Seat No.: _____ Enrolment No.______

GUJARAT TECHNOLOGICAL UNIVERSITY

M.E Sem-I Remedial Examination January/ February 2011

Subject code: 710401

Subject Name: Statistical Signal Analysis

Date: 31 /01 /2011 **Time: 02.30 pm – 05.00 pm**

Total Marks: 60

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q.1 **(a)** Define the following terms **04**

(1) Random experiment (3) Sample space

(2) Random variable (4) Random process

(b) Consider an experiment of tossing a fair coin twice. Let (X, Y) be a bivariate random variable, where X is the number of heads that occurs in the two tosses and Y is the number of tails that occurs in the two tosses. **04**

(a) What is the range Rx of X?

(b) What is the range Ry of Y?

(c) Find and sketch the range Rxy of (X, Y).

(d) Find $P(X = 2, Y = 0)$, $P(X = 0, Y = 2)$, and $P(X = 1, Y = 1)$.

(c) Suppose a random process X(t) has a mean square derivative X'(t). **04**

(a) Find E[X'(t)].

(b) Find the cross-correlation function of X(t) and X'(t).

(c) Find the autocorrelation function of X'(t).

Q.2 **(a)** **A** committee of 5 persons is to be selected randomly from a group of 5 men and 10 women. **06**

(a) Find the probability that the committee consists of 2 men and **3** women.

(b) Find the probability that the committee consists of all women.

(b) Prove the following statements **06**

1. For any event A, $P(A) = 1 - P(A')$
2. For any events A & B, $P(AUB) = P(A) + P(B) - P(A \cap B)$
3. If $A_1, A_2, A_3, \ldots, A_n$ are pair wise mutually exclusive, then $P(A_1) \cup P(A_2) \cup \ldots \cup P(A_n) = P(A_1) + P(A_2) + \ldots + P(A_n)$

OR

(b) Two dice are tossed. One die is regular and the other is biased with probabilities $P(1) = P(6) = 1/6$, $P(2) = P(4) = 0$, $P(3) = P(6) = 1/3$. **06**

Determine the probabilities of obtaining a sum (1) 4 (2) 5 (3) 6

Q.3 **(a)** 1. If random variables X and Y are uncorrelated and have zero means than prove that $\overline{(X+Y)^2} = \overline{X^2} + \overline{Y^2}$ **03**

2. Given $X = \cos\theta$ and $Y = \sin\theta$, where θ is a random variable uniformly distributed in the range $(0, 2\pi)$. Show that X and Y are uncorrelated but are not independent. **03**

(b) Explain the central limit theorem and prove that under certain conditions, the sum of a large numbers of independent random variables tends to be a gaussian random variable, independent of the probability densities of the variables added. **06**

OR

Q.3	**(a)**	Determine mean, mean square and the variance of the random variable X for the following $P_X(X) = 0.5\,|X|\,e^{-|X|}$	**06**
	(b)	A player chooses any 6 numbers out of 49 numbers. Six balls are drawn randomly without replacement from 49 balls numbered 1 through 49. (a) Find the probability of matching all 6 balls to the 6 numbers chosen by the player (b) Find the probability of matching exactly 4 balls (c) Find the probability of matching exactly 4 balls	**06**
Q.4	**(a)**	The joint cdf of a bivariate random variable (X, Y) is given by $F_{XY}(x, y) = \begin{cases}(1 - e^{-\alpha x})(1 - e^{-\beta y}) & x \geq 0,\ y \geq 0,\ \alpha, \beta > 0 \\ 0 & \text{otherwise}\end{cases}$ (a) Find the marginal cdf's of X and Y. (b) Show that X and Y are independent. (c) Find $P(X \leq 1, Y \leq I)$, $P(X \leq I)$, $P(Y > I)$, and $P(X > x, Y > y)$.	**06**
	(b)	Explain CDF and PDF with its properties and prove all these properties.	**06**
		OR	
Q.4	**(a)**	Consider an experiment of drawing randomly three balls from an urn containing two red, three white, and four blue balls. Let (X, Y) be a bivariate random variable. Where X and Y denote, respectively, the number of red and white balls chosen. 1. Find the range of (X, Y). 2. Find the joint pmf's of (X, Y). 3. Find the marginal pmf's of X and Y. 4. Are X and Y independent?	**06**
	(b)	Explain in detail different laws of large numbers	**06**
Q.5	**(a)**	Classify the random processes explain each in detail	**06**
	(b)	Let $X_n = \cos(2\pi f_0 n + \theta)$, where θ is a uniformly distributed random variable in the interval $(0, 2\pi)$. Find $S_X(f)$.	**06**
		OR	
Q.5	**(a)**	Let $R_X(k) = 4(1/2)^{|k|} + 16(1/4)^{|k|}$. Find $S_X(f)$.	**06**
	(b)	A binary source generates digits 1 and 0 randomly with equal probability. Assign probabilities to the following events. In ten digits generated by the source (a) there are exactly two 1's and eight 0's (b) there are at least four 0's. (c) there are at least five 1's	**06**

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