SATHYABAMA UNIVERSITY

(Established under section 3 of UGC Act, 1956)

Course & Branch: B.E/B.TECH – Common to ALL Branches (Except Bio Groups)
Title of the paper: Engineering Mathematics - I
Semester: I Max. Marks: 80
Sub.Code: 6C0002(2006/2007/2008) Time: 3 Hours
Date: 08-12-2008 Session: FN

PART – A (10 x 2 = 20)

Answer All the Questions

1. Find the sum and product of eigen values of the matrix
$$\begin{pmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{pmatrix}$$

2. State Cayley-Hamilton theorem.

3. Prove that $\frac{1}{1.2}+\frac{1}{2.2^2}+\frac{1}{3.2^3}=\ldots=\log 2.$

4. Find the Coefficient of x^n in $(2 + 3x)^{-1}$.

5. Find the curvature of the circle x+ + y^2 = 25

6. Define evolute of a curve.

7. Find $\frac{\partial(r,\theta)}{\partial(x,y)}$ of x = cosθ , y = r sin θ.

8. Find the stationary points of $3x - x^2 - y^2$.

9. Solve $xy'' + y' = 0$

10. Solve $y'' + y = \sin x$.

PART – B (5 x 12 = 60)

Answer All the Questions

11. Verify Cayley-Hamilton theorem and hence find the inverse of
$$\begin{pmatrix} 1 & 2 & -1 \\ 3 & -3 & 1 \\ 2 & 1 & -2 \end{pmatrix}$$

(or)

12. Reduce the quadratic form $6x_1^2 + 3x_2^2 + 3x_3^2 - 4x_1x_2 - 2x_2x_3 + 4x_1x_3$ to canonical form by an orthogonal transformation.

13. (a) Find the sum of $1 - \frac{1}{4} + \frac{1.3}{48} - \frac{1.3.5}{4.8.12} + \infty$

(b) If x is small, prove that $\left(\frac{1-x}{1+x}\right)^{\frac{1}{2}} = 1 - x + \frac{x^2}{2}$

(or)

14. (a) Evaluate $\lim_{x \to 0} \left(\frac{a^x - b^x}{x}\right)$

(b) Show that $\frac{1}{1.2} - \frac{1}{2.3} + \frac{1}{3.4} \ldots = \log\left(\frac{4}{e}\right)$

15. (a) Find the radius of curvature for $y = \frac{(\log x)}{x} atx = 1$

(b) Find the envelope $(x - \alpha)^2 + y^2 = k\alpha$ where α is the parameter.

(or)

16. Find the evolute of $y^2 = 4ax$ as the envelope of normals.

17. (a) Expand by Taylor's series $f(x, y) = e^x \cos y$ at (0, 0)

    (b) Find the dimensions of the rectangular box without a top of maximum capacity whose surface area is 108 sq.cm.

(or)

18. Show that $\int_0^{\varepsilon} \frac{\log(1+ax)}{1+x^2}\,dx = \frac{1}{2}\log\left(1+\alpha^2\right)\tan^1 x$

    Hence deduce that $\int_0^1 \frac{\log(1+x)}{1+x^2}\,dx = \frac{\pi \log 2}{8}$

19. (a) Solve $x^2 y'' + xy'' + y = \cos(2 \log x)$

    (b) Solve $\frac{dx}{dt} + y = 0, x + \frac{dy}{dt} = 2\cos t$

(or)

20. (a) Using variation of parameters, solve $y'' + 4y = \tan 2x$.

    (b) Solve $(D^2 - 2D + 1)\, y = e^x (3x^2 - 1)$