system is in a lift that is going up with a constant velocity.

(r) 

A pulley Y of mass m_0 is fixed to a table through a clamp X. A block of mass M hangs from a string that goes over the pulley and is fixed at point P of the table. The whole system is kept in a lift that is going down with a constant velocity.

(s) 

A sphere Y of mass M is put in a nonviscous liquid X kept in a container at rest. The sphere is released and it moves down in the liquid.

(t) 

A sphere Y of mass M is falling with its terminal velocity in a viscous liquid X kept in a container.

60) A → P, T
B → Q, S, T
C → P, R, ~~S~~, T
D → Q