Seat No.: ________ Enrolment No.______________

GUJARAT TECHNOLOGICAL UNIVERSITY

M.E –Ist SEMESTER–EXAMINATION – JULY- 2012

Subject code: 710401N **Date: 05/07/2012**

Subject Name: Statistical Signal Analysis

Time: 2:30 pm – 05:00 pm **Total Marks: 70**

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q.1 **(a)** State and prove central limit theorem. **07**

(b) Define CDF. State and prove its properties. **07**

Q.2 **(a)** Define and classify random process. Explain each of them. **07**

(b) **i)** A system consists of a controller and three peripheral units. The system is said to be 'up' if the controller and at least two of the peripherals are functioning. Find the probability that the system is up, assuming that all components fail independently. **04**

ii) Two manufacturing plants produce similar parts. Plant I produces 1000 parts, 100 of which are defective. Plant II produces 2000 parts, 150 out of which are defective. A part is selected at random and found to be defective. What is the probability that it came from plant I ? **03**

OR

(b) **i)** A number is selected at a random from 1 to 100. Given that the number selected is divisible by 2, find the probability that it is divisible by 3 or 5. **04**

ii) A batch of 50 items contains 10 defective items. Suppose 10 items are selected at random and tested. What is the probability that exactly 5 of the items tested are defective ? **03**

Q.3 **(a)** **i)** Let X be the number of heads in three coin tosses. Find the PDF and CDF of X. Find $P[1<X\leq 2]$ and $p[2\leq X<3]$. **04**

ii) Find the value of constants a and b such that

$$F(x) = 1 - a\, e^{-x/b} \quad x \geq 0$$
$$= 0 \quad x < 0$$

is a valid CDF. **03**

(b) Let the random variable Y be defined by

$$Y = aX + b$$

Where a is a nonzero constant. Suppose that X has CDF $F_X(x)$, then find $F_Y(y)$. Also find $f_Y(y)$. **07**

OR

Q.3 **(a)** **i)** Let $Y = a\cos(\omega t + \theta)$ where a, ω and t are constants and θ is a uniform random variable in the interval $(0,2\pi)$. The random variable Y results from sampling the amplitude of a sinusoid with random phase θ. Find the expected values of Y and expected value of power of Y. **04**

ii) Find the variance of the random variable X that is uniformly distributed in the interval [a,b]. **03**

Q.3 **(b)** Write the PDF for Rayleigh random variable. Derive the corresponding CDF from it. Sketch and PDF and CDF for $\sigma = 1$. **07**

Q.4 **(a)** State and explain Markov and Chebyshew inequalities. **07**

(b) Consider a random process $X(t) = U \cos \omega t + V \sin\omega t \quad -\infty < t < \infty$

Where ω is constant and U and V are random variables.

i) Show that the condition E(U)=E(V)=0 is necessary for X(t) to be stationary.

ii) Show that X(t) is WSS if and only if U and V are uncorrelated with equal variance. **07**

OR

Q.4 **(a)** Let Y(t) = X(t-d), where d is a constant delay and where X(t) is WSS. Find $R_{YX}(\tau)$, $S_{YX}(f)$, $R_Y(\tau)$ and $S_Y(f)$. **07**

(b) Consider a random process X(t) is defined by

$$X(t) = A \cos(\omega t + \theta) \qquad -\infty < t < \infty$$

Where A and ω are constants and θ is uniform random variable over $(-\pi, \pi)$. Show that X(t) is WSS. **07**

Q.5 **(a)** Find the normalization constant c and the marginal PDF's for the following joint PDF :

$$f_{X,Y}(x,y) = c\, e^{-x} e^{-y} \qquad 0 \le y \le x < \infty$$
$$= 0 \qquad \text{elsewhere}$$

Find $P[X + Y \le 1]$. **07**

(b) Consider an experiment of drawing randomly three balls from an urn containing two red, three white and four blue balls. Let (X, Y) be a random variable, where X and Y denotes respectively the number of red and white balls chosen,

Find range of (X,Y), joint pmf's of (X,Y), marginal pmf's of X and Y. Are X and Y independent ? **07**

OR

Q.5 **(a)** The joint CDF for the vector of random variables X =(X,Y) is given by

$$F_{X,Y}(x,y) = (1 - e^{-\alpha x})(1 - e^{-\beta y}) \qquad x \ge 0, y \ge 0$$
$$= 0 \qquad \text{elsewhere}$$

Find the marginal CDF's. **07**

Find the probability of events $A = \{X \le 1, Y \le 1\}$, $B = \{X > x, Y > y\}$.

(b) Consider an experiment of tossing two coins three times. Coin A is fair, but coin B is not fair with P(H) = 0.25 and P(T) = 0.75. Consider a random variable (X,Y), where X denotes number of heads resulting from coin A and Y denotes the numbers of heads resulting from coin B.

Find range of (X,Y), joint pmf's of (X,Y), find P(X=Y), P(X>Y) and $P(X+Y \le 4)$. **07**

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