1. Let x, y, z be non-zero real numbers. Suppose α, β, γ are complex numbers such that $|\alpha| = |\beta| = |\gamma| = 1$. If $x+y+z = 0 = \alpha x+\beta y+\gamma z$, then prove that $\alpha = \beta = \gamma$.

2. Let c be a fixed real number. Show that a root of the equation

$$x(x+1)(x+2)\cdots(x+2009) = c$$

can have multiplicity at most 2. Determine the number of values of c for which the equation has a root of multiplicity 2.

3. Let $1, 2, 3, 4, 5, 6, 7, 8, 9, 11, 12, \cdots$ be the sequence of all the positive integers which do not contain the digit zero. Write $\{a_n\}$ for this sequence. By comparing with a geometric series, show that $\sum_n \frac{1}{a_n} < 90$.

4. Find the values of x, y for which $x^2 + y^2$ takes the minimum value where $(x+5)^2 + (y-12)^2 = 14^2$.

5. Let p be a prime number bigger than 5. Suppose, the decimal expansion of $1/p$ looks like $0.\overline{a_1 a_2 \cdots a_r}$ where the line denotes a recurring decimal. Prove that 10^r leaves a remainder of 1 on dividing by p.

6. Let a, b, c, d be integers such that $ad - bc$ is non-zero. Suppose b_1, b_2 are integers both of which are multiples of $ad - bc$. Prove that there exist integers simultaneously satisfying both the equalities $ax + by = b_1, cx + dy = b_2$.

7. Compute the maximum area of a rectangle which can be inscribed in a triangle of area M.

8. Suppose you are given six colours and, are asked to colour each face of a cube by a different colour. Determine the different number of colourings possible.

9. Let $f(x) = ax^2 + bx + c$ where a, b, c are real numbers. Suppose $f(-1), f(0), f(1) \in [-1, 1]$. Prove that $|f(x)| \leq 3/2$ for all $x \in [-1, 1]$.

10. Given odd integers a, b, c, prove that the equation $ax^2 + bx + c = 0$ cannot have a solution x which is a rational number.