KVS Junior Mathematics Olympiad (JMO) – 2005

M.M. 100   Time : 3 hours

Note : (i)   Attempt all questions. Each question carries ten marks.

   (ii)   Please check that there are two printed pages and ten

           Questions in the question paper.

1.   Fill in the blanks:

   (a)   If four times the reciprocal of the circumference of a circle equals the

          diameter of the circle, then the area of the circle is …………..

   (b)   If $1 - \dfrac{4}{x} + \dfrac{4}{x^2} = 0$ then $\dfrac{2}{x}$ equals………

   (c)   If a=1000, b=100, c=10, and d=1, then

          (a+b+c-d) + (a+b-c+d) + (a-b+c+d)+(-a+b+c+d) is equal to …….

   (d)   When the base of a triangle is increased by 10% and the altitude to the

          base is decreased by 10%, the change in area is ……….

   (e)   If the sum of two numbers is 1, and their product is 1, then the sum of

   their cubes is ……………

2.   (a)   If $x = \left(\log_8^2 \right)^{\log_2^8}$ find the value of $\log_3 x$.

   (b)   If $\dfrac{4^x}{2^{x+y}} = 8$ and $\dfrac{9^{x+y}}{3^{5y}} = 243$ find the value of x-y.

3.   (a)   Find the number of digits in the number $2^{2005} \times 5^{2000}$ when

written in full.

(b)   Find the remainder when 2^{2005} is divided by 13.

4.   (a)   A polynomial p (x) leaves a remainder three when divided by x – 1 and a remainder five when divided by x-3. Find the remainder when p(x) is divided by (x-1) (x-3).

(b)   Find two numbers, both lying between 60 and 70, each of which is exactly divides 2^{43}-1.

5.   In triangle ABC the medians AM and CN to the sides BC and AB, respectively intersect in the point O.P is the mid-point of side AC, and MP intersects CN in Q. If the area of triangle OMQ is 24 cm^2, find the area of triangle ABC.

6.   The base of a pyramid is an equilateral triangle of side length 6 cm. The other edges of the pyramid are each of length $\sqrt{15}$ cm. Find the volume of the pyramid.

7.   Chords AB and CD of a circle (see figure) intersect at E and are perpendicular to each other segments AE. EB and ED are of lengths 2cm, 6cm and 3cm respectively. Find the length of the diameter of the circle.
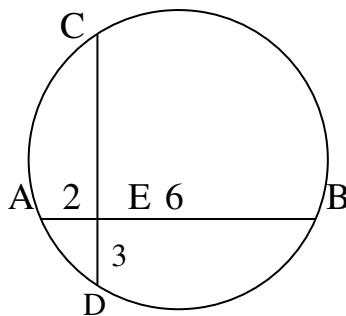
8.　Three men A, B and C working together, do a job in 6 hours less time than A alone, in 1 hour less time than B alone, and in one half the time needed by C when working alone. How many hours will be needed by A and B working together, to do the job ?

9.　Pegs are put on a board 1 unit apart both horizontally and vertically. A rubber band is stretched over 4 pegs as shown in the figure forming a quadrilateral. Find the area of the quadrilateral in square units.
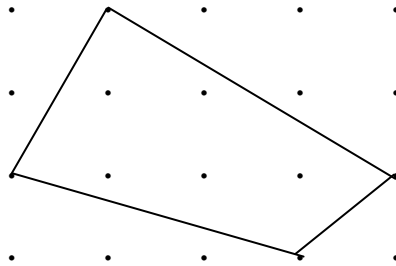
10.　The odd positive integers 1, 3, 5, 7 ….. are arranged in five columns continuing with the pattern shown on the right. Counting from the left, in which column (I, II, III, IV or V) does the number 2005 appear ? (Justify your answer)

I	II	III	IV	V
	1	3	5	7
15	13	11	9	
	17	19	21	23
31	29	27	25	
	33	35	37	39
47	45	43	41	
	49	51	53	55
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