MATHEMATICS - JUNE 2008

PART - A

Answer *all* the *ten* questions. $10 \cdot 1 = 10$

- 1. Find the number of incongruent solutions of $9x \equiv 21 \pmod{30}$.
- 2. Evaluate 4321 4322 4323 4324
- 3. In a group $\left(Z_6, + \mod 6\right)$, find $2 + 6 4^{-1} + 6 3^{-1}$.
- 4. Find the position vector of the point P which is the mid-point \overrightarrow{AB} where the position vectors of A and B are $\hat{i} + \hat{j} + 2\hat{k}$ and $3\hat{i} + 3\hat{j} + 2\hat{k}$.
- 5. Find the equation to a circle whose centre is (a, 0) and touching the y-axis.
- 6. Find the equation to directrix of $(x + 1)^2 = -4 (y 3)$.
- 7. Find the value of \cos^{-1} ($\sin 330^{\circ}$)
- 8. If 1, ω , ω^2 are the cube roots of unity, find the value of (1 + ω ω^2)
- 9. If $y = e^{\sqrt{x}} + x^{\sqrt{e}}$, find $\frac{dy}{dx}$
- 10. Evaluate $\int e^{x} \left(\frac{1}{\cos x} + \tan x \right) dx$.

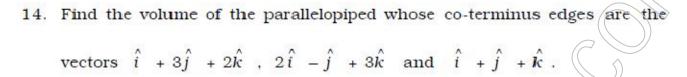
PART - B

Answer any ten questions.

$$10 \times 2 = 20$$

11. Find the G.C.D. of 352 and 891.

- 12. Find the characteristic roots of the matrix $\begin{bmatrix} 1 & 4 \\ 3 & 2 \end{bmatrix}$
- 13. Prove that a group of order three is Abelian.



- 15. Find the equation to the parabola whose focus is (3, 2) and its directrix is x = 1.
- 16. Prove that

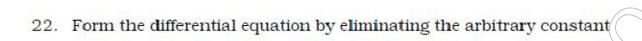
$$\sin \left[2 \tan^{-1} \sqrt{\frac{1-x}{1+x}} \right] = \sqrt{1-x^2}$$

17. Find the equation of a circle passing through the origin, having its centre on the line y = x and cutting orthogonally the circle

$$x^2 + y^2 - 4x - 6y + 10 = 0.$$

- 18. Prove that $(1-i)^9 = 16 16i$.
- 19. If $y = \log \left(\frac{1 \cos x}{1 + \cos x}\right)$, then prove that $\frac{dy}{dx} = 2 \csc x$.
- 20. Find the point on the curve $y^2 = x$ the tangent at which makes an angle of 45° with the x-axis.

21. Evaluate $\int_{0}^{1} x(1-x)^{7} dx$.



$$(y-2)^2 = 4a(x+1).$$

I. Answer any three questions:

$$3 \times 5 = 15$$

- 23. a) Find the number of positive divisors and sum of all such positive divisors of 756.
 - b) If a/bc and (a, b) = 1, then prove that a/c.
- 24. Solve by matrix method:

$$3x + y + 2z = 3$$

$$2x - 3y - z = -3$$

$$x + 2y + z = 4.$$

- 25. Prove that the set z of integers is an Abelian group under binary operation * defined by a*b=a+b+3, $\forall a,b\in z$.
 - 26. a) If $\vec{a} = \hat{i} 2\hat{j} 3\hat{k}$, $\vec{b} = 2\hat{i} + \hat{j} \hat{k}$ and $\vec{c} = \hat{i} + \hat{j} + \hat{k}$

 $3\hat{j}$ – $2\hat{k}$, find a unit vector perpendicular to \vec{a} and in the

same plane on \overrightarrow{b} and \overrightarrow{c} .

3

b) Find the area of a parallelogram whose diagonals are the vectors

$$2\hat{i} + \hat{j} + \hat{k}$$
 and $\hat{i} - 2\hat{j} + 3\hat{k}$.

2

II. Answer any two questions:

2 × 5 = 10

27. a) Find the length of the tangent from the point (x_1, y_1) to the

circle
$$x^2 + y^2 + 2gx + 2fy + c = 0$$
.

3

b) Find the equations of tangent to the circle

 $x^2 + y^2 - 2x - 4y - 4 = 0$, which are perpendicular to

$$3x - 4y + 6 = 0$$
.

2

28. a) Find the focus and equation to the directrix of the ellipse

$$9x^2 + 5y^2 - 36x + 10y - 4 = 0.$$

3

b) Find the equation to the hyperbola in the standard form

 $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, given that length of latus rectum = $\frac{14}{3}$ and

$$e = \frac{4}{3}$$
.

2

29. a) If $\tan^{-1} x + \tan^{-1} y + \tan^{-1} z = \frac{\pi}{2}$, prove that

$$xy + yz + zx = 1$$

3

b) Find the general solution of $\sin^2 \theta - \cos 2\theta = \frac{5}{4}$.

2

III. Answer any three of the following questions:

 $3 \times 5 = 15$

30. a) Differentiate a^x w.r.t. x by first principles.

3

b) If
$$y = \tan^{-1}\left(\frac{4x}{4-x^2}\right)$$
, prove that $\frac{dy}{dx} = \frac{4}{4+x^2}$.

- 2
- 31. a) If $y = (\sin^{-1} x)^2 + (\cos^{-1} x)^2$, prove that
 - $(1-x^2)$ $y_2 xy_1 4 = 0$.



3

- b) If $x = 3 \sin 2\theta + 2 \sin 3\theta$, and
 - $y = 2\cos 3\theta 3\cos 2\theta$
 - prove that $\frac{dy}{dx} = -\tan\frac{\theta}{2}$.



- 32. a) Prove that in the curve $y = e^{\frac{\pi}{a}}$ the subnormal varies as the
 - square of the ordinate and subtangent is constant.
 - b) Evaluate $\int \frac{\sin x \cdot \cos x}{1 + \sin^4 x} dx.$ 2
- 33. a) Evaluate $\int \frac{2-3 \tan x}{1+2 \tan x} dx.$ 3
 - b) Evaluate $\frac{1}{(1+e^x)(1-e^{-x})} dx.$ 2
- 34. Find the area of the ellipse $9x^2 + 16y^2 = 144$ by integration. 5

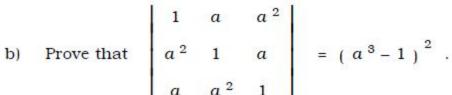
PART - D

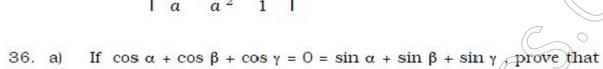
Answer any two of the following questions:

 $2 \times 10 = 20$

Define hyperbola as a locus and derive the standard equation of the 35. a)

hyperbola in the form
$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$
.





i)
$$\cos 2\alpha + \cos 2\beta + \cos 2\gamma = 0$$

$$\sin 2\alpha + \sin 2\beta + \sin 2\gamma = 0$$

$$\sin 2\alpha + \sin 2\beta + \sin 2\gamma = 0$$
ii)
$$\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma = \frac{3}{2}$$

$$\sin^2 \alpha + \sin^2 \beta + \sin^2 \gamma = \frac{3}{2} .$$

b) Prove that
$$\begin{bmatrix} \vec{a} \times \vec{b} & \vec{b} \times \vec{c} & \vec{c} \times \vec{a} \end{bmatrix} = \begin{bmatrix} \vec{a} & \vec{b} & \vec{c} \end{bmatrix}^2$$
.

The surface area of a sphere is increasing at the rate of 8 sq.cm/sec. 37. a)

> Find the rate at which the radius and the volume of the sphere are increasing when the volume of the sphere is $\frac{500 \, \pi}{2}$ c.c. 6

b) Find the general solution of
$$\sin \theta + \sin 2\theta + \sin 3\theta = 0$$
.

36. a)

38. a) Prove that
$$\int_{0}^{\pi/2} \frac{\cos^2 x}{1 + \sin x \cos x} dx = \frac{\pi}{3\sqrt{3}}.$$



b) Find the general solution of the differential equation

$$xy \frac{dy}{dx} = \frac{1+y^2}{1+x^2} (1+x+x^2)$$
.



PART - E

Answer any one of the following questions:

$$1 \times 10 = 10$$

- 39. a) If $\vec{a} + \vec{b} + \vec{c} = 0$ and $|\vec{a}| = 3$, $|\vec{b}| = 5$ and $|\vec{c}| = 7$ find the angle between \vec{a} and \vec{b} .
 - b) Find the cube roots of a complex number $\sqrt{3}$ i and represent them in argand diagram.
 - c) Find the remainder when 2^{202} is divided by 11 (least positive remainder).
- 40. a) The sum of the lengths of a hypotenuse and another side of a right angled triangle is given. Show that the area of the triangle is maximum when the angle between these sides is $\frac{\pi}{8}$.
- b) Evaluate $\int \cot^4 (3x) dx$.
- c) Differentiate w.r.t. x:

$$y = \log_5 \sqrt{1 - x^2} .$$

2