

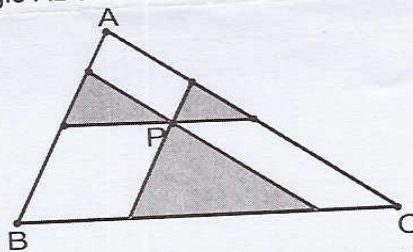
Junior Mathematics Olympiad - 2006

M.M.100

Time 3 Hours

NOTE: Attempt all questions, all questions carry equal marks.

1. a, b, c are three distinct real numbers and there are real numbers x, y such that $a^3 + ax + y = 0$, $b^3 + bx + y = 0$ and $c^3 + cx + y = 0$. Show that $a + b + c = 0$.
2. The triangle ABC has $CA = CB$. P is a point on the circumcircle between A and B (and on the opposite side of the line AB to C). D is the foot of the perpendicular from C to PB . Show that $PA + PB = 2 \cdot PD$.
3. Given reals x, y with $(x^2 + y^2)/(x^2 - y^2) + (x^2 - y^2)/(x^2 + y^2) = k$, find $(x^8 + y^8)/(x^8 - y^8) + (x^8 - y^8)/(x^8 + y^8)$ in terms of k .
4. In a right triangle ABC right angled at B , a point P is taken on the side AB such that $AP = h$ and $PB = b$. If $BC = d$ and $AC = y$ such that $h + y = b + d$. Prove that $h = bd/(2b+d)$.
5. P is a point inside the triangle ABC . Lines are drawn through P Parallel to the sides of the triangle. The areas of the three resulting triangles with a vertex at P , have areas 4, 9 and 49. What is the area of triangle ABC ?



6. A lotus plant in a pool of water is $\frac{1}{2}$ cubit above water level. When propelled by air, the lotus sinks in the pool 2 cubits away from its position. Find the depth of water in the pool.
7. Let C_1 be any point on side AB of a triangle ABC . Join C_1C . The lines through A and B parallel to CC_1 meet BC and AC produced at A_1 and B_1 respectively. Prove that $1/AA_1 + 1/BB_1 = 1/CC_1$.
8. The triangle ABC has angle $B = 90^\circ$. When it is rotated about AB it gives a cone of volume 800π sq. When it is rotated about BC it gives a cone of volume 1920π sq. Find the length AC .
9. A number when divided by 7, 11 and 13 (the prime factor of 1001) successively leave the remainders 6, 10 and 12 respectively. Find the remainder if the number is divided by 1001.
10. Two candles of the same height are lighted together. First one gets burnt up completely in 3 hours while the second in 4 hours. At what point of time, the length of second candle will be double the length of the first candle.