Second PUC July – 2007 Question paper

MATHEMATICS

PART – A

Answer *all* the *ten* questions : $10 \times 1 = 10$

1. If $3^{127} \equiv x \pmod{10}$, find x.

2. If $A = \begin{pmatrix} 4 & 3 \\ 1 & 2 \end{pmatrix}$, $B = \begin{pmatrix} -4 \\ 3 \end{pmatrix}$, find AB.

3. In a group $(G, *)$, if $a * x = e \ \forall \ a \in G$, find x.

4. Find the value of $(j - 3k) \times (i - j + 2k)$.

5. Find the centre of the circle passing through $(0, 0)$, $(3, 0)$ and $(0, 5)$.

6. Find the vertex of parabola $(y-2)^2 = -8x$.

7. If $\cos^{-1} x - \sin^{-1} x = 0$, prove that $x = \frac{1}{\sqrt{2}}$.

8. Find amplitude of $2i - 4$.

9. If $y = 3^{-x}$, find $\frac{dy}{dx}$.

10. Evaluate : $\int_0^{\pi/2} \sqrt{1 - \cos 2x}\, dx$.

PART – B

Answer any *ten* questions : $10 \times 2 = 20$

11. If $a \equiv b \pmod m$ and $n | m \; \forall \; n \in I$, prove that $a \equiv b \pmod n$.

12. Without expansion, find the value of

$$\begin{vmatrix} \sin^2 x & \cos^2 x & 1 \\ \cos^2 x & \sin^2 x & 1 \\ -10 & 12 & 2 \end{vmatrix}$$

13. If Q^+ is the set of all positive rationals w.r.t. $*$.

define $a * b = \dfrac{2ab}{3} \; \forall \; a, b \in Q^+$. Find

a) Identity element.

b) Inverse of a under $*$.

14. For any vector $\vec{a}$, prove that

$\vec{a} = (\vec{a} \cdot i)\, i + (\vec{a} \cdot j)\, j + (\vec{a} \cdot k)\, k.$

15. Find the length of tangent from the centre of circle $x^2 + y^2 - 8x = 0$ to the circle $3x^2 + 3y^2 = 7$.

16. Find the centre of ellipse whose vertices are $(2, -2)$ and $(2, 4)$. Also find the length of major axis.

17. If $\tan^{-1} x + \tan^{-1} y = \frac{\pi}{2}$, prove that $xy = 1$.

18. If $x = \text{cis}\, \alpha$ and $y = \text{cis}\, \beta$,

prove that $\sin(\alpha-\beta) = \frac{1}{2i}\left(\frac{x}{y}-\frac{y}{x}\right)$.

19. If $y \log_e x = y - x$, prove that

$$\frac{dy}{dx} = \frac{2-\log_e x}{(1-\log_e x)^2}.$$

20. Prove that x^x is minimum at $x = \frac{1}{e}$.

21. Evaluate : $\int \frac{1}{5e^{3x}+1}\,dx$.

22. Form a differential equation for the equation $x^2 + y^2 + 2ky = 0$.

PART – C

I. Answer any *three* questions : $3 \times 5 = 15$

23. a) Find the G.C.D. of 48 and 18. If $6 = 48m + 18n$, find m and n. 3

b) Solve $51x \equiv 32 \pmod 7$. Write the solution set. 2

24. If

$$\begin{bmatrix} 7 & 6 & -5 \\ 3 & -4 & 1 \\ 1 & 2 & -3 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 30 \\ 0 \\ 10 \end{bmatrix},$$

find x, y and z using Cramer's Rule. 5

25. Prove that the set $G = \{ \ldots\ldots\ldots, 5^{-2}, 5^{-1}, 5^{0}, 5^{1}, 5^{2}, \ldots\ldots\ldots \}$ is an Abelian group under usual multiplication. 5

26. a) Find the area of the triangle ABC where position vectors of A, B, C are $i - j + 2k$, $2j + k$, $j + 3k$ respectively. 3

b) Prove that

$$\vec{a} \times (\vec{b} \times \vec{c}) + \vec{b} \times (\vec{c} \times \vec{a}) + \vec{c} \times (\vec{a} \times \vec{b}) = \vec{0}.$$ 2

II. Answer any *two* questions : $2 \times 5 = 10$

27. a) Obtain the condition for two circles

$$x^2 + y^2 + 2g_1 x + 2f_1 y + c_1 = 0$$

$$x^2 + y^2 + 2g_2 x + 2f_2 y + c_2 = 0$$

to intersect orthogonally. 3

b) The radical axis of two circles is $x - 2y + 6 = 0$. The equation of one of the circles is $2x^2 + 2y^2 - 8x - 4y - 22 = 0$. If the second circle passes through the point $(1, 6)$, find its equation. 2

28. a) Find the centre and the foci of ellipse

$$4x^2 + 9y^2 + 16x - 18y - 11 = 0.$$ 3

b) Find the focal distance of any point (x, y) on the parabola

$y^2 = 4ax$. 2

29. a) Prove that

$$\tan\left\{\frac{1}{2}\sin^{-1}\left(\frac{2x}{1+x^2}\right)+\frac{1}{2}\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)\right\}=\frac{2x}{1-x^2}.$$ 3

b) Find the general solution of

$\tan m\theta = \tan n\theta$. 2

III. Answer any *three* of the following questions : $3 \times 5 = 15$

30. a) Differentiate cosec $4x$ with respect to x from first principles. 3

b) If $y = \tan^{-1}\left[\frac{2+5\tan x}{5-2\tan x}\right]$, find $\frac{dy}{dx}$. 2

31. a) If $y = \left[x+\sqrt{1+x^2}\right]^m$, prove that

$$(1+x^2)\frac{d^2y}{dx^2}+x\frac{dy}{dx}-m^2y=0.$$ 3

b) Find a point on the curve $y = x^3 - 3x$, where tangent is parallel to the line joining the points $(1, -2)$ and $(2, -5)$. 2

32. a) A circular blot of ink in a blotting paper increases in area in such a way that the radius r cm at time t seconds is given by $r = 2t^2 - \frac{t^3}{4}$. Find the rate of increase of area when $t = 2$. 3

b) Prove that $\int uv'\,dx = uv - \int vu'\,dx$

where $u' = \frac{du}{dx}$ and $v' = \frac{dv}{dx}$. 2

33. a) Evaluate : $\int \frac{x \sin^{-1} x}{\sqrt{1 - x^2}}\,dx$. 3

b) Evaluate : $\int \frac{1}{\sqrt{1 - 4x - 4x^2}}\,dx$. 2

34. Find the area enclosed between the parabolas $y^2 = 4ax$ and $x^2 = 4ay$. 5

PART – D

Answer any *two* of the following questions : $2 \times 10 = 20$

35. a) Define director circle of a hyperbola. Derive the equation of director circle of the hyperbola. 6

b) Using $A(x) = \begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix}$

find $adj\,[A(x)]$. Prove that $adj\,[A(x)] = A(-x)$. 4

36. a) Find the fourth roots of $\left(\sqrt{3}-i\right)^3$. Also find their continued product. 6

b) Prove by vector method,

$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$. 4

37. a) Show that the height of a right circular cylinder of the greatest volume which is inscribed in a sphere of radius a is $\frac{2a}{\sqrt{3}}$. Find the radius of the right circular cylinder. 6

b) Find the general solution of

$\sec x - \tan x + \sqrt{3} = 0$ 4

38. a) Prove that $\displaystyle\int_0^{\pi} \frac{x\,dx}{a^2\cos^2 x + b^2\sin^2 x} = \frac{\pi^2}{2ab}$. 6

b) Solve the differential equation

$\frac{dy}{dx} = \tan^2(x+y)$ 4