Mathematics

Class X

Past Year Paper - 2013

Time: 2½ hour Total Marks: 80

Solution

SECTION – A (40 marks)

Sol. 1

(a) A + 2X = 2B + C

$$\begin{bmatrix}2 & -6\\2 & 0\end{bmatrix}+2X=2\begin{bmatrix}-3 & 2\\4 & 0\end{bmatrix}+\begin{bmatrix}4 & 0\\0 & 2\end{bmatrix}$$

$$\begin{bmatrix}2 & -6\\2 & 0\end{bmatrix}+2X=\begin{bmatrix}-6+4 & 4+0\\8+0 & 0+2\end{bmatrix}$$

$$\begin{bmatrix}2 & -6\\2 & 0\end{bmatrix}+2X=\begin{bmatrix}-2 & 4\\8 & 2\end{bmatrix}$$

$$2X=\begin{bmatrix}-2 & 4\\8 & 2\end{bmatrix}-\begin{bmatrix}2 & -6\\2 & 0\end{bmatrix}$$

$$2X=\begin{bmatrix}-4 & 10\\6 & 2\end{bmatrix}$$

$$X=\frac{1}{2}\begin{bmatrix}-4 & 10\\6 & 2\end{bmatrix}=\begin{bmatrix}-2 & 5\\3 & 1\end{bmatrix}$$

(b) P = ₹ 4000, C.I. = ₹ 1324, n = 3 years

Amount, A = P + C.I. = ₹ 4000 + ₹ 1324 = ₹ 5324

$$A=P\left(1+\frac{r}{100}\right)^n$$

$$5324=4000\left(1+\frac{r}{100}\right)^3$$

$$\left(1+\frac{r}{100}\right)^3=\frac{5324}{4000}=\frac{1331}{1000}$$

$$\left(1+\frac{r}{100}\right)^3=\left(\frac{11}{10}\right)^3$$

$$1+\frac{r}{100}=\frac{11}{10}$$

$$\frac{r}{100}=\frac{11}{10}-1=\frac{1}{10}$$

$$r=10\%$$

(c) The observations in ascending order are:

11, 12, 14, (x - 2), (x + 4), (x + 9), 32, 38, 47

Number of observations = 9 (odd)

Median = $\left(\frac{n+1}{2}\right)^{th}$ observation = 5^{th} observation

$\therefore$ x + 4 = 24

$\Rightarrow$ x = 20

Thus, the observations are:

11, 12, 14, 18, 24, 29, 32, 38, 47

Mean = $\frac{11+12+14+18+24+29+32+38+47}{9}$

$= \frac{225}{9}$

= 25

Sol. 2

(a) Let the number added be x.

$\therefore$ (6 + x) : (15 + x) :: (20 + x) (43 + x)

$\frac{6+x}{15+x} = \frac{20+x}{43+x}$

$(6+x)(43+x) = (20+x)(15+x)$

$258 + 6x + 43x + x^2 = 300 + 20x + 15x + x^2$

$49x - 35x = 300 - 258$

$14x = 42$

$x = 3$

Thus, the required number which should be added is 3.

(b) Let $p(x) = 2x^3 + ax^2 + bx - 14$

Given, (x - 2) is a factor of p(x)

$\Rightarrow$ Remainder = p(2) = 0

$\Rightarrow 2(2)^3 + a(2)^2 + b(2) - 14 = 0$

$\Rightarrow$ 16 + 4a + 2b - 14 = 0

$\Rightarrow$ 4a + 2b + 2 = 0

$\Rightarrow$ 2a + b + 1 = 0 ...(1)

Given, when p(x) is divided by (x - 3), it leaves a remainder 52.

$\therefore$ p(3) = 52

$\Rightarrow 2(3)^3 + a(3)^2 + b(3) - 14 = 52$

$\Rightarrow$ 54 + 9a + 3b - 14 - 52 = 0

$\Rightarrow$ 9a + 3b - 12 = 0

$\Rightarrow$ 3a + b - 4 = 0 ...(2)

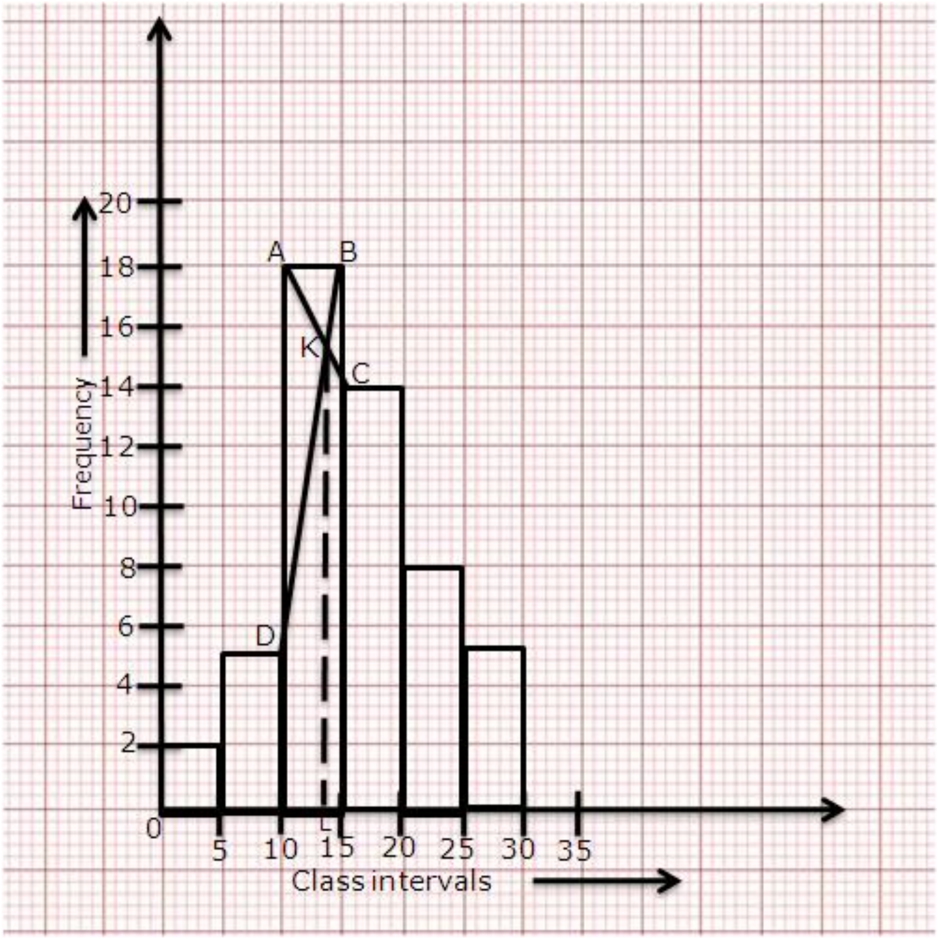
Subtracting (1) from (2), we get,

a - 5 = 0 $\Rightarrow$ a = 5

From (1),

10 + b + 1 = 0 $\Rightarrow$ b = -11

(c)



Steps for calculation of mode.

(i) Mark the end points of the upper corner of rectangle with maximum frequency as A and B.

(ii) Mark the inner corner of adjacent rectangles as C and D.

(iii) Join AC and BD to intersect at K. From K, draw KL perpendicular to x-axis.

(iv) The value of L on x- axis represents the mode.

$\therefore$ Mode = 13

Sol. 3

(a) $3\cos 80^\circ . \operatorname{cosec} 10^\circ + 2\sin 59^\circ \sec 31^\circ$

$= 3\cos(90^\circ - 10^\circ) \operatorname{cosec} 10^\circ + 2\sin(90^\circ - 31^\circ) \sec 31^\circ$

$= 3\sin 10^\circ \operatorname{cosec} 10^\circ + 2\cos 31^\circ, \sec 31^\circ$

$[\because \sin(90^\circ - \theta) = \cos\theta, \cos(90^\circ - \theta) = \sin\theta]$

$= 3 \times 1 + 2 \times 1 \qquad [\because \sin\theta . \operatorname{cosec}\theta = 1, \cos\theta . \sec\theta = 1]$

$= 3 + 2 = 5$

(b) (i) In Δ ABD,

$\angle DAB + \angle ABD + \angle ADB = 180^\circ$

$\Rightarrow 65^\circ + 70^\circ + \angle ADB = 180^\circ$

$\Rightarrow \angle ADB = 180^o - 70^o - 65^o = 45^o$

Now, $\angle ADC = \angle ADB + \angle BDC = 45^o + 45^o = 90^o$

$\angle ADC$ is the angle of semi-circle so AC is a diameter of the circle.

(ii) $\angle ACB = \angle ADB$ (angle subtended by the same segment)

$\Rightarrow \angle ACB = 45^o$

(c)

(i) Radius AC = $\sqrt{(3+2)^2 + (-7-5)^2}$

$$= \sqrt{(5)^2 + (-12)^2}$$
$$= \sqrt{25 + 144}$$
$$= \sqrt{169}$$
$$= 13 \text{ units}$$

(ii) Let the coordinates of B be (x, y)

Using mid-point formula, we have

$$-2 = \frac{3+x}{2} \qquad 5 = \frac{-7+y}{2}$$
$$-4 = 3 + x \qquad 10 = -7 + y$$
$$x = -7 \qquad y = 17$$

Thus, the coordinates of points B are (-7, 17).

Sol. 4

(a) x^2-5x-10 = 0

Use quadratic formula: a = 1, b = -5, c = -10

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(1)(-10)}}{2(1)}$$
$$x = \frac{5 \pm \sqrt{25+40}}{2}$$
$$x = \frac{5 \pm \sqrt{65}}{2}$$
$$x = \frac{5 \pm 8.06}{2}$$
$$x = \frac{13.06}{2}, \frac{-3.06}{2}$$
$$x = 6.53, -1.53$$

(b) (i) In $\triangle ABC$ and $\triangle DEC$,

$\angle ABC = \angle DEC = 90°$ (perpendiculars to BC)

$\angle ACB = \angle DCE$ (Common)

$\therefore \Delta ABC \sim \Delta DEC$ (AA criterion)

(ii) Since $\Delta ABC \sim \Delta DEC$,

$$\frac{AB}{DE} = \frac{AC}{CD}$$

$$\Rightarrow \frac{6}{4} = \frac{15}{CD}$$

$$\Rightarrow 6 \times CD = 60$$

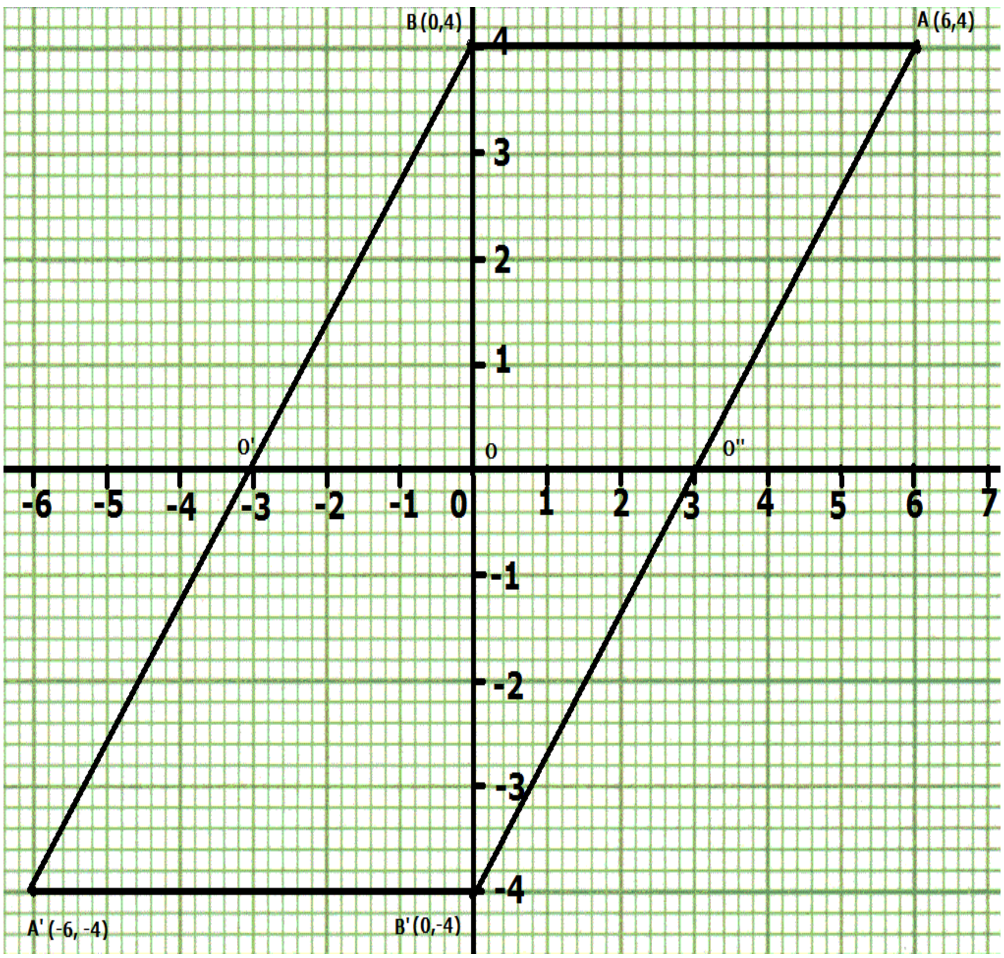
$$\Rightarrow CD = \frac{60}{6} = 10 \text{ cm}$$

(iii) It is known that the ratio of areas of two similar triangles is equal to the square of the ratio of their corresponding sides.

ar(ΔABC): ar(ΔDEC) = $AB^2 : DE^2 = 6^2 : 4^2 = 36 : 16 = 9 : 4$

(c)

(i)



(ii) Co-ordinates of A' = (-6, -4)

Co-ordinates of B' = (0, -4)

(iii) ABA'B' is a parallelogram.

(iv) AB = A'B' = 6 units

In $\triangle OBO'$,

$OO' = 3$ units

$OB = 4$ units

$$BO' = \sqrt{OB^2 + OO'^2}$$
$$= \sqrt{4^2 + 3^2}$$
$$= \sqrt{25}$$
$$= 5 \text{ units}$$

Since BO' = 5 units

BA' = 10 units = AB'

Perimeter of ABA'B' = (6 + 10 + 6 + 10) units = 32 units

SECTION – B (40 marks)

Sol. 5

(a) The given inequation is $-\frac{x}{3} \le \frac{x}{2} - 1\frac{1}{3} < \frac{1}{6}, x \in R$

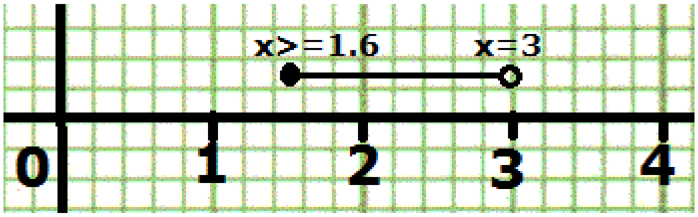
$$-\frac{x}{3} \le \frac{x}{2} - 1\frac{1}{3}$$
$$-\frac{x}{3} - \frac{x}{2} \le -\frac{4}{3}$$
$$\frac{2x + 3x}{6} \ge \frac{4}{3}$$
$$\frac{5x}{6} \ge \frac{4}{3}$$
$$5x \ge 8$$
$$x \ge \frac{8}{5}$$
$$x \ge 1.6$$

and

$$\frac{x}{2} - 1\frac{1}{3} < \frac{1}{6}$$
$$\frac{x}{2} < \frac{1}{6} + \frac{4}{3}$$
$$\frac{x}{2} < \frac{1+8}{6}$$
$$\frac{x}{2} < \frac{9}{6}$$
$$x < \frac{18}{6}$$
$$x < 3$$

The solution set is {x: x $\in$ R and 1.6 $\le$ x < 3}

It can be represented on a number line as follows:



(b) Maturity amount = ₹ 8088

Period (n) = 3 yrs = 36 months

Rate = 8% p.a.

Let x be the monthly deposit.

$$S.I. = P \times \frac{n(n+1)}{2 \times 12} \times \frac{r}{100}$$

$$= P \times \frac{36 \times 37}{24} \times \frac{8}{100} = 4.44x$$

Total amount of maturity $= 36x + 4.44x = 40.44x$

Now,

$40.44x = 8088$

$\Rightarrow x = 200$

Thus, the value of monthly installment is ₹ 200.

(c)

(i) No. of shares = 50

Market value of one share = ₹ 132

Salman's investment = ₹ (132 x 50) = ₹ 6600

(ii) Dividend on one share = 7.5% of ₹ 100 = ₹ 7.50

His annual income = 50 × ₹ 7.50 = ₹ 375

(iii) Salman wants to increase his income by ₹ 150.

Income on one share = ₹ 7.50

$$\text{No. of extra shares he buys} = \frac{150}{7.50} = 20$$

Sol. 6

(a)

LHS

$$= \sqrt{\frac{1 - \text{CosA}}{1 + \text{CosA}}}$$

$$= \sqrt{\frac{(1 - \text{CosA})(1 + \text{CosA})}{(1 + \text{CosA})(1 + \text{CosA})}}$$

$$= \sqrt{\frac{1 - \text{Cos}^2 A}{(1 + \text{CosA})^2}}$$

$$= \sqrt{\frac{\text{Sin}^2 A}{(1 + \text{CosA})^2}}$$

$$= \frac{\text{SinA}}{1 + \text{CosA}} = \text{RHS}$$

(b) In cyclic quadrilateral ABCD,

$\angle B + \angle D = 180^\circ$ (opp. angles of cyclic quad are supplementary)

$\Rightarrow 100^\circ + \angle ADC = 180^\circ$

$\Rightarrow \angle ADC = 80^\circ$

Now, in ΔACD,

$\angle ACD + \angle CAD + \angle ADC = 180°$

$40° + \angle CAD + 80° = 180°$

$\angle CAD = 180° - 120° = 60°$

Now $\angle DCT = \angle CAD$ (angles in the alternate segment)

$\therefore \angle DCT = 60°$

(c)

On completing the given table, we get:

Date	Particulars	Withdrawls	Deposit	Balance
Feb8	B/F	-	-	₹ 8500
Feb 18	To self	₹ 4000	-	₹ 4500
April 12	By cash	-	₹ 2230	₹ 6730
June 15	To self	₹ 5000	-	₹ 1730
July 8	By cash	-	₹ 6000	₹ 7730

Principal for the month of Feb = ₹ 4500

Principal for the month of March = ₹ 4500

Principal for the month of April = ₹ 4500

Principal for the month of May = ₹ 6730

Principal for the month of June = ₹ 1730

Principal for the month of July = ₹ 7730

Total Principal for 1 month

= ₹ (4500+4500+4500+6730+1730+7730) = ₹ 29690

P = ₹ 29690, T = $\frac{1}{12}$ years, R = 6%

$$\therefore \text{Interest} = \frac{P \times R \times T}{100}$$

$$= \frac{29690 \times 6 \times 1}{100 \times 12}$$

$$= \text{Rs } 148.45$$

Sol. 7

(a) The vertices of $\triangle$ ABC are A(3, 5), B(7,8) and C(1, -10).

Coordinates of the mid-point D of BC are

$$\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$

$$=\left(\frac{7+1}{2}, \frac{8+(-10)}{2}\right)$$

$$=\left(\frac{8}{2}, \frac{-2}{2}\right)$$

$$=(4,-1)$$

$$\text{Slope of AD} = \frac{y_2-y_1}{x_2-x_1}$$

$$=\frac{-1-5}{4-3}$$

$$=\frac{-6}{1}=-6$$

Now, the equation of median is given by:

$y - y_1 = m(x - x_1)$

$y - 5 = -6(x - 3)$

$y - 5 = -6x + 18$

$6x + y - 23 = 0$

(b) Since, the shopkeeper sells the article for ₹ 1500 and charges sales-tax at the rate of 12%.

$\therefore$ Tax charged by the shopkeeper = 12% of ₹ 1500 = ₹ 180

VAT = Tax charges - Tax paid

₹ 36 = ₹ 180 - Tax paid

Tax paid = ₹ 144

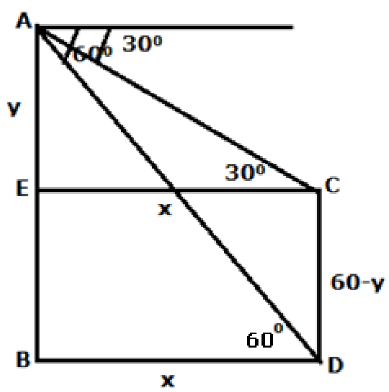
If the shopkeeper buys the article ₹ x.

Tax on it = 12% on ₹ x = ₹ 144

$\Rightarrow x =$ ₹ $144 \times \frac{100}{12} =$ ₹ 1200

Thus, the price (inclusive of tax) paid by the shopkeeper = ₹ 1200 + ₹ 144 = ₹ 1344.

(c)



(i) In Δ AEC,

$\tan 30^0 = \frac{AE}{EC}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{y}{x}$

$\Rightarrow y = \frac{x}{\sqrt{3}}$......(1)

In Δ DBA,

$\cot 60^0 = \frac{BD}{AB}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{x}{60}$

$\Rightarrow x = \frac{60}{\sqrt{3}} = 20\sqrt{3} = 34.64\ \text{m}$

Therefore, the horizontal distance between AB and CD = 34.64 m.

(ii) Substituting the value of x in (1),

$\Rightarrow y = \frac{20\sqrt{3}}{\sqrt{3}} = 20\ \text{m}$

$\therefore$ Height of the lamp post = CD = (60 - 20) m = 40 m

Sol. 8

(a) $\begin{bmatrix} x & 3x \\ y & 4y \end{bmatrix}\begin{bmatrix} 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 5 \\ 12 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 2x + 3x \\ 2y + 4y \end{bmatrix} = \begin{bmatrix} 5 \\ 12 \end{bmatrix}$

$\Rightarrow$ 2x + 3x = 5 $\Rightarrow$ x = 1

and 2y + 4y = 12 $\Rightarrow$ y = 2

(b) Sphere : R = 15 cm

Cone : r = 2.5 cm, h = 8 cm

Let the number of cones recasted be n.

$\therefore$ n $\times$ Volume of one cone = Volume of solid sphere

$\Rightarrow n \times \frac{1}{3}\pi r^2 h = \frac{4}{3}\pi R^3$

$\Rightarrow n \times (2.5)^2 \times (8) = 4 \times (15)^3$

$\Rightarrow n = \frac{4 \times 15 \times 15 \times 15}{2.5 \times 2.5 \times 8}$

$\Rightarrow n = 270$

Thus, 270 cones were recasted.

(c) $x^2 + (p - 3)x + p = 0$

Here, A = 1, B = (p - 3), C = p

Since, the roots are real and equal, D = 0

$\Rightarrow B^2 - 4ac = 0$

$\Rightarrow (p - 3)^2 - 4(1)(p) = 0$

$\Rightarrow p^2 + 9 - 6p - 4p = 0$

$\Rightarrow p^2 - 10p + 9 = 0$

$\Rightarrow (p - 1)(p - 9) = 0$

$\Rightarrow p = 1$ or $p = 9$

Sol. 9

(a) Area of the quadrant OACB =

$\frac{1}{4} \times \pi r^2$

$= \frac{1}{4} \times \frac{22}{7} \times 3.5 \times 3.5$

$= 9.625\ cm^2$

Area of the triangle OAD =

$\frac{1}{2} \times \text{base} \times \text{height}$

$= \frac{1}{2} \times 3.5 \times 2$

$= 3.5\ cm^2$

Shaded Area = Area of quadrant OACB - area of triangle OAD

$= 9.625 - 3.5\ cm^2$

$= 6.125\ cm^2$

(b) Number of white balls in the bag = 30

Let the number of black balls in the box be x.

$\therefore$ Total number of balls = x + 30

P(drawing a black ball) = $\frac{x}{x+30}$

P(drawing a white ball) = $\frac{30}{x+30}$

It is given that:

P(drawing a black ball) = $\frac{2}{5}$ x P(drawing a white ball)

$\Rightarrow \frac{x}{x+30} = \frac{2}{5} \times \frac{30}{x+30}$

$\Rightarrow \frac{x}{x+30} = \frac{12}{x+30}$

$\Rightarrow x = 12$

Therefore, number of black balls in the box is 12.

(c)

Class interval	Frequency (f)	Class mark (x)	$d = \frac{x - A}{h}$ (A = 55)	fd
20-30	10	25	-3	-30
30-40	6	35	-2	-12
40-50	8	45	-1	-8
50-60	12	A=55	0	0
60-70	5	65	1	5
70-80	9	75	2	18
Total	50			-27

Here, A = 55, h = 10

$$\text{Mean} = A + \frac{\sum fd}{\sum f} \times h$$
$$= 55 + \frac{-27}{50} \times 10$$
$$= 55 - 5.4$$
$$= 49.6$$

Sol. 10

(a)

(i) Steps of constructions:

(a) Draw a line segment BC = 6 cm.

(b) At B, draw a ray BX making an angle of 120^o with BC.

(c) From point B cut an arc of radius 3.5 cm to meet ray BX at C.

(d) Join AC.

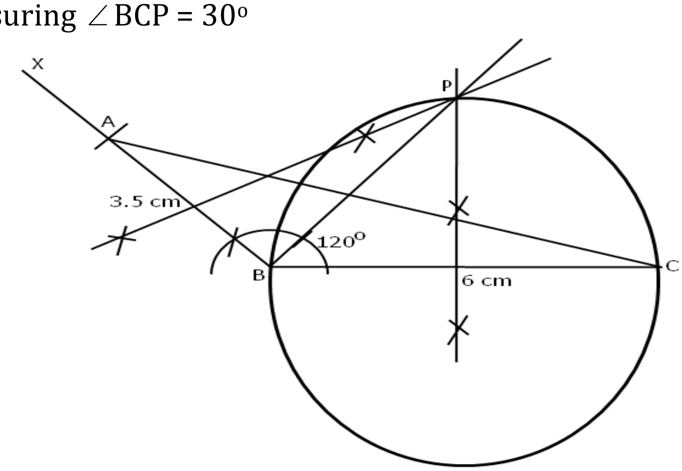
ABC is the required triangle.

(ii)

(a) Bisect BC and draw a circle with BC as diameter.

(b) Draw perpendicular bisectors of AB. Let the two bisectors meet the ray of angle bisector of $\angle ABC$ at point P. P is equidistant from AB and BC.

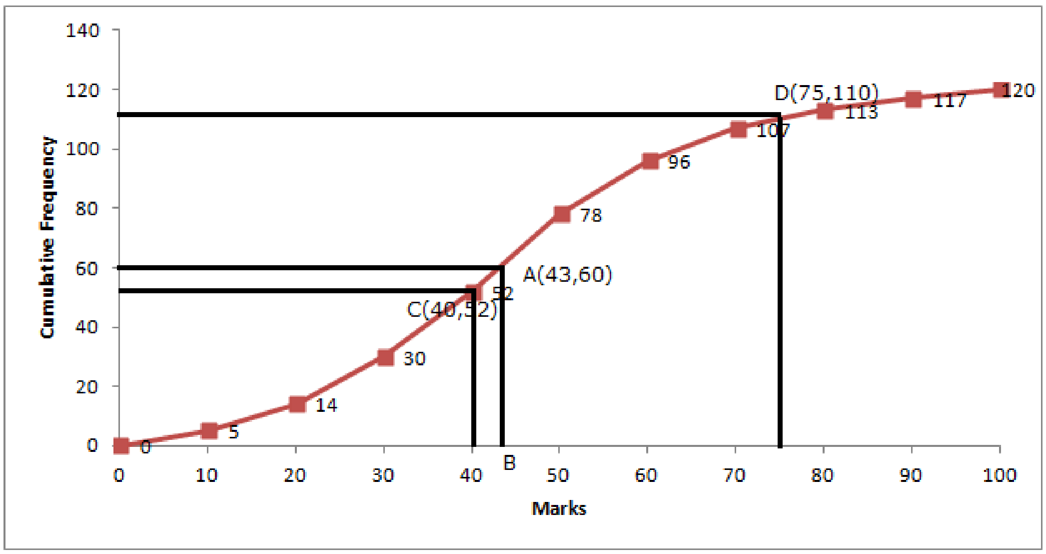
(iii) On measuring $\angle BCP = 30^o$



(b)

Marks	No. of Students	Cumulative Frequency
0-10	5	5
10-20	9	14
20-30	16	30
30-40	22	52
40-50	26	78
50-60	18	96
60-70	11	107
70-80	6	113
80-90	4	117
90-100	3	120

$n = \frac{120}{2} = 60$



(i) Through marks 60, draw a line segment parallel to x-axis which meets the curve at A. From A, draw a line perpendicular to x-axis meeting at B.

Median = 43

(ii) Through marks 75, draw a line segment parallel to y-axis which meets the curve at D. From D, draw a line perpendicular to y-axis which meets y-axis at 110.

Number of students getting more than 75% = 120 - 110 = 10 students

(iii) Through marks 40, draw a line segment parallel to y-axis which meets the curve at C. From C, draw a line perpendicular to y-axis which meets y-axis at 52.

Number of students who did not pass = 52.

Sol. 11

(a) Let the coordinates of A and B be (x, 0) and (0, y) respectively.

Given P divides AB is the ratio 2:3,

Using section formula, we have :

$$-3 = \frac{2\times 0 + 3\times x}{2+3} \qquad 4 = \frac{2\times y + 3\times 0}{2+3}$$

$$-3 = \frac{3x}{5} \qquad 4 = \frac{2y}{5}$$

$$-15 = 3x \qquad 20 = 2y$$

$$x = -5 \qquad y = 10$$

Thus, the coordinates of A and B are (-5, 0) and (0, 10) respectively.

(b) $\frac{x^4+1}{2x^2} = \frac{17}{8}$

Using componendo and dividendo,

$$\frac{x^4+1+2x^2}{x^4+1-2x^2} = \frac{17+8}{17-8}$$

$$\Rightarrow \frac{(x^2+1)^2}{(x^2-1)^2} = \frac{25}{9}$$

$$\Rightarrow \left(\frac{x^2+1}{x^2-1}\right)^2 = \left(\frac{5}{3}\right)^2$$

$$\Rightarrow \frac{x^2+1}{x^2-1} = \frac{5}{3}$$

$$\Rightarrow \frac{x^2+1+x^2-1}{x^2+1-x^2+1} = \frac{5+3}{5-3} \quad \text{(Using componendo and dividendo)}$$

$$\Rightarrow \frac{2x^2}{2} = \frac{8}{2}$$

$$\Rightarrow x^2 = 4$$

$$\Rightarrow x = \pm 2$$

(c) Original cost of each book = ₹ x

$\therefore$ Number of books brought for ₹ 960 = $\frac{960}{x}$

In 2nd case:

The cost of each book = ₹ (x - 8)

Number of books bought for ₹ 960 = $\frac{960}{x-8}$

From the given information, we have:

$$\frac{960}{x-8}-\frac{960}{x}=4$$

$$\Rightarrow \frac{960x-960x+960\times 8}{x(x-8)}=4$$

$$\Rightarrow x(x-8)=\frac{960\times 8}{4}=1920$$

$\Rightarrow x^2 - 8x - 1920 = 0$

$\Rightarrow (x - 48)(x + 40) = 0$

$\Rightarrow x = 48$ or -40

But x can't be negative.

$\therefore x = 48$

Thus, the original cost of each book is ₹ 48.