KVS Junior Mathematics Olympiad (JMO) – 2001

M.M. 100   Time : 3 hours

Note : (i)   Please check that there are two printed pages and ten question in all.

(ii)   Attempt all questions. All questions carry equal marks.

1.   Fill in the blanks :

   (a)   If $x + y = 1$, $x^3 + y^3 = 4$, then $x^2 + y^2 = $ ……..

   (b)   After 15 litres of petrol was added to the fuel tank of a car, the tank was 75% full. If the capacity of the tank is 28 litres, then the number of litres in the tank before adding the petrol was ……

   (c)   The perimeter of a rectangle is 56 metres. The ratio of its length to width is 4:3. The length of the diagonal in metres is ……..

   (d)   If April 23 falls on Tuesday, then March 23 of the same year was a ……..
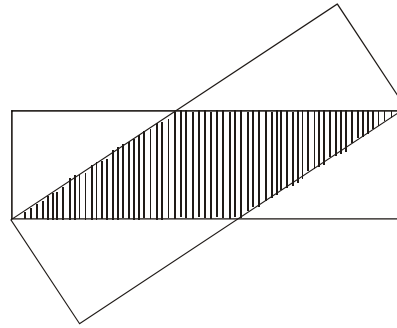
   (e)   The sum of the digits of the number $2^{2000}5^{2004}$ is ….

2.   (a)   Arrange the following in ascending order :

   2^{5555}, 3^{3333}, 6^{2222}

   (b)   Two rectangles, each measuring 3 cm x 7 cm, are placed as in the adjoining figure :

Find the area of the overlapping portion (shaded) in cm^2.

3. (a) Solve :

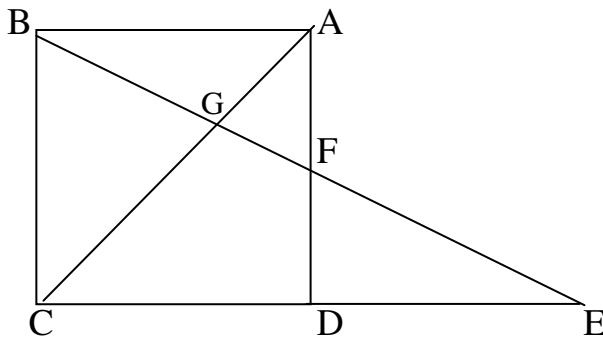$$\frac{\log_{10}(35 - x^3)}{\log_{10}(5 - x)} = 3$$

(b) Simplify :

$$\frac{a-b}{a+b} + \frac{b-c}{b+c} + \frac{c-a}{c+a} + \frac{(a-b)(b-c)(c-a)}{(a+b)(b+c)(c+a)}$$

4. (a) Factorize :

$$(x\text{-}y)^3 + (y\text{-}z)^3 + (z\text{-}x)^3$$

(b) If x^2-x-1=0, then find the value of x^3-2x +1

5. ABCD is a square. A line through B intersects CD produced at E, the side AD at F and the diagonal AC at G.

If BG = 3, and GF=1, then find the length of FE,

6.   (a)   Find all integers n such that $(n^2-n-1)^{n+2} = 1$

   (b)   If $x = \dfrac{4ab}{a+b}$, find the value of $\dfrac{x+2a}{x-2a} + \dfrac{x+2b}{x-2b}$

7.   (a)   Find all the positive perfect cubes that divide 9^9.

   (b)   Find the integer closest to $100\,(12-\sqrt{143}\,)$

8.   In a triangle ABC, $\angle BCA = 90^\circ$. Points E and F lie on the hypotenuse AB such that AE=AC and BF = BC. Find $\angle ECF$.
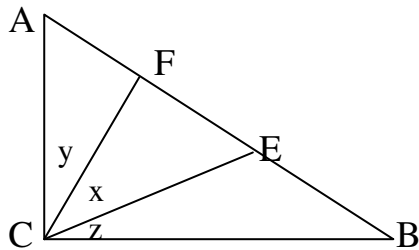
9.   An ant crawls 1 centimetre north, 2 centimetres west, 3 centimetres south, 4 centimetres east, 5 centimetres north and so on, at 1 centimetre per second. Each segment is 1 centimetre longer than the preceding one, and at the end of a segment, the ant makes a left turn. In which direction is the ant moving 1 minute after the start ?

10.   Find the lengths of the sides of a triangle with 20, 28 and 35 as the lengths of its altitudes.