Second PUC March/April – 2007 Question paper

MATHEMATICS

PART – A

Answer *all* the *ten* questions : $10 \times 1 = 10$

1. Find an integer x, satisfying $5x \equiv 4 \pmod{13}$.

2. If the matrix $\begin{bmatrix} 6 & x-2 \\ 3 & x \end{bmatrix}$ is singular, find x.

3. On the set Z of integers if "O" is defined by $a \circ b = a + b + 1$, $\forall\ a, b \in Z$, find the identity element.

4. If $\vec{a} = 2\hat{i} + 3\hat{j}$ and $\vec{b} = 3\hat{i} + 4\hat{j}$, find the magnitude of $\vec{a} + \vec{b}$.

5. Write the condition (in terms of g, f and c) under which $x^2 + y^2 + 2gx + 2fy + c = 0$ becomes a point circle.

6. Find the equation of the directrix of the parabola $y^2 = -8x$.

7. Find the value of $\sin\left[\frac{\pi}{2} - \sin^{-1}\left(\frac{-\sqrt{3}}{2}\right)\right]$.

8. Find the modulus of the complex number $\frac{2-i}{5i}$.

9. If $f(x) = x^2 + \frac{1}{x^2}$, find $f'(1)$.

10. Evaluate $\int_0^{\pi/4} \sin^3 x \cos x \, dx$.

PART – B

Answer any *ten* questions : $10 \times 2 = 20$

11. If $a \equiv b \pmod{m}$ and $n > 1$ is a positive divisor of m, prove that $a \equiv b \pmod{n}$.

12. Evaluate

$$\begin{vmatrix} -a^2 & ab & ac \\ ab & -b^2 & bc \\ ac & bc & -c^2 \end{vmatrix}$$

13. Define the binary operation, on a non-empty set S. Give an example to show that, on Z, the operation $*$, defined by $a * b = a^b$, is not binary.

14. Find the angle between the vectors $2\hat{i} - 2\hat{j} + \hat{k}$ and $2\hat{i} - \hat{j} - 2\hat{k}$.

15. Examine whether the point (1, 5) lies outside, inside or on the circle $x^2 + y^2 + 4x + 2y + 3 = 0$.

16. The two ends of the major axis of an ellipse are (5, 0) and (– 5, 0). If $3x - 5y - 9 = 0$ is a focal chord, find the eccentricity of the ellipse.

17. Prove that $2\tan^{-1}\frac{1}{2} + \sin^{-1}\frac{3}{5} = \frac{\pi}{2}$.

18. If $x = \cos\alpha + i\sin\alpha$ and

$y = \cos\beta + i\sin\beta$

prove that $\dfrac{y^3}{x^2} + \dfrac{x^2}{y^3} = 2\cos(3\beta - 2\alpha)$.

19. If $x^y = a^x$, prove that $\dfrac{dy}{dx} = \dfrac{x\log a - y}{x\log x}$.

20. Find the length of the subtangent to the curve $y = \sqrt{x^2 + x + 1}$ at the point $(1, \sqrt{3})$ on it.

21. Integrate $\sin 3x \cos x$ with respect to x.

22. Form the differential equation of the family of straight lines passing through the origin of Cartesian plane.

PART – C

I. Answer any *three* questions : $3 \times 5 = 15$

23. Find the G.C.D. of 408 and 1032 using Euclidean algorithm. Express it in two ways in the form $408m + 1032n$ where m, n are integers. 5

24. a) Find x and y if

$$\begin{bmatrix} x & 2 & -3 \\ 5 & y & 2 \\ 1 & -1 & 1 \end{bmatrix}\begin{bmatrix} 3 & -1 & 2 \\ 4 & 2 & 5 \\ 2 & 0 & 3 \end{bmatrix} = \begin{bmatrix} 5 & 3 & 3 \\ 19 & -5 & 16 \\ 1 & -3 & 0 \end{bmatrix}$$

3

b) Solve by Cramer's rule :

$$2x - y = 10$$

$$x - 2y = 2.$$

2

25. a) Given that H is a non-empty subset of a set G and $(G, *)$ is a group. If for all $a, b \in H$, $a * b^{-1} \in H$, prove that $(H, *)$ is a subgroup of $(G, *)$. 3

b) If, in a group G, every element is its own inverse, prove that G is an Abelian group. 2

26. a) Using vector method, find the area of the triangle whose vertices are $(1, 2, 3)$, $(2, -1, 1)$ and $(1, 2, -4)$. 3

b) Find the volume of the parallelopiped whose co-terminal edges are $2\hat{i} + \hat{j} - \hat{k}$, $3\hat{i} - 2\hat{j} + 2\hat{k}$ and $\hat{i} - 3\hat{j} - 3\hat{k}$. 2

II. Answer any *two* questions : $2 \times 5 = 10$

27. a) Find the equation of the circle which passes through the point $(2, 3)$, has its centre on $x + y = 4$ and cuts orthogonally the circle $x^2 + y^2 - 4x + 2y - 3 = 0$. 3

b) Find the radical centre of the circles $x^2 + y^2 + 2x - 4 = 0$,

$x^2 + y^2 + 4y - 4 = 0$ and $x^2 + y^2 - 2x - 5 = 0$. 2

28. a) Find the centre and the eccentricity of the hyperbola

$x^2 - 3y^2 - 4x - 6y - 11 = 0$. 3

b) Find the equation of the parabola with vertex $(-4, 2)$, axis $y = 2$ and passing through the point $(0, 6)$. 2

29. a) If $x \geq 0$ and $y \geq 0$, prove that

$\sin^{-1} x - \sin^{-1} y = \sin^{-1}\left(x\sqrt{1-y^2} - y\sqrt{1-x^2} \right)$. 3

b) Find the general solution of the equation

$\cos x - \cos 7x = \sin 4x$. 2

III. Answer any *three* of the following questions : $3 \times 5 = 15$

30. a) Differentiate e^x with respect to x from first principles. 3

b) Differentiate $\log_{10}(\log x)$ with respect to x. 2

31. a) If $y = x \cosh x$, prove that

$xy_2 - 2y_1 - xy + 2\cosh x = 0$. 3

b) Prove that x^x function has a minimum value at $x = \frac{1}{e}$. 2

32. a) Find $\int \frac{x+1}{x^2 - 4x + 6}\, dx$. 3

b) A stone is thrown up vertically and the height x feet reached by it in time "t" seconds is given by $x = 80t - 16t^2$. Find the time for the stone to reach its maximum height. Also find the maximum height reached by the stone. 2

33. a) If $x = a(\theta + \sin\theta)$ and $y = a(1 - \cos\theta)$, find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$. 3

b) Find $\int \frac{xe^x}{(1+x)^2}\, dx$ 2

34. Find the area bounded by the curves $4y^2 = 9x$ and $3x^2 = 16y$. 5

PART – D

Answer any *two* of the following questions : $2 \times 10 = 20$

35. a) Define ellipse as the locus of a point. Derive the equation of the ellipse in the form

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \quad (a > b).$$ 6

b) Using Caley-Hamilton theorem, find the inverse of the matrix $\begin{bmatrix} 1 & 2 \\ 3 & 1 \end{bmatrix}$. 4

36. a) Find all the cube roots of the complex number $\sqrt{3} + i$. Represent them in the Argand diagram. Find their product. 6

b) Prove by vector method that the medians of a triangle are concurrent. 4

37. a) A man 6 feet tall moves away from a source of light 20 feet above the ground level and his rate of walking being 4 miles/hour. At what rate, is the length of the shadow changing ? At what rate is the tip of the shadow moving ? 6

b) Find the general solution of

$\sqrt{3} \cos x + \sin x - \sqrt{2}$. 4

38. a) Evaluate $\int_0^{\pi/2} \log \sin x \, dx$ 6

b) Find the general solution of the differential equation

$y \log x . \log y \, dx + dy - 0$. 4