Reg. No. : ....

D 1083 **Q.P. Code : [D 07 PMA 06]**

(For the candidates admitted from 2007 onwards)

M.Sc. DEGREE EXAMINATION, DECEMBER 2013.

Second Year

Mathematics

MECHANICS

Time : Three hours Maximum : 100 marks

Answer any FIVE questions.

All questions carry equal marks.

$(5 \times 20 = 100)$

1. (a) Derive the equation of motion of a single particle using plane polar coordinates.

   (b) Define Atwood's machine. Derive its equation of motion.

2. (a) Derive a necessary condition for the stationary value of the functional

$$J(\alpha) = \int_{x_1}^{x_2} f(y(x,\alpha), \dot{y}(x,\alpha), x)\, dx.$$

(b) Find the shape of the curve passing between two fixed points (x_1, y_1) and (x_2, y_2) which when resolved about the y axis generates a minimum surface of revolution.

3. (a) Prove that the generalized momentum conjugate to a cyclic coordinate is conserved.

(b) A heavy particle is placed at the top of a vertical hoop. Calculate the reaction of the hoop on the particle by means of the Lagrange's undetermined multipliers and Lagrange's equation. Find the height at which the particle falls off.

4. (a) State and prove the Jacobi's form of the least action principle.

(b) Derive the Hamilton's equation from variational principle.

5. (a) Derive the sympletic condition for a canonical transformation.

(b) Define Poisson brackets and prove that the fundamental Poisson brackets are invariant under canonical transformation.

6. (a) Prove directly that the transformation :

$Q_1 = q_1,\ p_1 = p_1 - 2p_2$

$Q_2 = p_2,\ p_2 = -2q_1 - q_2$

is canonical and find a generating function.

(b) Determine whether the transformation

$$Q_1 = q_1 q_2, \; p_1 = \frac{p_1 - p_2}{q_2 - q_1} + 1$$

$$Q_2 = q_1 + q_2, \; p_2 = \frac{q_2 p_2 - q_1 p_1}{q_2 - q_1} - (q_2 + q_1) \text{ is}$$

canonical.

7. (a) Show that when solving the Hamilton-Jacobi equation we are at the same time obtaining a solution to the mechanical problem.

(b) Derive the Hamilton-Jacobi equation for Hamilton's characteristic function.

8. (a) For a conservative system show that by solving an appropriate partial differential equation one can construct a canonical transformation such that the new Hamiltonian is a function of the new coordinate only.

(b) Set up the problem of the heavy symmetrical top, with one point fixed, in the Hamilton-Jacobi method and obtain the formed solution to the motion.

———————

Reg. No. : ....

D 1084 **Q.P. Code : [D 07 PMA 07]**

(For the candidates admitted from 2007 onwards)

M.Sc. DEGREE EXAMINATION, DECEMBER 2013.

Second Year

Mathematics

OPERATIONS RESEARCH

Time : Three hours Maximum : 100 marks

Answer any FIVE questions.

Each question carries 20 marks.

$(5 \times 20 = 100)$

1. (a) Maximize $z = 4x_1 + 5x_2$

Subject to $x_1 + x_2 \geq 1$

$-2x_1 + x_2 \leq 1$

$4x_1 - x_2 \geq 1$

$x_1, x_2 \geq 0.$

(b) Solve by Simplex method :

Maximize $z = 2x_1 + x_2$

Subject to $x_1 + x_2 \leq 6$

$x_1 + 2x_2 \leq 10$

$x_1 - x_2 \leq 2$

$x_1 - 2x_2 \leq 1$

$x_1, x_2 \geq 0.$

2. (a) Solve the following transportation problem :

		Ware house				
		W_1	W_2	W_3	W_4	Supply
	F_1	14	25	45	5	6
Factory	F_2	65	25	35	55	8
	F_3	35	3	65	15	16
Requirement		4	7	6	13	

(b) How different jobs can be done on four different machines. The set up and take down time costs are consumed to be prohibitively high for change over. The matrix below give the cost in rupee of producing job i on machine j.

		Machines			
		m_1	m_2	m_3	m_4
	J_1	5	7	11	6
Jobs	J_2	8	5	9	6
	J_{13}	4	7	10	7
	J_4	10	4	8	3

How should to jobs be consigned to the various machine so that the total cost is minimized?

3. (a) Consider the following LP :

Maximize $z = 4x_1 + 14x_2$

Subject to $2x_1 + 7x_2 + x_3 = 21$

$7x_1 + 2x_2 + x_4 = 21$

$x_1, x_2, x_4 \geq 0.$

Check the optimality and feasibility of the basic solution :

$$X_0 = \begin{bmatrix} x_2 \\ x_4 \end{bmatrix}; \; B^{-1} = \begin{bmatrix} 1/7 & 0 \\ -2/7 & 1 \end{bmatrix}$$

(b) Solve the following problem by dual simplex algorithm.

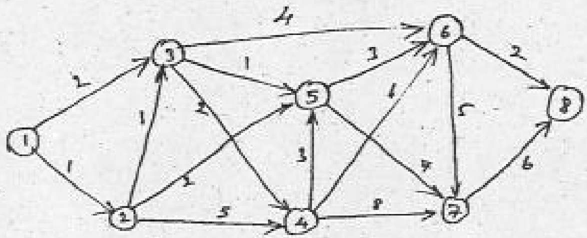
Minimize $z = 4x_1 + 2x_2$

Subject to $x_1 + x_2 = 1$

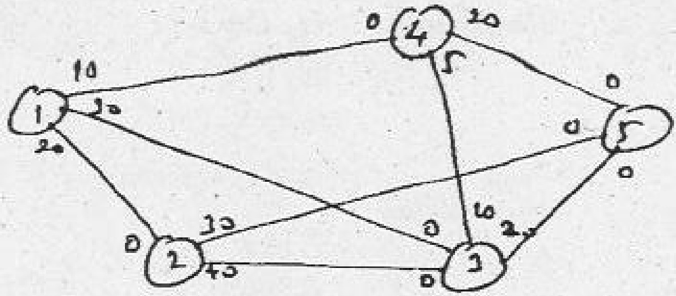
$3x_1 - x_2 \geq 2$

$x_1, x_2 \geq 0.$

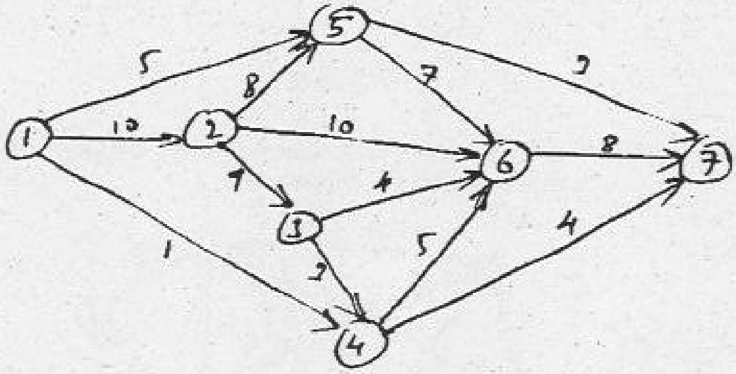
4. (a) Find the shortest route between cities 1 and 8 in the following networks.



(b) Determine the maximal flow in the network given below :



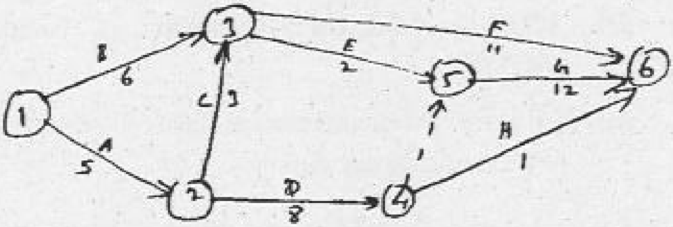
5. (a) Determine the critical path for the project network given below :



(b) For the following project.

(i) Determine the critical path.

(ii) Determine the total and free floats for the network and identify to red-flagged activities.



6. (a) Solve by Revised simplex method

Minimize $z = 2x_1 + x_2$

Subject to $3x_1 + x_2 = 3$

$$4x_1 + 3x_2 \geq 6$$

$$x_1 + 2x_2 \leq 3$$

$$x_1, x_2 \geq 0.$$

(b) Consider the LP maximize

$z = 50x_1 + 30x_2 + 10x_3$

Subject to $2x_1 + x_2 = 1$

$$2x_2 = -5$$

$$4x_1 + x_3 = 6$$

$$x_1, x_2, x_3 \geq 0$$

(i) Write the dual.

(ii) Show by inspection that the primal is infeasible.

(iii) Show that dual is unbounded.

7. (a) Write a note on Generation of Random numbers.

(b) Explain the mechanics of discrete simulation with suitable example.

8. (a) Classify the Markov chain and find its stationary distribution $\begin{bmatrix} \frac{1}{4} & \frac{1}{4} & \frac{1}{2} \\ \frac{1}{4} & \frac{3}{4} & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} \end{bmatrix}$.

(b) Find the mean recurrence time for each state of the following Markov chain.

$$\begin{bmatrix} \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ \frac{1}{2} & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{5} & \frac{3}{5} & \frac{1}{5} \end{bmatrix}$$

Reg. No. : ....

D 1085 **Q.P. Code : [D 07 PMA 08]**

(For the candidates admitted from 2007 onwards)

M.Sc. DEGREE EXAMINATION, DECEMBER 2013.

Second Semester

Mathematics

TOPOLOGY

Time : Three hours Maximum : 100 marks

Answer any FIVE questions.

Each question carries 20 marks.

($5 \times 20 = 100$)

1. (a) State and prove the existence theorem of a choice function.

   (b) State and prove the well-ordering theorem.

2. (a) Prove that the Cartesian product of a connected space is connected.

   (b) Prove that a space X is locally connected if and only if for every open set U of X each component of U is open in X.

3. (a) Prove that every metrizable space is normal.

   (b) Prove that every regular space with a countable basis is normal.

4. State and prove the Tychonoff theorem.

5. (a) Prove that in a simply connected space X any two paths having the same initial and final points are path homotopic.

   (b) State and prove the special Van kampen theorem.

6. (a) Prove that every compact subset of a Hausdorff spuce is closed.

   (b) State and prove the tube lemma.

7. (a) Prove that every well-ordered set X is normal in the order topology.

   (b) State and prove the Tiet≠e extension theorem.

8. (a) Prove that the map $P : R \to S'$ given by the equation $p(x) = (\cos 2\pi x, \sin 2\pi x)$ is a covering map.

   (b) Let A be a strong deformation retract of X. Let $a_0 \in A$. Then prove that the inclusion map $j : (A, a_0) \to (X, a_0)$ induces a isomorphism of fundamental groups.

Reg. No. : ....

D 1086 **Q.P. Code : [D 07 PMA 09]**

(For the candidates admitted from 2007 onwards)

M.Sc. DEGREE EXAMINATION, DECEMBER 2013.

Second Year

Mathematics

COMPUTER PROGRAMMING (C++ THEORY)

Time : Three hours Maximum : 100 marks

Answer any FIVE questions.

All questions carry equal marks.

$(5 \times 20 = 100)$

1. (a) Explain the benefits of object oriented programming.

   (b) Write the application of C++ program.

2. (a) Explain user defined data type.

   (b) Write a short notes on Identifiers.

3. (a) What are component selection and class member binary operation?

   (b) Write the order of precedence of the operators.

4. (a) What are the classification of C++ data types?

   (b) Write a program to compute $n!$.

5. (a) List out the purpose of declaration statement.

   (b) What are the rules for implicit conversion?

6. (a) Write a short notes on virtual function.

   (b) Explain unformatted I/O operations.

7. (a) Write the characteristics of member functions.

   (b) Explain two dimensional array with example.

8. (a) List out the advantages of operator overloading.

   (b) Write a short notes on virtual base classes.

Reg. No. : ....

D 1087 **Q.P. Code : [D 07 PMA 10]**

(For the candidates admitted from 2007 onwards)

M.Sc. DEGREE EXAMINATION, DECEMBER 2013.

Second Year

Mathematics

FUNCTIONAL ANALYSIS

Time : Three hours Maximum : 100 marks

Answer any FIVE questions.

All questions carry equal marks.

($5 \times 20 = 100$)

1. (a) State and prove the Hahn-Banach theorem.

   (b) State and prove the closed Graph theorem.

2. (a) If N_1 and N_2 are normal operators on H with the property that either commutes with the adjoint of the other, then prove that $N_1 + N_2$ and $N_1 N_2$ are normal.

   (b) Prove that a closed convex subset C of a Hilbert space H contains a unique vector of smallest norm.

3. (a) If P and Q are the projections on closed linear subspces M and N of H then prove that $M \perp N \Leftrightarrow PQ = 0 \Leftrightarrow QP = 0$.

(b) If T is normal then prove that each M_i reduces T.

4. (a) If G is an open set then prove that S is a closed set.

(b) Prove that the boundary of S is a subset of z.

5. (a) State and prove the Gelfand-Neumark theorem.

(b) If x is a normal element in a B^* – algebra then prove that $\|x^2\| = \|x\|^2$.

6. (a) Let M be a linear subspace of a normed linear space N and let f be a functional defined on M. If x_0 is a vector not in M, and if $M_0 = M + [x_0]$ is the linear subspace spanned by M and x_0 then prove that f can be extended to a functional f_0 defined on M_0 such that $\|f_0\| = \|f\|$.

(b) Prove that a non-empty subset X of a normed linear space N is bounded iff $f(X)$ is a bounded set of numbers for each f in N^*.

7. (a) If $\{e_i\}$ is an orthonormal set in a Hilbert space H and if x is any vector in H then prove that the set $S = \{e_i : (x, e_i) \neq 0\}$ is either empty or countable.

(b) If T is an operator on H for which $(Tx, x) = 0$ for all x then prove that $T = 0$.

8. (a) If 0 is the only topological divisor of zero in A then prove that $A = C$.

(b) If r is an element of R then prove that $1 - r$ is regular.
