INTERNATIONAL BACCALAUREATE
BACCALAURÉAT INTERNATIONAL
BACHILLERATO INTERNACIONAL

FURTHER MATHEMATICS
STANDARD LEVEL
PAPER 2

Tuesday 16 May 2000 (morning)

2 hours

INSTRUCTIONS TO CANDIDATES

- Do not open this examination paper until instructed to do so.
- Answer all the questions.
- Unless otherwise stated in the question, all numerical answers must be given exactly or to three significant figures, as appropriate.

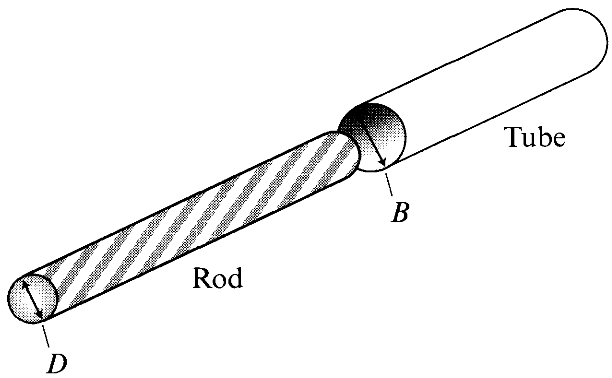
- Write the make and model of your calculator on the front cover of your answer booklets *e.g.* Casio *fx-7400G*, Sharp EL-9400, Texas Instruments TI-80.

You are advised to start each new question on a new page. A correct answer with **no** *indication of the method used will usually receive* **no** *marks. You are therefore advised to show your working. In particular, where graphs from a graphic display calculator are being used to find solutions, you should sketch these graphs as part of your answer.*

Statistics

1. *[Maximum mark: 23]*

In an industrial process, a steel rod has to be constructed so as to fit into another metallic tube as shown in the diagram. At room temperature the diameter D of the rod is normally distributed with mean 4.00 and standard deviation 0.10 and the internal diameter B of the tube is normally distributed with mean 4.02 and standard deviation 0.10 . Assume that the two variables B and D are independent.



(a) Describe the distribution of the variable $B–D$. *[3 marks]*

(b) With the conditions as above, what percentage of the rods will fit into the tubes? *[3 marks]*

(c) The material of each part expands on usage due to heat. The rods expand by 3% while the tubes by 5% . After heating, what percentage of the rods will fit into the tubes? *[6 marks]*

(d) The producers are unhappy with these results and wish to improve the production. They change the production process so as to reduce the standard deviations by half. They also develop a quality control procedure in which they choose 10 rods and 10 tubes and measure the diameters. The mean of each of these two samples is calculated, and these means are compared. If the difference is lower than a certain number l , the sample is rejected. Find the value of l which results in rejecting the sample 10% of the time. *[6 marks]*

(e) The management of this factory wants to test whether, in the process of improving the variation of the products, the difference of means of the tubes and rods has been altered from its historic 0.02 level and the process has been rendered unstable. For that purpose, they collected the samples given below, randomly and independently. Test, at the 5% level of significance, whether there is any change. Give all details.

Rods	Tubes
4.03	4.16
4.02	4.08
3.94	3.98
4.00	4.05
4.05	4.05
4.13	3.98
4.03	4.12
3.94	4.06
4.02	4.04
3.92	4.02
4.02	4.05
3.97	4.11
4.00	3.99
3.92	3.93
	4.00

[5 marks]

Sets Relations and Groups

2. *[Maximum mark: 20]*

(i) Consider the set of matrices

$$S = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \mid a, b, c, d \in \mathbb{R}, ad - bc = 1 \right\}$$

Show that $(S, \otimes)$ forms a group where $\otimes$ is matrix multiplication. *[8 marks]*

(ii) Determine whether the relation R, defined by zRw if and only if $\frac{z}{w}$ is real, is an equivalence relation on the set of non-zero complex numbers. *[4 marks]*

(iii) Let $(G, \odot)$ be a group and H a **non-empty** subset of G. Show that if $a \odot b^{-1}$ is in H whenever a and b are in H, then $(H, \odot)$ is a subgroup of $(G, \odot)$. *[8 marks]*

Discrete Mathematics

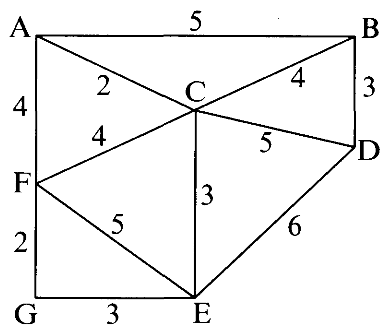
3. *[Maximum marks: 17]*

(i) Solve the following difference equation:

$$b_n = 7b_{n-1} - 12b_{n-2}\ ,\ b_1 = 1\ ,\ b_2 = 7$$

[6 marks]

(ii) Describe the difference between Prim's and Kruskal's algorithms and use one of them to find a minimal spanning tree for the graph shown below. List the edges in the order in which they are chosen.



[4 marks]

(iii) (a) Show that any integral power of 10 leaves a remainder of 1 when divided by 3. *[3 marks]*

It is given that any number $y \in \mathbb{N}$ can be written in expanded form as

$$y = a_n 10^n + a_{n-1} 10^{n-1} \ldots + a_1 10 + a_0$$

(b) Show that $y = 3k$ + sum of digits of y, for some $k \in \mathbb{N}$. *[3 marks]*

(c) Show that 3 divides y if 3 divides the sum of digits of y. *[1 mark]*

Analysis and Approximations

4. *[Maximum mark: 24]*

(i) (a) State Rolle's theorem. *[2 marks]*

(b) Let $f(x) = x^2 - \cos x$.

(i) Prove that $f'(x)$ is an increasing function. Hence, using Rolle's theorem, prove that $f(x)$ has precisely two zeros. *[5 marks]*

(ii) Use Taylor's expansion of $\cos x$ as far as the term in x^2 to show that the solutions of $f(x) = 0$ are approximately $\pm \frac{\sqrt{6}}{3}$. *[3 marks]*

(iii) Find the bounds on the error in the estimates of the solutions of $f(x) = 0$ in part (ii) above. *[5 marks]*

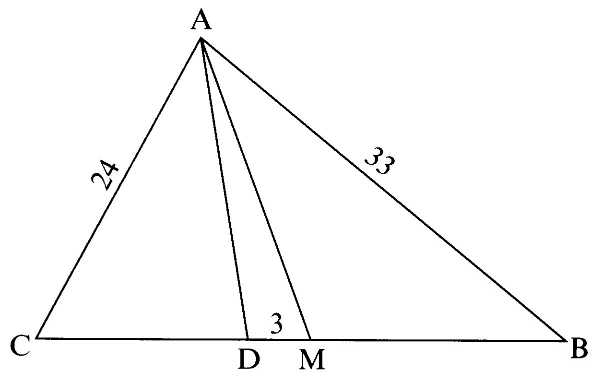
(iv) Use the Newton-Raphson method to find an approximation of the solution to the equation $f(x) = 0$ to three decimal places. *[4 marks]*

(ii) Find the radius of convergence of the series $\sum_{n=1}^{\infty} \frac{(nx)^n}{n!}$. *[5 marks]*

Euclidean Geometry and Conic Sections

5. *[Maximum mark: 16]*

In the triangle ABC , [AM] is a median and [AD] is the internal bisector of angle A . AC = 24 , AB = 33 , and DM = 3 .



(a) Find $\frac{\text{CD}}{\text{DB}}$. *[2 marks]*

(b) Let CD = $8k$. Express DB and CM in terms of k . *[3 marks]*

(c) Hence or otherwise, show that $k = 2$. *[3 marks]*

Let E be the point where the bisector of angle ACB intersects the side [AB] , and let F be the point where the line (ME) intersects the side [CA] produced.

(d) Find the length of the line segment [AF] . *[8 marks]*
