B4.2-R3: DISCRETE STRUCTURES

NOTE:

1.	**Answer question 1 and any FOUR questions from 2 to 7.**
2.	**Parts of the same question should be answered together and in the same sequence.**

Time: 3 Hours **Total Marks: 100**

1.

a) i) Determine the power sets of {ϕ , { ϕ }}.

ii) Let S={2, 5, $\sqrt{2}$, 25, π, 5/2} and T={4, 25, $\sqrt{2}$, 6, 3/2}
Find $S \cap T$ and $T \times (S \cap T)$.

b) For integers a,b define a ~ b if and only if 2a +3b = 5n for some integer n. Show that ~ defines an equivalence relation on Z.(set of Integers).

c) Define a Monoid.

d) Draw the Hassediagrams for each of the following partial orders.

i) ({ 1,2,3,4,5,6},$\leq$)

ii) ({ {a},{a,b},{a,b,c},{a,b,c,d},{a,c},{c,d} },$\subseteq$)

e) What is a Spanning tree?

f) Write the converse, inverse and contrapositive of $P \rightarrow Q$.

g) Show that the functions $f : R \rightarrow (1, \infty)$ and $g : (1, \infty) \rightarrow R$

Defined by $f(x)=3^{2x}+1$, $g(x) = \dfrac{\log_3(x-1)}{2}$ are inverse of each others.

(7x4)

2.

a) Find the principal disjunctive normal form of $(P \wedge Q) \vee (\sim P \wedge R) \vee (Q \wedge R)$.

b) Show that $\sim (P \wedge Q)$ follows from $\sim P \wedge \sim Q$.

c) In a group of 25 students, 12 have taken Mathematics,8 have taken Mathematics but not Biology. Find the number of students who have taken Mathematics and biology and those who have taken Biology but not Mathematics.

(6+6+6)

3.

a) For any a, b, c, d in a lattice (A, $\leq$), if a $\leq$ b and c $\leq$ d then Prove that $(a \vee c) \leq (b \vee d)$ and $(a \wedge c) \leq (b \wedge d)$ (where $\vee$ is join and $\wedge$ is meet operation).

b) Prove that if the meet operation is distributive over the join operation in a lattice, then the join operation is also distributive over the meet operation.

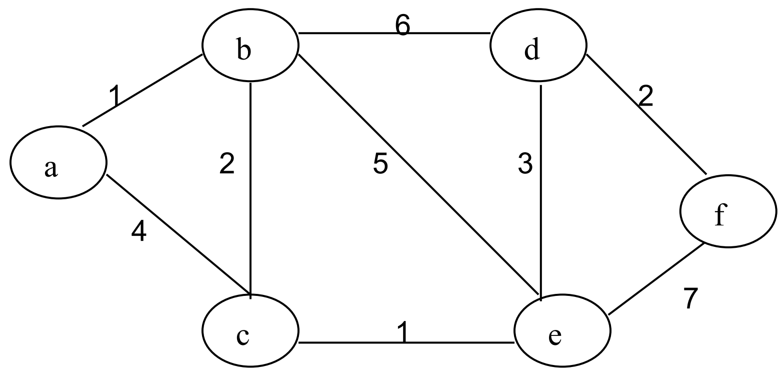
c) Minimize the following expressions using Karnaugh map.

$$F = AB\overline{C} + \overline{A}B\overline{C} + A\overline{B}\overline{C} + \overline{A}\overline{B}\overline{C}$$

(6+6+6)

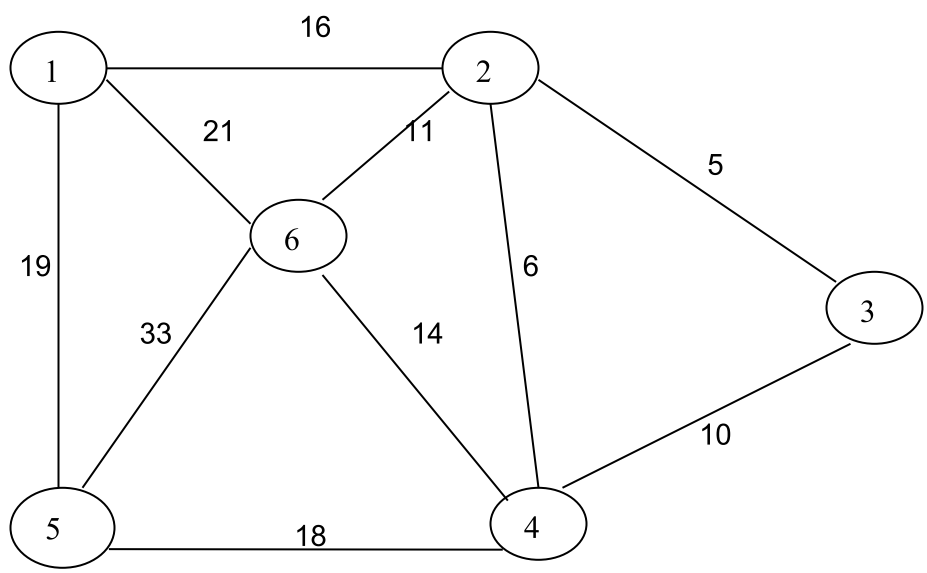
4.

a) Apply Dijkstra's algorithm to the graph given below and find the shortest path from a to f. (Show all the steps)



b) Define briefly the following:
   i) Cut set
   ii) Hamiltonian path
   iii) Bipartite Graph
   iv) Isomorphic graph

c) For the following graph, find its spanning tree of minimal Cost using Kruskal algorithm.



(8+4+6)

5.

a) In how many ways 7 women and 3 men are arranged in a row if the 3 men must always stand next to each other.

b) i) State pigeonhole principle.
   ii) Suppose that a patient is given a prescription of 45 pills with the instruction to take at least one pill per day for 30 days. Then prove that there must be a period of consecutive days during which the patient takes a total of exactly 14 pills.

c) If Fn satisfies the Fibonacci relation for the Fibonacci series (1,1,2,3…) defined by the recurrence relation, $F_n = F_{n-1}+F_{n-2}$, $F_0=F_1=1$ then prove that nth Fibonacci number is given by (for n = 0,1,2,3, -----------).

$$F_n = \frac{1}{2^n\sqrt{5}}[(1+\sqrt{5}^n) - (1-\sqrt{5}^n)]$$

(6+6+6)

6.

a) Prove that for any a and b in a Boolean algebra

$\overline{A \vee B} = \overline{A} \wedge \overline{B}$ and

$\overline{A \wedge B} = \overline{A} \vee \overline{B}$

b) Define the following terms:

i) Permutation of a set
ii) Abelian group
iii) Subgroup
iv) Group Homomorphism.

c) Prove that every finite group of order n is isomorphic to a permutation group of degree n.

(4+8+6)

7.

a) Prove by mathematical induction the following, $3^n>n^3$ for n>3.

b) Find the regular expressions for a Valid Identifier of any length in C language:
(The rule of an Identifier in C language is that first character is an alphabet or an Underscore and the consequent letters are alphabet and/or digit and/or underscore, no extra symbols are allowed except defined above).

c) Define a finite State Machine.

d) Calculate the greatest common divisor of 240 and 70(Step wise) by using Euclid's algorithm.

(6+4+4+4)