Seat No.: _____ **Enrolment No._____**

GUJARAT TECHNOLOGICAL UNIVERSITY

ME Semester –I Examination Feb. - 2012

Subject code: 710901N **Date: 11/02/2012**

Subject Name: Theory of Elasticity

Time: 10.30 am – 01.00 pm **Total Marks: 70**

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q.1 **(a)** The displacement field in a solid elastic body is given as **07**

$u_x = (3x^2 z + 60x) \times 10^{-3}$ mm, $u_y = (5z^2 + 20xy) \times 10^{-3}$ mm

$u_z = (6z^2+2xyz) \times 10^{-3}$ mm.

Evaluate the components of strain tensor at a point (3, 4, 0.5). Also determine the principal strains and principal axes.

$J_1 = \varepsilon_{xx} + \varepsilon_{yy} + \varepsilon_{zz}$

$$J_2 = \begin{vmatrix} \varepsilon_{xx} & \frac{1}{2}\gamma_{xy} \\ \frac{1}{2}\gamma_{xy} & \varepsilon_{yy} \end{vmatrix} + \begin{vmatrix} \varepsilon_{xx} & \frac{1}{2}\gamma_{xz} \\ \frac{1}{2}\gamma_{xz} & \varepsilon_{zz} \end{vmatrix} + \begin{vmatrix} \varepsilon_{yy} & \frac{1}{2}\gamma_{yz} \\ \frac{1}{2}\gamma_{yz} & \varepsilon_{zz} \end{vmatrix}$$

$$J_3 = \begin{vmatrix} \varepsilon_{xx} & \frac{1}{2}\gamma_{xy} & \frac{1}{2}\gamma_{xz} \\ \frac{1}{2}\gamma_{yx} & \varepsilon_{yy} & \frac{1}{2}\gamma_{yz} \\ \frac{1}{2}\gamma_{zx} & \frac{1}{2}\gamma_{zy} & \varepsilon_{zz} \end{vmatrix}$$

(b) Differentiate between pressure and stress. Define tensor and show that pressure is not a second order tensor whereas the stress is a second order tensor. **07**

Q.2 **(a)** Explain with a neat schematic sketch the interpretation of shear strain γ_{zx} in x-z plane for two fibers PQ and PR mutually perpendicular to each other and parallel to x and z axes. Also explain rigid body rotation about y axis for two fibers PQ and PR mutually perpendicular to each other and parallel to x and z axes. **07**

(b) Explain with a neat schematic sketch the state of stress in the following cases: **07**

1. A retaining wall of the dam as a long prismatic body with the two ends of the wall prevented from moving. The vertical or horizontal line segments on any cross section are prevented from tilting to the left or right and up or down.
2. A long thin walled cylindrical pressure vessel closed at both the ends and subjected to internal pressure.

Justify your answer with reason.

OR

(b) Draw and comment on the nature of the Mohr's circle diagram for the following cases where in the three principal stresses σ_1, σ_2 and σ_3 are given as: (i) unequal (ii) equal (iii) any two of them are equal **07**

Q.3 **(a)** Define bulk Modulus K and express it in terms of the Lame's coefficient λ and μ. For steel having E = 207 x 10^6 kPa and ν = 0.3 calculate λ, μ and K. **07**

(b) A cubical element is subjected to the following state of stresses. **07**
σ_{xx} = 100 MPa, σ_{yy} = - 20 MPa, σ_{zz} = - 40 MPa,
$\tau_{xy} = \tau_{yz} = \tau_{xy} = 0$
Assuming the material to be homogeneous and isotropic determine the octahedral shear strain, if E = 2 x 10^5 MPa and ν = 0.25.

OR

Q.3 **(a)** A thin rubber is enclosed between two fixed hard steel plate. Friction between rubber and plate is negligible. If the rubber plate is subjected to compressive stresses σ_{xx} , σ_{yy} and σ_{zz} in x, y and z direction respectively determine strains ε_{xx} , ε_{yy} and ε_{zz}.. **07**

(b) The component of a strain tensor at a point in a body are given by ε_{xx} = 0.005 , ε_{yy} = 0.004, and ε_{zz}= -0.002, γ_{xy} = 0.001, γ_{yz} = 0.0005 γ_{zx} = 0.002 , if E = 2 x 10^5 MPa and ν = 0.25, determine the components of stress tensor. **07**

Q.4 **(a)** The end of a semi-circular member is subjected to torque T as shown in Fig. 1. What is the twist at thee end of A? The member is circular in section. **07**

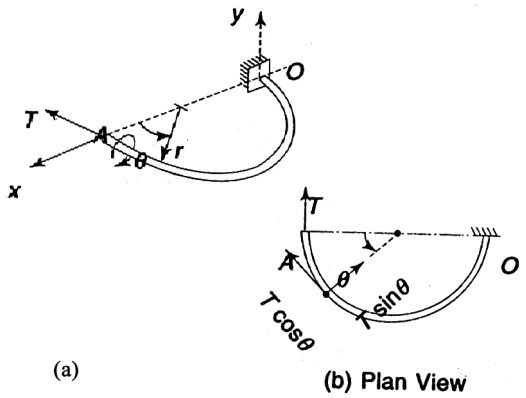

Fig. 1

(b) For the structure shown in Fig. 2 determine the vertical deflection at end A? **07**

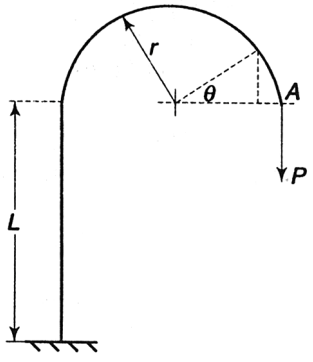

Fig. 2

OR

Q.4 **(a)** Determine the slope at the end A of the cantilever in Fig 3 which is subjected to load P. **07**

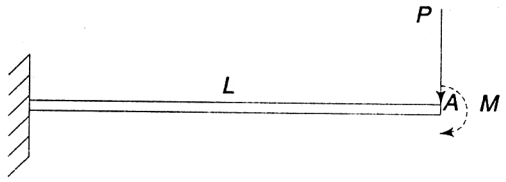

Fig. 3

(b) Determine the support reaction for the propped cantilever as shown in Fig. 4 **07**

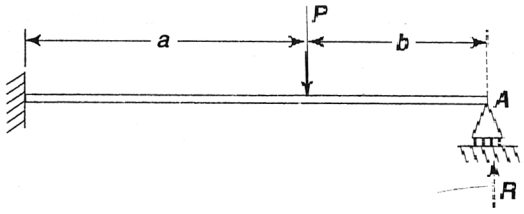

Fig. 4

Q.5 **(a)** Determine the radial displacement $u_{r.}$ when a thin disc subjected to a temperature distribution which varies with its radius '**r**' and is independent of the '**θ** 'direction. The temperature distribution is not varying along the thickness direction. **07**

(b) Determine the radial displacement u_r for a solid sphere subjected to purely radial temperature variation. **07**

OR

Q.5 **(a)** Determine the radial displacement u_r in a long circular cylinder when the temperature distribution is symmetrical about an axis and does not vary along the axis. The z – axis is the axis of the cylinder and r the radius and temperature T is the function of r alone and independent of z. **07**

(b) Determine the radial and angular stresses for a solid sphere subjected to purely radial temperature variation. **07**

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