FIITJEE Solutions to IITJEE–2005 Mains Paper

Physics

Time: 2 hours

Note: Question number 1 to 8 carries ***2 marks*** each, 9 to 16 carries ***4 marks*** each and 17 to 18 carries ***6 marks*** each.

Q1. A whistling train approaches a junction. An observer standing at junction observes the frequency to be 2.2 KHz and 1.8 KHz of the approaching and the receding train. Find the speed of the train (speed of sound = 300 m/s)

Sol. While approaching

$$f' = f_0\left(\frac{v}{v - v_s}\right)$$

$$2200 = f_0\left(\frac{300}{300 - v_s}\right)$$

While receding

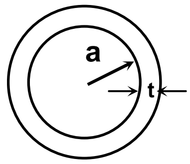
$$f'' = f_0\left(\frac{v}{v + v_s}\right)$$

$$1800 = f_0\left(\frac{300}{300 + v_s}\right)$$

On solving velocity of source (train) v_s = 30 m/s

Q2. A conducting liquid bubble of radius **a** and thickness **t** (t <<a) is charged to potential **V**. If the bubble collapses to a droplet, find the potential on the droplet.

Sol. Potential of the bubble $(V) = \frac{1}{4\pi\varepsilon_0}\frac{q}{a}$



by conservation of volume

$$4\pi a^2 t = \frac{4}{3}\pi R^3$$

$$R = \left(3a^2 t\right)^{1/3}$$

Hence, potential on the droplet

$$V' = \frac{1}{4\pi\varepsilon_0}\frac{q}{R} \text{ (as charge is conserved)}$$

$$\Rightarrow V' = \left(\frac{a}{3t}\right)^{1/3}.V$$

Q3. The potential energy of a particle of mass **m** is given by

$$V(x) = \begin{cases} E_0 & 0 \le x \le 1 \\ 0 & x > 1 \end{cases}$$

λ_1 and λ_2 are the de–Broglie wavelengths of the particle, when $0 \le x \le 1$ and $x > 1$ respectively. If the total energy of particle is $2E_0$, find λ_1/λ_2.

Sol. K.E. = $2E_0 - E_0 = E_0$ (for $0 \le x \le 1$)

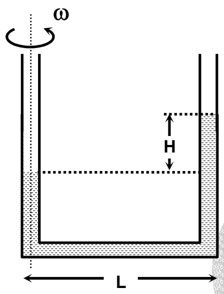
$$\lambda_1 = \frac{h}{\sqrt{2mE_0}}$$

KE = $2E_0$ (for $x > 1$)

$$\lambda_2 = \frac{h}{\sqrt{4mE_0}}$$

$$\frac{\lambda_1}{\lambda_2} = \sqrt{2}$$

Q4. A **U** tube is rotated about one of it's limbs with an angular velocity ω. Find the difference in height H of the liquid (density ρ) level, where diameter of the tube d << L.

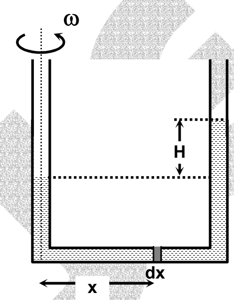


Sol.

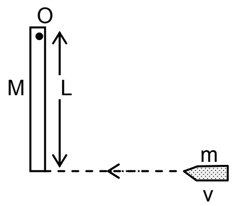
$$\Delta PA = \int_0^L dm x\omega^2$$

$$\rho gHA = \frac{\omega^2 L^2 \rho A}{2}$$

$$H = \frac{L^2\omega^2}{2g}$$



Q5. A wooden log of mass **M** and length **L** is hinged by a frictionless nail at O. A bullet of mass **m** strikes with velocity **v** and sticks to it. Find angular velocity of the system immediately after the collision about O.

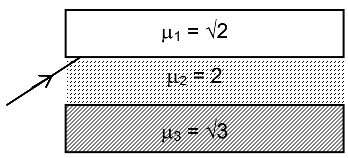


Sol. Apply conservation of angular momentum about O

$$(mv)L = (mL^2 + \frac{ML^2}{3})\omega$$

$$\omega = \frac{3mv}{(3m + M)L}$$

Q6. What will be the minimum angle of incidence such that the total internal reflection occurs on both the surfaces?



Sol. For first surface

$2 \sin c_1 = \sqrt{2} \sin 90°$

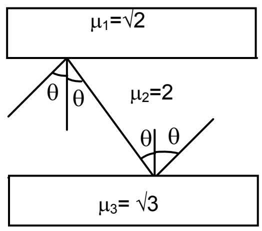
$\Rightarrow$ $c_1 = 45^0$

For second surface

$2 \sin c_2 = \sqrt{3} \sin 90°$

$\Rightarrow$ $c_2 = 60°$

$\therefore$ Minimum angle of incidence = Max $\{c_1, c_2\} = 60°$



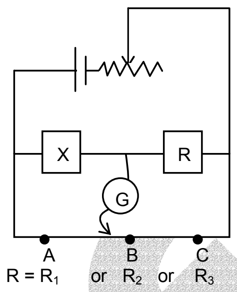
Q7. The side of a cube is measured by vernier callipers (10 divisions of a vernier scale coincide with 9 divisions of main scale, where 1 division of main scale is 1 mm). The main scale reads 10 mm and first division of vernier scale coincides with the main scale. Mass of the cube is 2.736 g. Find the density of the cube in appropriate significant figures.

Sol. Least count of vernier callipers = $(1 - \frac{9}{10})$mm = 0.1mm

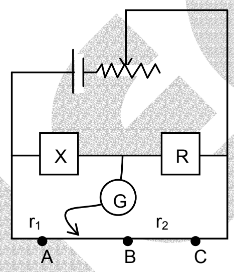
Side of the cube = 10 mm + 1 $\times$ 0.1 mm = 10.1 mm = 1.01 cm

Density = $\frac{2.736}{(1.01)^3} = 2.66 \text{g/cm}^3$

Q8. An unknown resistance **X** is to be determined using resistances **R_1, R_2** or **R_3**. Their corresponding null points are **A, B** and **C**. Find which of the above will give the most accurate reading and why?



Sol. $X = \frac{r_1 R}{r_2}$



$$\left|\frac{\delta X}{X}\right| = \left|\frac{\delta r_1}{r_1}\right| + \left|\frac{\delta r_2}{r_2}\right|$$

$$|\delta r_1| = |\delta r_2| = \Delta$$

$$\left|\frac{\delta X}{X}\right| = \left(\frac{r_1 + r_2}{r_1 r_2}\right)\Delta$$

For $\left|\frac{\delta X}{X}\right|$ to be minimum, $r_1 r_2$ should be maximum and as $r_1 + r_2$ is constant.

This is true for $r_1 = r_2$.

So R_2 gives most accurate value.

Q9. A transverse harmonic disturbance is produced in a string. The maximum transverse velocity is 3 m/s and maximum transverse acceleration is 90 m/s^2. If the wave velocity is 20 m/s then find the waveform.

Sol. If amplitude of wave is A and angular frequency is ω,

$$\frac{\omega A}{\omega^2 A} = \frac{3}{90} \Rightarrow \quad \omega = 30 \text{ rad/s}$$

$$v = \frac{\omega}{k} \quad \Rightarrow \quad k = \frac{3}{2} m^{-1}$$

A = 10 cm

Considering sinusoidal harmonic function

$$\therefore \quad y = (10 \text{ cm}) \sin(30t \pm \frac{3}{2}x + \phi)$$

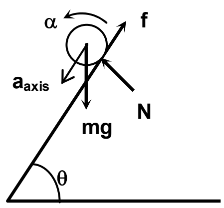
Q10. A cylinder of mass **m** and radius **R** rolls down an inclined plane of inclination θ. Calculate the linear acceleration of the axis of cylinder.

Sol. $mg\sin\theta - f = ma_{axis}$ (1)

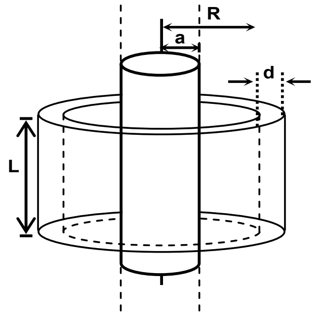
$fR = I_{axis}\alpha$ (2)

$a_{axis} = R\alpha$ (3)

$$a_{axis} = \frac{2g\sin\theta}{3}$$

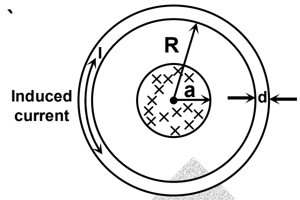


Q11. A long solenoid of radius **a** and number of turns per unit length **n** is enclosed by cylindrical shell of radius **R**, thickness **d** (d<<R) and length **L**. A variable current $i = i_0 \sin \omega t$ flows through the coil. If the resistivity of the material of cylindrical shell is ρ, find the induced current in the shell.



Sol. $\phi = (\mu_0 n i_0 \sin \omega t)\pi a^2$

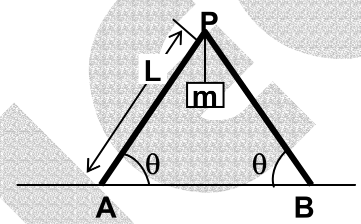
$$\varepsilon = \left|\frac{d\phi}{dt}\right| = (\mu_0 n i_0 \omega \cos \omega t)\pi a^2$$



Resistance = $\frac{\rho 2\pi R}{Ld}$

$$I = \frac{(\mu_0 n i_0 \omega \cos \omega t)\pi a^2 (Ld)}{\rho 2\pi R}$$

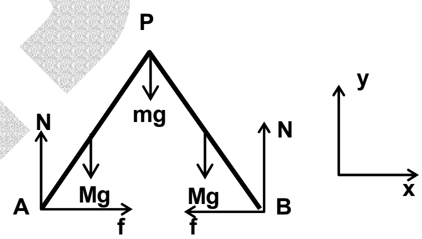
Q12. Two identical ladders, each of mass **M** and length **L** are resting on the rough horizontal surface as shown in the figure. A block of mass **m** hangs from P. If the system is in equilibrium, find the magnitude and the direction of frictional force at A and B.



Sol. For equilibrium of whole system,

$\Sigma F_y = 0$

$\Rightarrow \quad N = \left(\frac{2M+m}{2}\right)g$



For rotational equilibrium of either ladder

Calculating torque about **P**

$NL \cos \theta - Mg\frac{L}{2} \cos\theta - fL \sin\theta = 0$

$\Rightarrow \quad f = (M+m)g\frac{\cot \theta}{2}$

Q13. Highly energetic electrons are bombarded on a target of an element containing 30 neutrons. The ratio of radii of nucleus to that of Helium nucleus is $(14)^{1/3}$. Find

(a) atomic number of the nucleus.

(b) the frequency of K_α line of the X–ray produced. ($R = 1.1 \times 10^7 \text{ m}^{-1}$ and $c = 3 \times 10^8$ m/s)

Sol. $r = r_0 A^{1/3}$

$$\frac{r}{r_{He}} = \left(\frac{A}{4}\right)^{1/3} = 14^{1/3}$$

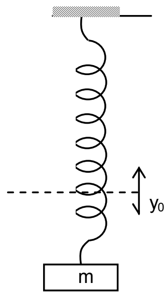
$\Rightarrow \quad A = 56$ and $Z = (56 - 30) = 26$

for K_α–line,

$$\sqrt{\nu} = \sqrt{\frac{3Rc}{4}}(Z-1)$$

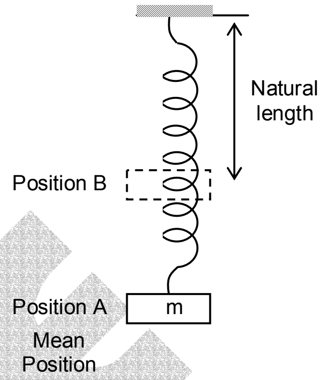
$\Rightarrow \quad \nu = 1.546 \times 10^{18}$ Hz

Q14. A small body attached to one end of a vertically hanging spring is performing SHM about it's mean position with angular frequency ω and amplitude **a**. If at a height **y*** from the mean position the body gets detached from the spring, calculate the value of y* so that the height **H** attained by the mass is maximum. The body does not interact with the spring during it's subsequent motion after detachment. ($a\omega^2 > g$)

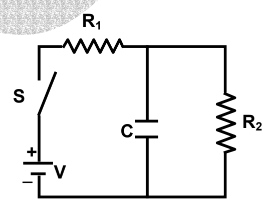


Sol. At position B as the potential energy of the spring will be zero, the total energy (Gravitational potential energy + Kinetic energy) of the block at this point will be maximum and therefore if the block gets detached at this point, it will rise to maximum height,

$$\therefore \quad y^* = \frac{mg}{k} = \frac{g}{\omega^2} < a$$



Q15. In the given circuit, the switch S is closed at time t = 0. The charge **Q** on the capacitor at any instant **t** is given by $Q(t) = Q_0\,(1-e^{-\alpha t})$. Find the value of $\mathbf{Q_0}$ and α in terms of given parameters shown in the circuit.

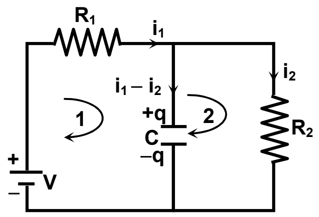


Sol. Applying KVL in loop 1 and 2,

$$V - i_1R_1 - \frac{q}{C} = 0 \qquad \ldots (1)$$

$$\frac{q}{C} - i_2R_2 = 0 \qquad \ldots (2)$$

$$i_1 - i_2 = \frac{dq}{dt} \qquad \ldots (3)$$



On solving we get,

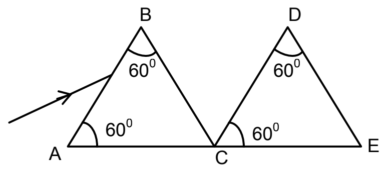
$$q = \frac{CVR_2}{R_1+R_2}\left(1-e^{-\frac{t(R_1+R_2)}{CR_1R_2}}\right)$$

$$\Rightarrow \quad Q_0 = \frac{CVR_2}{R_1+R_2} \quad \text{and} \quad \alpha = \frac{R_1+R_2}{CR_1R_2}$$

Q16. Two identical prisms of refractive index $\sqrt{3}$ are kept as shown in the figure. A light ray strikes the first prism at face AB. Find,

(a) the angle of incidence, so that the emergent ray from the first prism has minimum deviation.

(b) through what angle the prism DCE should be rotated about C so that the final emergent ray also has minimum deviation.

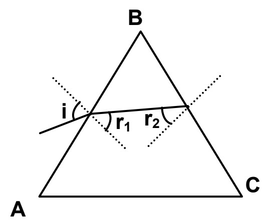


Sol. (a) For minimum deviation

$$r_1 = r_2 = \frac{\angle B}{2}$$

$$\frac{\sin i}{\sin 30^0} = \sqrt{3}$$

$$\Rightarrow \quad i = 60^0$$



(b) Prism DCE should be rotated about C in anticlockwise direction through 60° so that the final emergent ray is parallel to the incident ray and angle of deviation is zero (minimum)

Q17. A cylinder of mass 1 kg is given heat of 20000J at atmospheric pressure. If initially temperature of cylinder is 20^0C, find
(a) final temperature of the cylinder.
(b) work done by the cylinder.
(c) change in internal energy of the cylinder.
(Given that Specific heat of cylinder= 400 J kg^{-1} $^0C^{-1}$, Coefficient of volume expansion $= 9 \times 10^{-5}$ °C^{-1}, Atmospheric pressure $= 10^5$ N/m^2 and Density of cylinder = 9000 kg/m^3)

Sol. (a) $\Delta Q = ms\Delta T$

$$\Rightarrow \Delta T = \frac{20000J}{1kg \times (400J/kg\ ^0C)} = 50°C$$

$T_{final} = 70°C$

(b) $W = P_{atm}\ \Delta V = P_{atm}\ V_0\gamma\Delta T$

$$= (10^5\ N/m^2)\left(\frac{1}{9\times10^3}m^3\right)(9 \times 10^{-5}/°C)\ (50°C) = 0.05\ J$$

(c) $\Delta U = \Delta Q - W = 20000\ J - 0.05\ J = 19999.95\ J$

Q18. In a moving coil galvanometer, torque on the coil can be expressed as $\tau = ki$, where **i** is current through the wire and **k** is constant. The rectangular coil of the galvanometer having numbers of turns **N**, area **A** and moment of inertia **I** is placed in magnetic field **B**. Find
(a) k in terms of given parameters N, I, A and B.
(b) the torsional constant of the spring, if a current $\mathbf{i_0}$ produces a deflection of $\pi/2$ in the coil.
(c) the maximum angle through which coil is deflected, if charge **Q** is passed through the coil almost instantaneously. (Ignore the damping in mechanical oscillations)

Sol. (a) $\tau = iNAB\sin\alpha$
For a moving coil galvanometer $\alpha = 90°$
$ki = iNAB \quad \Rightarrow k = NAB$

(b) $\tau = C\theta$

$$i_0NAB = C\pi/2 \quad \Rightarrow C = \frac{2i_0NAB}{\pi}$$

(c) Angular impulse $= \int\tau.\ dt = \int NABidt = NABQ$

$\Rightarrow NABQ = I\omega_0$

$$\Rightarrow \omega_0 = \frac{NABQ}{I}$$

Using energy of conservation

$$\frac{1}{2}I\omega_0^2 = \frac{1}{2}C\theta_{max}^2$$

$$\Rightarrow \theta_{max} = \omega_0\sqrt{\frac{I}{C}} = Q\sqrt{\frac{NAB\pi}{2Ii_0}}$$