**B. Sc.(Honours) in Mathematics and Computing**
**INSTITUTE OF MATHEMATICS AND APPLICATIONS**
**BHUBANESWAR**

**DATE : 29.06.2008**

**FULL MARKS : 100**                                **TIME : 11 A.M.-1 P.M.**

**Answer *all* questions.**
**Wrong answers are liable to be penalized**.

1. If the sets $A$ and $B$ are defined as $A = \{(x,y) : y = e^x, x \in \mathbb{R}\}$ and
   $B = \{(x,y) : y = x, x \in \mathbb{R}\}$, then
   (a) $A \subset B$      (b) $B \subset A$      **(c)** $A \cap B = \emptyset$      (d) $A \cup B = A$

2. $A$ and $B$ are two sets having 3 and 4 elements respectively and 2 elements in common.
   Then the number of relations which can be defined from the set $A$ to $B$ is
   (a) $2^5$      (b) $2^{10}$      (c) $2^{12} - 1$      (d) None of these

3. The complex number $\dfrac{2^{50}}{(1-i)^{100}} + \dfrac{(1+i)^{100}}{2^{50}}$ is equal to
   (a) 0      (b) 1      (c) $i$      (d) $-2$

4. The locus of the complex numbers $z$ satisfying the equation $\left| \dfrac{z - 5i}{z + 5i} \right| = 1$ is
   (a) $X$-axis.                              (b) the straight line $y = 5$.
   (c) the straight line $x + y = 5$.      (d) a circle passing through the origin.

5. Which of the following is/are incorrect ?
   (i) Adjoint of a symmetric matrix is symmetric.
   (ii) Adjoint of a unit matrix is a unit matrix.
   (iii) Adjoint of a diagonal matrix is a diagonal matrix.
   (iv) If $A$ is a singular matrix, then $A(\text{adj}) A$ is the null matrix.
   (a) (i)      (b) (ii)      (c) (ii) and (iv)      (d) (iii)

6. The system of equations : $2x - y + z = 0$, $x - 2y + z = 0$, $\alpha x - y + 2z = 0$ have non-trivial
   solutions for
   (a) $\alpha = -5$      (b) $\alpha = 1$      (c) $\alpha = 5$      (d) no real value of $\alpha$.

7. If $x$ satisfies the inequality: $|x - 1| + |x - 2| + |x - 3| \geq 6$, then
   (a) $0 \leq x \leq 4$.      (b) $x \leq 0$ or $x \geq 4$.      (c) $x \leq -2$ or $x \geq 4$.      (d) None of these.

8. If $x^2 + px + q = 0$ and $x^2 + qx + p = 0$ have a common root, then
   (a) $p = q$ only.  (b) $p + q + 1 = 0$ only.
   (c) either $p = q$ or $p + q + 1 = 0$.  (d) None of these.

9. Let $n(> 1)$ be a positive integer. Then the largest integer $k$ such that $(n^k + 1)$ divides $1 + n + n^2 + \cdots + n^{255}$ is
   (a) 16  (b) 32  (c) 64  (d) 128

10. Total number of ways in which six $'+'$ and four $'-'$ signs can be arranged in a line such that no two $'-'$ signs occur together is
    (a) 27  (b) 35  (c) 42  (d) 57

11. Let $A$ and $B$ be two sets such that $n(B) = 2$ and the number of mappings from $A$ to $B$ which are onto is 30. Then the number of elements in the set $A$ is
    (a) 3  (b) 5  (c) 7  (d) 8

12. For two events $A$ and $B$, if $P(A) = P(A|B) = \dfrac{1}{4}$ and $P(B|A) = \dfrac{1}{2}$, then
    (a) $A$ and $B$ are independent events.
    (b) $A$ and $B$ are mutually exclusive events.
    (c) $P(A'|B) = \dfrac{1}{2}$.
    (d) $P(B'|A') = \dfrac{3}{4}$.

13. The function $f(x) = (x + 2)e^{-x}$ $(x \in \mathbb{R})$ is
    (a) decreasing for all $x$.  (b) decreasing in $(-\infty, -1)$ and increasing in $(-1, \infty)$.
    (c) increasing for all $x$.  (d) decreasing in $(-1, \infty)$ and increasing in $(-\infty, -1)$.

14. The value of $\lim_{x \to 0} \dfrac{\int_0^x \cos t^2 dt}{x}$ is
    (a) $-1$  (b) 0  (c) 1  (d) None of these.

15. The values of $\alpha$ and $\beta$ for which the function

$$f(x) = \begin{cases} -2\sin x, & x \leq -\pi/2 \\ \alpha \sin x + \beta, & -\pi/2 < x < \pi/2 \\ \cos x, & x \geq \pi/2 \end{cases}$$

is continuous everywhere are
    (a) $\alpha = 1, \beta = 1$  (b) $\alpha = 0, \beta = 1$  (c) $\alpha = -1, \beta = 0$  (d) $\alpha = -1, \beta = 1$

16. The function $f(x) = \begin{cases} x^k \sin \dfrac{1}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases}$ is continuous but not differentiable at $x = 0$ if
    (a) $k \in (0, 1]$  (b) $k \in [1, \infty)$  (c) $k \in (-\infty, 0)$  (d) $k = 0$

17. The function $f(x) = |x| + |x - 1|$ is

    (a) continuous at $x = 0, 1$ but not differentiable at $x = 0, 1$.
    (b) both continuous at $x = 0, 1$ and differentiable at $x = 0, 1$.
    (c) continuous at $x = 0, 1$, differentiable at $x = 0$ but not differentiable at $x = 1$.
    (d) not continuous at $x = 0, 1$ but differentiable at $x = 0, 1$.

18. Let $f$ be a continuous function in $\mathbb{R}$ such that $f(x + y) = f(x) + f(y)$. Then the value of $\int_{-2}^{2} f(x)dx$ is

    (a) 0      (b) $f(2)$      (c) $f(-2)$      (d) $2 \int_{0}^{2} f(x)dx$

19. $\int_{0}^{\pi/2} \dfrac{e^{\sin x}}{e^{\sin x} + e^{\cos x}} dx =$

    (a) $\pi$      (b) $\pi/2$      (c) $\pi/4$      (d) $\pi/8$

20. The area between the curve $y = 1 - |x|$ and the $X$-axis is

    (a) 1/2 sq. unit      (b) 1 sq. unit      (c) 2 sq. unit      (d) None of these.

21. The equation of the tangent to the curve $y = 2e^{-x/3}$, where it crosses the $Y$-axis is

    (a) $\dfrac{x}{3} + \dfrac{y}{2} = 1$.      (b) $\dfrac{x}{2} + \dfrac{y}{3} = 1$.      (c) $3x - 2y = 1$.      (d) $2x + 3y = 1$.

22. The value of $\theta$ satisfying the equation $\cos\theta + \sqrt{3}\sin\theta = 2$ is

    (a) $\pi/3$      (b) $2\pi/3$      (c) $\pi$      (d) $4\pi/3$

23. The coordinates of the points which lie on the line $x + y = 4$ and whose distance from the line $4x + 3y = 10$ is unity, are

    (a) $(3, 1), (-9, 13)$      (b) $(3, 1), (-7, 11)$      (c) $(5, -1), (-9, 13)$      (d) $(1, 3), (-7, 11)$

24. The circles $x^2 + y^2 - 10x + 16 = 0$ and $x^2 + y^2 = r^2$ intersect each other at two distinct points provided

    (a) $r < 2$      (b) $r > 8$      (c) $2 < r < 8$      (d) $2 \le r \le 8$

25. The number of common tangents to the circles $x^2 + y^2 - x = 0$ and $x^2 + y^2 + x = 0$ is

    (a) 1      (b) 2      (c) 3      (d) 4