Seat No.: _____ Enrolment No.______

GUJARAT TECHNOLOGICAL UNIVERSITY

M. E. IST Semester–Remedial Examination – July- 2011

Subject code: 710401N

Subject Name: Statistical Signal Analysis

Date:07/07/2011 **Time: 10:30 am – 01:00 pm**

Total Marks: 70

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q.1 (a) Give answer of following questions. **07**

(i) What is the significance of probability theory? Describe statistical approach & axiomatic approach to a theory of probability.

(ii) Differentiate between Independent event & Mutually Exclusive event. Let A and B be events with probabilities P[A] and P[B].

1. Find $P[A \cup B]$ if A and B are independent.
2. Find $P[A \cup B]$ if A and B are mutually exclusive.

(b) Explain Total probability and Bayes' theorem. **07**

A firm has three machines A, B and C which generate items in the proportion 2:6:3. 50%, 70% and 90% of the items generated by A, B and C respectively are known to have standard quality. An item selected at random from a day's production is known to have standard quality. What is the chance that it came from machine C?

Q.2 (a) Explain Cumulative Distribution Function & Probability Density Function for random variable with their properties. Verify whether the following is a probability density function or not. **07**

$$f_x(x) = \begin{cases} 3x^2, & 0 \le x \le 1 \\ 0 & otherwise \end{cases}$$

(b) Give answer of following questions. **07**

(i) Discuss statistical independence and uncorrelation of random variables.

(ii) A communication system accepts a positive voltage V as a input and outputs a voltage Y=αV+N, where $\alpha=10^{-2}$ and N is a Gaussian random variable with parameters m = 0 and σ=2. Find the value of V that gives $P[Y<0] = 10^{-6}$.

OR

(b) Give answer of following questions. **07**

(i) The probability of a bit error in a communication line is 10^{-3}. Find the probability that a block of 1000 bits has five or more errors.

(ii) Show that E[X] for the Cauchy random variable X with pdf

$f_X(x)=\left(\pi\left(1+x^2\right)\right)^{-1}$ does not exist.

Q.3 **(a)** Describe characteristics function and moment generating function of random variable. Find the mean and variance of exponentially distributed random variable X having probability density function $f_X(x) = \lambda e^{-\lambda x}$, $x \geq 0$ and $\lambda > 0$ by applying the moment theorem. **07**

(b) Give answer of following questions. **07**

(i) If the random variable X is uniformly distributed over [-1 , 1], Find the pdf of random variable $Y = \sin\left(\frac{\pi x}{2}\right)$.

(ii) Show that the Gaussian PDF integrates to one.

OR

Q.3 **(a)** State the Markov and Chebyshev's inequalities for random variable. A symmetric die is thrown 720 times. Use Chebyshev's inequality to find the lower bound for the probability of getting 100 to 140 sixes. **07**

(b) Give answer of following questions. **07**

(i) Find mean and variance of the geometric random variable given by $p_k = p(1-p)^k, k=0,1,\ldots,n$.

(ii) The random variable X has the pdf $f_X(x) = e^{-x}u(x)$. Find the pdf of random variable $Y=X^2$

Q.4 **(a)** The joint pdf of two random variables X and Y is given by **07**

$$f_{(X,Y)}(x,y) = \begin{cases} ce^{-x}e^{-y} & 0 \leq y \leq x < \infty \\ 0 & elsewhere \end{cases}$$

(i) Find the normalization constant c and the marginal pdf's.

(ii) Are the random variable X and Y independent?

(iii) Find conditional pdf's $f_{X/Y}(x/y)$ & $f_{Y/X}(y/x)$

(b) Give answer of following questions. **07**

(i) Let Z = X + Y. Find $F_Z(z)$ and $f_Z(z)$ in terms of the joint pdf of X and Y.

(ii) Let X and Y be jointly Gaussian random variables. Derive the joint characteristic function of X and Y using conditional expectation.

OR

Q.4 **(a)** Explain central limit theorem with proof. The lifetime of a cheap light bulb is an exponential random variable with mean 36 hours. Suppose that 16 light bulbs are tested and their lifetimes measured. Use the central limit theorem to estimate the probability that the sum of lifetimes is less than 600 hours. **07**

(b) State the conditions for sure convergence and absolute sure convergence for a sequence of random variables $\{X_n(\zeta)\}$. **07**

Let U_n be a sequence of independent, identically distributed (iid) zero-mean, unit-variance Gaussian random variables. A "low-pass filter" takes the sequence U_n and produces the sequence $X_n = \frac{1}{2}(U_n + U_{n-1})$

(i) Does this sequence converge in the mean square sense?

(ii) Does it converge in distribution?

Q.5 **(a)** Define and classify Stochastic processes. Let X(t) and Y(t) be independent, wide-sense stationary random processes with zero means and the same covariance function $C_X(\tau)$. Let Z(t) be given by **08**

$$Z(t) = X(t) \cos \omega t + Y(t) \sin \omega t.$$

(i) Determine whether Z(t) is a wide-sense stationary random process.
(ii) Find the pdf of Z(t) if X(t) and Y(t) are also jointly Gaussian random processes.

(b) Give answer of following questions. **06**
(i) Give the condition for which random process will become the Ergodic random process.
(ii) Let Z(t) = X(t) – aX(t-s), where X(t) is Wiener process. Find the pdf, mean $m_Z(t)$ and autocovariance $C_Z(t_1,t_2)$ of Z(t).

OR

Q.5 (a) Give answer of following questions. **06**
(i) Is Wiener process mean square continuous?
(ii) Does the Wiener process have a mean square derivative?
(ii) Find the power spectral density of Z(t) = X(t) +Y(t), where X(t) and Y(t) are jointly Wide Sense Stationary processes.

(b) Give answer of following questions. **08**
(i) Explain the response of linear system to random signals in brief. Let X(t) be a differentiable WSS random process, and define $Y(t)=\frac{d}{dt}X(t)$. Find an expression for $S_Y(f)$ and $R_Y(\tau)$.
(Hint: For this system H(f) =j2πf.)
(ii) Discuss linear least square estimation in brief.

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