Seat No.: __________ Enrolment No.____________

GUJARAT TECHNOLOGICAL UNIVERSITY
ME Semester –I Examination Feb. - 2012

Subject code: 710401N **Date: 11/02/2012**
Subject Name: Statistical Signal Analysis
Time: 10.30 am – 01.00 pm **Total Marks: 70**

Instructions:
1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

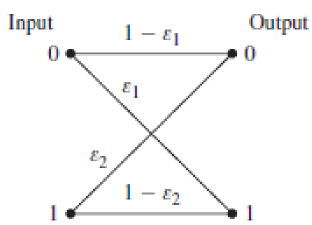
Q.1 **(a)** Let X be a continuous random variable with PDF **07**

$$f_x(x) = \begin{cases} kx & 0 < x < 1 \\ 0 & otherwise \end{cases}$$

(a) Determine the value of k and sketch $f_x(x)$.
(b) Find and sketch corresponding CDF $F_x(x)$.
(c) Find $P(¼ < X \le 2)$

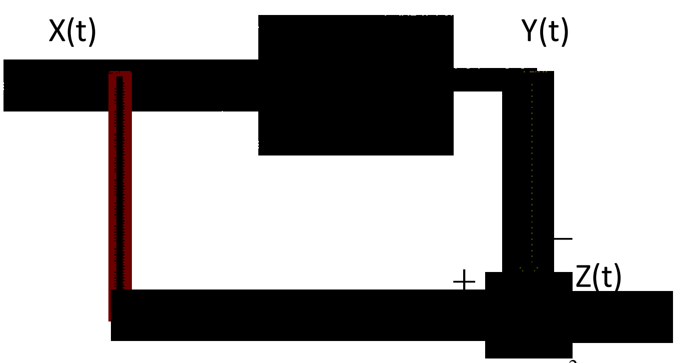
(b) A non symmetric binary communication channel is shown in Figure below. Assume the input is "0" with probability p and "1" with probability $1-p$ **07**

(a) Find the probability that the output is 0.
(b) Find the probability that the input was 0 given that the output is 1. Find the probability that the input is 1 given that the output is 1. Which input is more probable?



Q.2 **(a)** For a given system model as shown in figure below, **07**

Let $Y(t) = h(t) * X(t)$ and $z(t) = X(t) - Y(t)$



Find the power spectral density of Z(t) and $E[Z^2(t)]$. Here X(t) is Wide Sense Stationary random process.

(b) Let X be a Poisson random variable with parameter λ. Prove that $\mu_x = E(X) = \lambda$ and $\sigma_x^2 = Var(X) = \lambda$. **07**

OR

(b) Suppose X has continuous uniform distribution over the interval [3, 7.5] **07**
(a) Determine the mean, variance and standard deviation of X.
What is a probability P(X<2.5)?

Q.3 **(a)** A noisy channel has a per- digit error probability of 0.01. Find the probability of more than one error in 10 received digits. Also find same value of probability by Poisson distribution. **07**

(b) If $X_1, X_2, \ldots, X_n$. are jointly Gaussian with Probability Density Function (PDF) of $f_{\underline{x}}(\underline{x}) = \frac{1}{\sqrt{2\pi}} \frac{1}{\sqrt{|R|}} \exp\left[-\frac{1}{2}\underline{x}^T R^{-1} \underline{x}\right]$ **07**

Where R is covariance matrix defined as

$R = E\left[\underline{x}\ \underline{x}^T\right]$

If All X_i for i=1 to n are statistically independent then zero mean and uncorrelated random variable then find the characteristic function of Vectored random variable.

OR

Q.3 **(a)** What is Moment generating function? What is the use of this function the functions. Compare it with Characteristics function. **07**

(b) Find E[E[Y|X]]. Where E[.] is expected operator. **07**

Q.4 **(a)** Consider a random process X(t)=U cos t + V sin t, where U and V are independent random variable each of which assumes the value -2 and 1 with the probability 1/3 and 2/3 respectively. Show that X(t) is Wide Sense Stationary but not Strict Sense Stationary. **07**

(b) Two random processes X(t) and Y(t) are given by **07**

$X(t) = A\cos(wt + \theta)$ $Y(t) = A\sin(wt + \theta)$

Where A and W are constants and θ is a uniform random variable over (0, 2π). Find cross correlation function of X(t) and Y(t) also find cross power spectral density.

OR

Q.4 **(a)** Two IT companies are working in same domain. Companies have given their stock performance index in terms of random process with some parameters as described below. **07**

Company -A Share price (in $) modeled as Random Process X(t)

(1) Average value of 130. (2) Mean Square variation above average value is 180 .

(3) $X_{r1}(t)$ and $X_{r1}(t+\tau)$ are statistically independent for $|\tau| \leq 365$ days.

(4) $R(\tau)$ decreases with constant slop when $|\tau| \leq 365$ days.

Company -B Share price (in $) modeled as Random Process X(t)

(1) Average value of 100 .

(2) Mean Square variation above average value is 210 .

(3) $X_{r1}(t)$ and $X_{r1}(t+\tau)$ are independent for $|\tau| \leq$ (any observation window of 100 days in year).

(4) $R(\tau)$ decreases with constant slop when $|\tau| \leq$ (any observation window of 100 days in year).

As an expert in SSA plot auto correlation function of stock price of both the companies.

Your friend is interested to invest in one of the company advice your friend for selecting right company for investment

(b) Sketch the ensemble of the random process **07**

$x(t) = \mathbf{a} \cos(\mathbf{w}t + \boldsymbol{\theta})$ where **a** and **w** are constants and **θ** is an RV uniformly distributed in the range $(-\pi, \pi)$.

Just by observing the ensemble, determine whether this is a stationary or a non stationary process.

Determine $\overline{x(t)}$ and $R_x(t_1,t_2)$ for this random process and determine whether this is a wide-sense stationary process.

Q.5 **(a)** Are the following covariance functions are valid covariance functions of a real stationary process? Given answer with proper reasons. **07**

(a) $R(\tau) = \cos(t)$ (b) $R(\tau) = \begin{cases} \tau & |\tau| \leq 1 \\ 0 & |\tau| > 1 \end{cases}$

(b) Noise impulses occurs on a telephone line are according to a Poisson random process at rate λ. Find the probability that no impulses occur during the transmission of a message that is t seconds long. **07**

OR

Q.5 **(a)** What is convergence of Random variable? Explain Mean square convergence and convergence in probability **07**

(b) The Bernoulli process is defined as, **07**

S(n) = X(1)+X(2)+….+X(n)

Where X(i) for all i is a success or failure type events.

(i.e X(i) = 0 or 1)

X(i) and X(j) are statistically independent when $i \neq j$

S(n) is integer valued process with distribution;

$$P[S(n)=k] = \binom{n}{k} p^k (1-p)^{n-k}$$

Show that this process S(n) has Independent Increments and Stationary Increments.

[hint: $S(n_i) - S(n_j)$ for $i \neq j$ is one increment]

****