Seat No.: _____ Enrolment No.______

GUJARAT TECHNOLOGICAL UNIVERSITY

M.E Sem-I Examination January 2010

Subject code: 710901 **Subject Name: Theory of Elasticity**

Date: 20 / 01 / 2010 **Time: 12.00 – 2.30 pm**

Total Marks: 60

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**
4. **Notations used have common meaning.**

Q.1 (a) A rectangular steel bar of 2 cm x 3 cm is subjected to a tensile force of 6000 N. Determine the normal and shear stresses on a plane whose normal has the following direction cosines. **06**

(i) $n_x = n_y = \sqrt{\frac{1}{2}}, \quad n_z = 0$

(ii) $n_x = n_y = n_z = \sqrt{\frac{1}{3}}$

Consider the normal stresses as

$$\sigma_n = n_x^2\sigma_x + n_y^2\sigma_y + n_z^2\sigma_z + 2n_xn_y\tau_{xy} + 2n_yn_z\tau_{yz} + 2n_xn_z\tau_{xz}$$

(b) Draw Mohr's circle diagram for the cases where the values of principal stress are **06**

$(i)\sigma_x = \sigma_y \neq \sigma_z \quad (ii)\sigma_x > \sigma_y > \sigma_z$

Q.2 (a) The displacement field imposed on a body is given below **06**

$$u_x = xy.10^{-2},\ u_y = 3x^2z.10^{-2}, u_z = 0.04$$

Consider a point P (2,1,3) and has a neighboring point Q where PQ has following direction cosines as

$$n_x = 0.2, n_y = 0.8, n_z = 0.55$$

If PQ = Δs, find the components of P'Q' after deformation.

(b) Show the graphical representation of **06**

(i) shear strain component γ_{xy}

(ii) A pure rigid body rotation of the element PQR consisting of the fiber PQ and PR at right angles and subjected through a small angular displacement.

OR

(b) What is plain stress and plain strain condition? Give suitable engineering examples where such conditions exist. Also show that for the plane strain condition if the strain is zero in z-direction the stress in z-direction need not be zero. **06**

Q.3 (a) Write a Hooke's law and extend this concept to derive generalized **06**

Hooke's law for relating six strain components with stress components assuming linear variation of stress with strain for homogeneous material. Using this law derive a stress strain relationship for linear, elastic, homogeneous and isotropic material.

(b) A rubber cube is inserted in the cavity of the same form and size in a steel block and top of the cube is pressed in a steel block with a pressure of 'p' Pascals. Considering the steel to be absolutely hard and assuming that there is no friction between steel and rubber, find the pressure of rubber against the box walls if the Poisson's ratio is 0.5. **06**

OR

Q.3 (a) Define modulus of rigidity, bulk modulus, and Poisson's ratio and show that for the bulk modulus to be positive, the value of Poisson's ratio can not exceed a value 0.5. Also prove that materials with Poisson's ratio 0.5 are incompressible. **06**

(b) Explain the limitations of considering maximum principal normal stress, principal shear stress, maximum strain energy or octahedral shear stress as a criterion for failure **06**

Q.4 (a) State the first theorem of Castigliano for linearly elastic body. Determine the support reaction for propped cantilever as shown in Fig.1 **06**

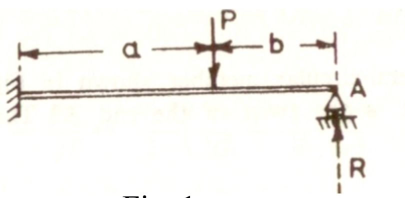

Fig. 1

(b) For the cantilever shown in Fig.2 determine the deflection by at end neglecting shear energy. **06**

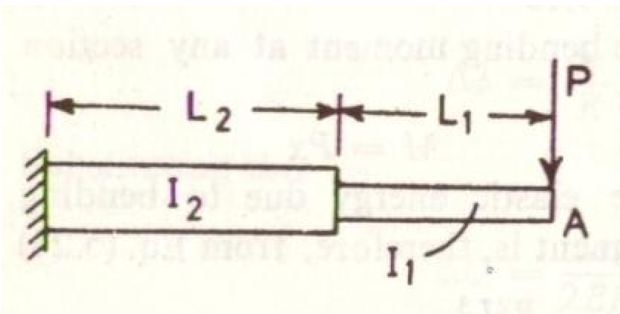

Fig. 2

OR

Q.4 (a) The Castigilano's theorem is extremely useful in determining the displacement of structures and solving statically indeterminate linearly elastic solids structure – Evaluate. **06**

(b) Consider Fig. 3 which shows two identical bars hinged together, carrying load W. Check Castigliano's the first theorem, using the elastic and complementary strain energy. **06**

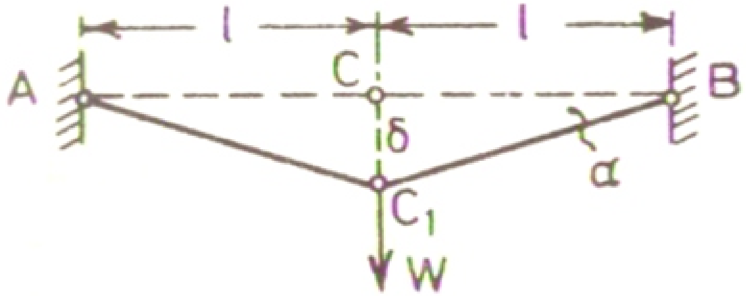

Fig 3

Q.5 **(a)** The inner surface of a hollow tube is at temperature T_i and the outer surface at zero temperature. Assuming steady state conditions, calculate the stresses. What are the values of σ_z and σ_θ near the inner and outer surfaces? Under steady state heat flow conditions, the temperature at any distance r from the centre is given by the expression **06**

$$T = \frac{T_i}{\log\left(\frac{b}{a}\right)} \log\left(\frac{b}{r}\right)$$

In usual notations σ_r, σ_θ and σ_z are given as

$$\sigma_r = \frac{\alpha E}{(1-\nu)r^2}\left[\frac{r^2 - a^2}{b^2 - a^2}\int_a^b Tr\,dr - \int_a^r Tr\,dr\right]$$

$$\sigma_\theta = \frac{\alpha E}{(1-\nu)r^2}\left[\frac{r^2 + a^2}{b^2 - a^2}\int_a^b Tr\,dr + \int_a^r Tr\,dr - Tr^2\right]$$

$$\sigma_z = \frac{\alpha E}{(1-\nu)}\left[\frac{2}{b^2 - a^2}\int_a^b Tr\,dr - T\right]$$

(b) Show that the resultant circumferential force across any radial section of a hollow disk is zero. Consider the disk of uniform thickness perpendicular to the plane of paper. **06**

$$\sigma_\theta = \frac{\alpha E}{r^2}\left[\frac{r^2 + a^2}{b^2 - a^2}\int_a^b Tr\,dr + \int_a^r Tr\,dr - Tr^2\right]$$

OR

Q.5 **(a)** Explain the advantages and limitation of finite element method as regards to discretization, heterogeneity, nature of domain of the problem and problem formulation. **06**

(b) Let the inner surface of hollow sphere be at temperature T_i and outer surface at temperature zero. Let the system be in steady heat flow condition. The temperature distribution is then given by **06**

$$T = \frac{T_i a}{b-a}\left(\frac{b}{r} - 1\right)$$

Determine the stress distribution if in usual notations σ_r, σ_θ and σ_ϕ are given as

$$\sigma_r = \frac{2\alpha E}{(1-\nu)}\left[\frac{r^3 - a^3}{(b^3 - a^3)r^3}\int_a^b Tr^2\, dr - \frac{1}{r^3}\int_a^r Tr^2\, dr\right]$$

$$\sigma_\phi = \sigma_\theta = \frac{2\alpha E}{(1-\nu)}\left[\frac{2r^3 + a^3}{2(b^3 - a^3)r^3}\int_a^b Tr^2\, dr - \frac{1}{2r^3}\int_a^r Tr^2\, dr - \frac{1}{2}T\right]$$

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