Errata for the ASM Study Manual for Exam P, Tenth Edition
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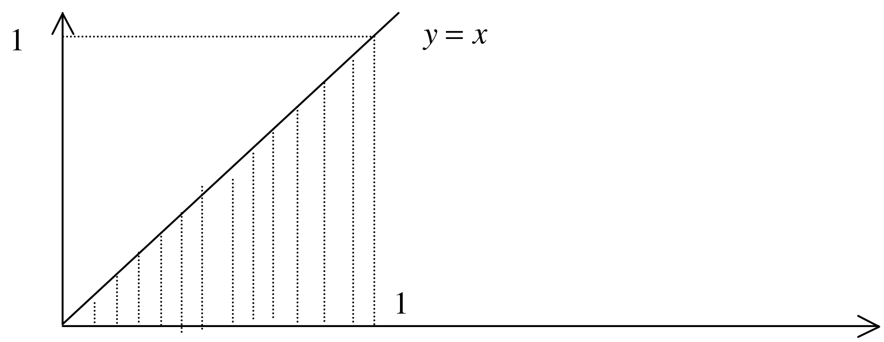
Posted September 21, 2011
The solution of Problem 29 in Practice Examination 19 should be:
Solution.
The joint density, where positive, for $0 < x < 1$ and $0 < y < x$, is

$$f_{X,Y}(x,y) = f_Y(y|X = x) \cdot f_X(x) = \frac{1}{x} \cdot 2x = 2.$$

The region where that joint density is positive is indicated with dotted lines in the graph below



Since the joint density is uniform, the conditional distribution of *X* given that $Y = y$ is uniform on the range of values of *x* determined by the condition $0 < y < x < 1$, i.e., the interval $(y,1)$, so that the variance is $\frac{(1-y)^2}{12}$.
Answer E.

Posted August 31, 2011
The first sentence of the solution of Problem 14 in Practice Examination 18 should be:
Chi-square distribution is obtained as a sum of squares of independent identically distributed standard normal random variables, so we need to standardize these variables, add squares of those standardized variables, and hope we get one of the answers.

Posted August 6, 2011
In the third line of Problem 21 in Practice Examination 15, the words “and integer” **should be** “an integer”.

Posted August 6, 2011
In the solution of Problem 6 in Practice Examination 9, in the first line, the words "for and" **should be "**for any".

Posted August 6, 2011
In the solution of Problem 1 in Practice Examination 9, the formula in the fifth line should be

$$\Pr\left(Y_{(1)}=3\right)=\Pr(E-F)=\Pr(E)-\Pr(F).$$

instead of

$$\Pr\left(Y_{(1)}=3\right)=\Pr(F)-\Pr(F).$$

Posted August 4, 2011
The solution of Problem 7 in Practice Examination 13 should be rephrased as follows:

Solution.
The event of at least one color not being represented is the complement of the event of all three colors being represented, and all colors being represented simply means that we pick one red ball out of 3, one green ball out of 2, and one yellow ball out of 1. Thus

$$\Pr(\text{At least one color not drawn})=$$

$$=1-\Pr(\text{All colors drawn})=1-\frac{\binom{3}{1}\cdot\binom{2}{1}\cdot\binom{1}{1}}{\binom{6}{3}}=0.70.$$

You could also argue as follows. We are looking for the probability that all three balls are of the same color, or of two colors only. We have

$$\Pr(3\text{ balls of one color})=\Pr(3\text{ red})=\frac{1}{\binom{6}{3}}=\frac{3!\ \cdot 3!}{6!}=\frac{6}{4\cdot 5\cdot 6}=\frac{1}{20},$$

and

$$\Pr(3\text{ balls of two colors only})=\Pr(2\text{ red}+1\text{ green})+\Pr(2\text{ red}+1\text{ yellow})+$$

$$+\Pr(2\text{ green}+1\text{ red})+\Pr(2\text{ green}+1\text{ yellow})=$$

$$=\frac{\binom{3}{2}\cdot\binom{2}{1}}{\binom{6}{3}}+\frac{\binom{3}{2}\cdot\binom{1}{1}}{\binom{6}{3}}+\frac{\binom{2}{2}\cdot\binom{3}{1}}{\binom{6}{3}}+\frac{\binom{2}{2}\cdot\binom{1}{1}}{\binom{6}{3}}=\frac{13}{20},$$

so that the total probability is $\frac{1}{20}+\frac{13}{20}=\frac{7}{10}$.

Answer B.

Posted August 3, 2011
In Practice Examination 20, Problem 6, in the solutions part of the examination, answer choices were not listed. They are listed in the questions part of the examination.

Posted August 3, 2011
The last sentence of Problem 11 in Practice Examination 16 should be:
Calculate the variance of Y given that $X > 3$ and $Y > 3$.
instead of
Calculate the variance of Y given that and $X > 3$ and $Y > 3$.

Posted August 3, 2011
In Problem 7 in Practice Examination 14, the solution should start with the words
Because you studied this manual
instead of
Because you studied his manual

Posted July 30, 2011
Problem 7 in Practice Examination 19 should start with
Let $X_1, X_2, ..., X_{36}$ and $Y_1, Y_2, ..., Y_{49}$ be independent random samples from distributions …
instead of
Let $x_1, x_2, ..., x_{36}$ and $y_1, y_2, ..., y_{49}$ be independent random samples from distributions …

Posted July 30, 2011
The first line of the formula in the solution of Problem 6 in Practice Examination 19 should be

$$\Pr\left(2X - X^2 > 0\right) = \Pr\left(X\left(2 - X\right) > 0\right) = \Pr\left(X\left(X - 2\right) < 0\right) =$$

instead of

$$\Pr\left(2X - X^2 > 0\right) = \Pr\left(X\left(2 - X\right) > 0\right) = \Pr\left(X\left(X - 2\right) > 0\right) =$$

Posted July 28, 2011
In the formula in the solution of Problem 26 in Practice Examination 17, the formula should be

$$f_X\left(x\right) = F_X'\left(x\right) = -\frac{d}{dx}\sum_{k=0}^{3}\frac{x^k \cdot e^{-x}}{k!} = -\frac{d}{dx}\left(e^{-x} + xe^{-x} + \frac{1}{2}x^2 \cdot e^{-x} + \frac{1}{6}x^3 \cdot e^{-x}\right) =$$

$$= -\left(-e^{-x} + \left(e^{-x} - xe^{-x}\right) + \left(xe^{-x} - \frac{1}{2}x^2 \cdot e^{-x}\right) + \left(\frac{1}{2}x^2 \cdot e^{-x} - \frac{1}{6}x^3 \cdot e^{-x}\right)\right) = \frac{1}{6}x^3 \cdot e^{-x}.$$

The formula was missing a minus sign in the second line just after the first parenthesis. The rest of the solution is unaffected.

Posted July 28, 2011
In the solution of Problem 16 in Practice Examination 17, the word "bad" **in the first sentence should be replaced by** "bag".

Posted July 28, 2011
The second sentence of the solution of Problem 12 in Practice Examination 16 should be:
Box 1 contains 1 blue and 4 red marbles, box 2 contains 2 blue and 3 red marbles and box 3 contains 3 blue and 2 red marbles.
instead of
Box 1 contains 1 blue and 4 red marbles, box 2 contains 2 blue and 3 red marbles and box 3 contains 3 red and 2 blue marbles.
The rest of the solution is unaffected by this typo.

Posted July 26, 2011
In Problem 9 in Practice Examination 14, the third condition should be:
(ii) The future lifetimes follow a Weibull distribution with $\alpha = 1.5$ and $\beta = 2.0$ for smokers, and $\alpha = 2.0$ and $\beta = 2.0$ for nonsmokers.
Also, the survival function of the Weibull distribution should be given as
$s_T(t) = e^{-\left(\frac{t}{\alpha}\right)^{\beta}}$.

Posted June 26, 2011
In Problem 29 in Practice Examination 19, the last sentence should be:
Find the variance of the conditional distribution of X, given $Y = y$.

Posted March 10, 2011
The second sentence of the solution of Problem 10 in Practice Examination 1 should be:
As the policy has a deductible of 1 (thousand), the claim payment is

$$Y = \begin{cases} 0, & \text{when there is no damage, with probability } 0.94, \\ \max(0, X-1), & \text{when } 0 < X < 15, \text{ with probability } 0.04, \\ 14, & \text{in the case of total loss, with probability } 0.02. \end{cases}$$

Posted January 25, 2011
The last two sentences of the solution of Problem 11 in Practice Examination 16 should be replaced by

But the memoryless property of the exponential distribution tells us that Y and $(Y-3|Y>3)$ have the same distribution. Note, however, that

$$(Y|Y>3) = 3 + (Y-3|Y>3),$$

so that

$$\operatorname{Var}(Y|Y>3) = \operatorname{Var}(Y-3|Y>3) = \operatorname{Var}(Y).$$

This implies that

$$\operatorname{Var}(Y|\{X>3\}\cap\{Y>3\}) = \operatorname{Var}(Y|Y>3) = \operatorname{Var}(Y) = \frac{1}{2^2} = 0.25.$$

Answer A.

Posted January 15, 2011
In Problem 16 in Practice Examination 6, the calculation of the second moment of X **should be:**

$$E(X^2) = \frac{1}{4}\cdot 0^2 + \frac{3}{4}\cdot \underbrace{(1+1)}_{\text{Second moment of } T} = \frac{3}{2}.$$

instead of

$$E(X^2) = E(X) = \frac{1}{4}\cdot 0^2 + \frac{3}{4}\cdot \underbrace{(1+1)}_{\text{Second moment of } T} = \frac{3}{2}.$$

Posted September 25, 2010
In Problem 22 in Practice Examination 19, answer choice C, the condition in the first line of the definition of $F_X(x)$ **should be** $x \le 2$, **not** $x \le 1$.

Posted August 2, 2010
In the solution of Problem 2 in Practice Examination 20, the sentence

Let X be the number of tails that are tossed that are tossed until the third head occurs.

should be

Let X be the number of tails that are tossed until the third head occurs.

Posted July 28, 2010
The first line of the first formula in the solution of Problem 13 in Practice Examination 20 has a typo in the denominator, it should say > instead of <.

Posted July 24, 2010
In the solution of Problem 21 in Practice Examination 6, the statement under the

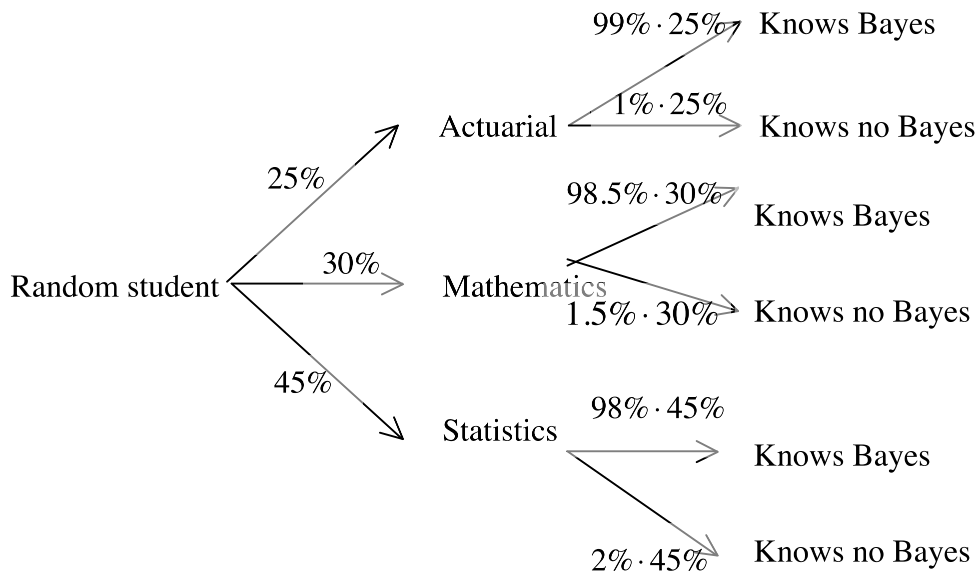
first expression on the right-hand side of the third to last formula should be:

number of ways to pick ordered samples of size 2 from population of size n **instead of** number of ways to pick ordered samples of size $n-2$ from population of size n

Posted July 17, 2010
Practice Examinations: An Introduction on page 109, the third sentence of the last section should be:
Practice examinations 6-20 are meant to be more challenging.

Posted July 3, 2010
In the solution of Problem 1, Practice Examination 5, at the end of the first part of the fourth sentence of the solution, 5/6 **is a typo, it should be** 5/36, **as used in the formula for** $\Pr(Y = 6)$.

Posted June 9, 2010
In the alternative solution of Problem 28, Practice Examination 11, the probability tree diagram should be:



Some numbers in the diagram were mistyped.

Posted February 25, 2010
In Problem 9, Practice Examination 20, answer D should be 0.6140, **and the third to last formula should be:**

$$\Pr\left(A^C \cap^C B \cap C \cap D^C\right) = \Pr\left(A^C\right) \cdot \Pr\left(B^C\right) \cdot \Pr(C) \cdot \Pr\left(D^C\right) =$$
$$= (1 - \Pr(A)) \cdot (1 - \Pr(B)) \cdot \Pr(C) \cdot (1 - \Pr(D)) = 0.4 \cdot 0.5 \cdot 0.4 \cdot 0.7 = 0.056,$$

while the last formula should be:

$$1-(0.084+0.126+0.084+0.056+0.036)=0.614.$$

Posted January 5, 2010
In the description of the gamma distribution in Section 2, the condition for the range of its MGF should be $t<\beta,$ **not** $0<t<\beta.$

Posted January 1, 2010
The Course P/1 syllabus updated for 2010 no longer contains direct references to chi-square, beta, Pareto, Weibull, and lognormal distributions. My interpretation of this change is that you do not need to memorize the details of chi-square, beta, Pareto and Weibull distributions, but you still should familiarize yourself with them. Since lognormal has a direct connection to normal, I think you should know that connection.

Posted November 20, 2009
Answers A and B in Problem 10, Practice Examination 9, have the symbol τ **mistyped as** r **in the numerator, and they should be:**

A. $f_Y(y)=\dfrac{\tau\theta y^{\tau-1}}{(y+\theta)^{\tau+1}}$ B. $f_Y(y)=\dfrac{\alpha\theta^{\alpha}\tau y^{\tau-1}}{(y^{\tau}+\theta)^{\alpha+1}}$

Posted November 17, 2009
In Problem 17 in Practice Examination 14, answer choice A should be

$$f_Y(y)=\begin{cases}0 & y<0,\\ e^{1-e^2}\left(e^{ey}+e^{-ey}\right) & 0<y<e,\\ e^{1-e^2}\cdot e^{-ey} & y\geq e.\end{cases}$$

and answer choice D should be:

$$f_Y(y)=\begin{cases}0 & y<0,\\ e^{e^2-1}\left(e^{ey}+e^{-ey}\right) & 0<y<e,\\ e^{e^2-1}\cdot e^{-ey} & y\geq e.\end{cases}$$

The last sentence of the solution should be:
Therefore, we can take

$$f_Y(y)=\begin{cases}0 & y<0,\\ e^{1-e^2}\left(e^{ey}+e^{-ey}\right) & 0<y<e,\\ e^{1-e^2}\cdot e^{-ey} & y\geq e.\end{cases}$$

Posted September 2, 2009
In Problem 17 in Practice Examination 17, this statement
and $f_N(n+1) > f_N(n)$ for $n = 0, 1, 2, 3, 4$.
should be removed.

Posted July 9, 2009
In the solution of Problem 26 in Practice Examination 19, the formula:

$$F_{X_2}(x) = 2F_X(x) - F_{X_1}(x) = \begin{cases} 0 & x < 0, \\ 0.5x & 0 \le x < 1, \\ 1 & x \ge 1. \end{cases}$$

should be:

$$F_{X_2}(x) = 2F_X(x) - F_{X_1}(x) = \begin{cases} 0 & x < 0, \\ 0.5x & 0 \le x < 2, \\ 1 & x \ge 2. \end{cases}$$

Posted July 1, 2009
In Section 2, the general definition of a percentile should be
the 100-p-th *percentile* of the distribution of X is the number x_p which satisfies both of the following inequalities: $\Pr\left(X \le x_p\right) \ge p$ and $\Pr\left(X \ge x_p\right) \ge 1 - p$.

Posted June 19, 2009
In the discussion of the mode of Poisson distribution on page 45, the expression
and as soon $n \ge \lambda - 1$,
should be:
and as soon as $n \ge \lambda - 1$,

Posted June 18, 2009
The last formula in the solution of Problem 19 of Practice Examination 5 should be
$\Pr(X \ge 10) \le \frac{1}{4^2} = \frac{1}{16}$ **instead of** $\Pr(X \ge 10) < \frac{1}{4^2} = \frac{1}{16}$.

Posted June 17, 2009
In the statement of Problem 9 in Practice Examination 7, $\Pr(X > 800)$ **should be** $\Pr(X \ge 800)$, **and in the solution, all inequalities should be changed accordingly.**

Posted May 24, 2009
Problem 7 in Practice Examination 20 needs, unfortunately, a major overhaul. My apologies. Here is what the problem should say:
The amount of damage X in a car accident is given by the exponential distribution with mean 10,000. The insurance policy covering that damage has a deductible of 500 and a policy limit of 100,000. Which of the following numbers is the closest to the coefficient of variation of the amount paid per payment (i.e., amount paid given that a payment is made by the insurance company).

A. 1.00 B. 1.10 C. 1.20 D. 1.30 E. 1.40

Solution.
Let us write Y for the amount paid per payment. Then

$$Y = \left(\min\left(X-500, 100000-500\right) \middle| X>500\right) = \left(\min\left(X-500, 99500\right) \middle| X>500\right).$$

Therefore, for $y>0$,

$$F_Y(y) = \Pr(Y \le y) = \Pr\left(\min\left(X-500, 99500\right) \le y \middle| X>500\right) =$$

$$= \left\{\begin{array}{ll} \dfrac{\Pr(500 < X < y+500)}{\Pr(X>500)}, & y < 99500, \\ 1, & y \ge 99500, \end{array}\right\} =$$

$$= \left\{\begin{array}{ll} \dfrac{e^{-0.05} - e^{-0.05-\frac{y}{10000}}}{e^{-0.05}}, & y < 99500, \\ 1, & y \le 99500, \end{array}\right\} = \left\{\begin{array}{ll} 1-e^{-\frac{y}{10000}}, & y < 99500, \\ 1, & y \ge 99500. \end{array}\right.$$

Note that the distribution of Y is mixed, with a point mass at $y = 99500$, where CDF jumps by $e^{-9.95}$. This gives us

$$f_Y(y) = \begin{cases} \dfrac{1}{10000} e^{-\frac{y}{10000}}, & 0 \le y \le 99500, \\ e^{-9.95}, & y = 99500, \\ 0, & \text{otherwise.} \end{cases}$$

Therefore

$$E(Y) = \int_0^{99500} \frac{ye^{-\frac{y}{10000}}}{10000} dy + 99500e^{-9.95} = \underbrace{\left|\begin{array}{ll} u = y & v = -e^{-\frac{y}{10000}} \\ du = dy & dv = \dfrac{1}{10000} e^{-\frac{y}{10000}} dy \end{array}\right|}_{\text{Integration by parts in the integral}} =$$

$$= -ye^{-\frac{y}{10000}}\Bigg|_{y=0}^{y=99500} + \int_0^{99500} e^{-\frac{y}{10000}} dy + 99500e^{-9.95} =$$

$$= -99500e^{-9.95} + 10000F_X(99500) + 99500e^{-9.95} = 10000\left(1-e^{-9.95}\right) \approx 9999.5227.$$

Furthermore,

$$E\left(Y^2\right)=\int_0^{99500}\frac{y^2e^{-\frac{y}{10000}}}{10000}dy+99500^2\cdot e^{-9.95}=\underbrace{\left|\begin{matrix} u=y^2 & v=-e^{-\frac{y}{10000}} \\ du=2ydy & dv=\frac{1}{10000}e^{-\frac{y}{10000}}dy \end{matrix}\right|}_{\text{Integration by parts in the integral}}=$$

$$=-y^2\cdot e^{-\frac{y}{10000}}\Bigg|_{y=0}^{y=99500}+\int_0^{99500}2ye^{-\frac{y}{10000}}dy+99500^2\cdot e^{-9.95}=$$

$$=-99500^2\cdot e^{-9.95}+20000\cdot\int_0^{99500}\frac{ye^{-\frac{y}{10000}}}{10000}dy+99500^2\cdot e^{-9.95}=$$

$$=20000\cdot\int_0^{99500}\frac{ye^{-\frac{y}{10000}}}{10000}dy=20000\left(E(Y)-99500e^{-9.95}\right)=$$

$$=20000\cdot\left(10000\left(1-e^{-9.95}\right)-99500e^{-9.95}\right)=$$

$$=20000\cdot\left(10000-109500e^{-9.95}\right)=10^6\cdot\left(200-2190e^{-9.95}\right)\approx 199895476.$$

Based on this

$$\operatorname{Var}(Y)=E\left(Y^2\right)-\left(E(Y)\right)^2=10^6\cdot\left(200-2190e^{-9.95}\right)-10^8\left(1-e^{-9.95}\right)^2\approx 99905021.8.$$

The coefficient of variation is

$$\frac{\sqrt{\operatorname{Var}(Y)}}{E(Y)}=\frac{\sqrt{10^6\cdot\left(200-2190e^{-9.95}\right)-10^8\left(1-e^{-9.95}\right)^2}}{10000\left(1-e^{-9.95}\right)}\approx 0.9996.$$

Answer A.

Posted May 17, 2009
Problem 25 in Practice Examination 18 and its solution should be:
P Sample Exam Questions, Problem No. 128, also Dr. Ostaszewski's online exercise posted November 15, 2008

An insurance agent offers his clients auto insurance, homeowners insurance and renters insurance. The purchase of homeowners insurance and the purchase of renters insurance are mutually exclusive. The profile of the agent's clients is as follows:

i) 17% of the clients have none of these three products.
ii) 64% of the clients have auto insurance.
iii) Twice as many of the clients have homeowners insurance as have renters insurance.
iv) 35% of the clients have two of these three products.
v) 11% of the clients have homeowners insurance, but not auto insurance.

Calculate the percentage of the agent's clients that have both auto and renters insurance.

A. 7% B. 10% C. 16% D. 25% E. 28%

Solution.
Let H be the event of a client having a homeowners insurance, R be the event of a client having a renters insurance, and A be the event of a client having an auto insurance. We are given that

$$\Pr\left(\left(H \cup R \cup A\right)^C\right) = 0.17,$$

so that $\Pr\left(H \cup R \cup A\right) = 0.83$. We are also given that $H \cap R = \varnothing$, so that $\Pr\left(H \cap R\right) = 0$. Note that $A \cap H \cap R \subset H \cap R$, so that $\Pr\left(A \cap H \cap R\right) = 0$, as well. Also, $\Pr\left(A\right) = 0.64$, $\Pr\left(H\right) = 2\Pr\left(R\right)$, and

$$\Pr\left(A \cap H\right) - \Pr\left(A \cap H \cap R\right) + \Pr\left(A \cap R\right) - \Pr\left(A \cap H \cap R\right) +$$
$$+ \Pr\left(H \cap R\right) - \Pr\left(A \cap H \cap R\right) = \Pr\left(A \cap H\right) + \Pr\left(A \cap R\right) = 0.35.$$

Finally, we are also given that

$$\Pr\left(H - A\right) = \Pr\left(H - \left(A \cap H\right)\right) = \Pr\left(H\right) - \Pr\left(A \cap H\right) = 0.11.$$

The quantity we are looking for is $\Pr\left(A \cap R\right)$. First note that

$$0.83 = \Pr\left(H \cup R \cup A\right) = \Pr\left(H\right) + \Pr\left(R\right) + \Pr\left(A\right) -$$
$$- \Pr\left(H \cap R\right) - \Pr\left(H \cap A\right) - \Pr\left(R \cap A\right) + \Pr\left(H \cap R \cap A\right) = 3\Pr\left(R\right) + 0.64 - 0.35$$

This means that $\Pr\left(R\right) = 0.18$ and consequently $\Pr\left(H\right) = 0.36$. But

$$0.36 = \Pr\left(H\right) = \Pr\left(H \cap A\right) + \Pr\left(H - A\right) = \Pr\left(H \cap A\right) + 0.11,$$

so that $\Pr\left(H \cap A\right) = 0.25$ and $\Pr\left(A \cap R\right) = 0.35 - 0.25 = 0.10$.
Answer B.

Posted April 23, 2009
In Problem 20, Practice Examination 14, the first sentence of the solution should be:
Let us define the following random variables:
X: time until death of Dwizeel by causes other than their private plane crash,
Y: time until death of Satellite Component by causes other than their private plane crash,
Z: time until death of Dwizeel and Satellite Component as a result of their private plane crash.
The solution has the words "of" **mistyped as** "od."
Posted April 23, 2009
In Problem 9, Practice Examination 14, the Greek letter τ in the statement of the problem should be replaced by the Greek letter α.

Posted April 6, 2009
In the text of Problem 4 in Practice Examination 11, the sentence:
We put that chip aside and pick a second chip from the same contained.
should be:
We put that chip aside and pick a second chip from the same container.

Posted April 6, 2009
In the text of Problem 7 in Practice Examination 3, the word "whwther" **should be** "whether".

Posted April 6, 2009
In the text of Problem 30 in Practice Examination 19 the expression:
α denotes the number of those variable added,
should be
α denotes the number of those variables added,

Posted April 6, 2009
The first sentence of Problem 28 in Practice Examination 19 should be:
A box contains 4 red balls and 6 white balls.

Posted April 6, 2009
The text of Problem 24 in Practice Examination 19 should be:
You are given a random sample X_1, X_2 from the exponential distribution with mean of 1. Let $Y_{(1)}, Y_{(2)}$ be the corresponding order statistics. Define $U = Y_{(2)} - Y_{(1)}$. Find the probability density function of *U,* where positive.

A. $2e^{-2u}$ B. $\frac{1}{2}e^{-\frac{1}{2}u}$ C. e^{-u} D. ue^{-u} E. $\frac{1}{2}u^2e^{-u}$

Posted April 6, 2009
The solution of Problem 15 in Practice Examination 19 should be:
Solution.
The amount the insurance company pays is

$$X = \begin{cases} 0 & \text{if there is no fire and no storm, i.e., with probability } 0.99 \cdot 0.97, \\ 900 & \text{if there is a fire and no storm, i.e., with probability } 0.01 \cdot 0.97, \\ 2900 & \text{if there is no fire and a storm, i.e., with probability } 0.99 \cdot 0.03, \\ 3900 & \text{if there is a fire and a storm, i.e., with probability } 0.01 \cdot 0.03. \end{cases}$$

Therefore,

$$E(X) = 0.01 \cdot 0.97 \cdot 900 + 0.99 \cdot 0.03 \cdot 2900 + 0.01 \cdot 0.03 \cdot 3900 = 96.03.$$

Also,

$$E(X^2) = 0.01 \cdot 0.97 \cdot 900^2 + 0.99 \cdot 0.03 \cdot 2900^2 + 0.01 \cdot 0.03 \cdot 3900^2 = 262197.$$

This gives us

$$\operatorname{Var}(X) = E(X^2) - (E(X))^2 \approx 252975.239$$

and

$$\sigma_X = \sqrt{\mathrm{Var}(X)} \approx 502.9664.$$

The coefficient of variation is

$$\frac{\sqrt{\mathrm{Var}(X)}}{E(X)} \approx 5.2376.$$

Answer A.

Posted April 6, 2009
In the text of Problem 13, Practice Examination 19, the expression

at attendant

should be

an attendant.

Posted April 6, 2009
In the second displayed formula in the solution of Problem 11 in Practice Examination 19, the expression

$$\frac{1 \cdot \Pr(X=1) + 2 \cdot \Pr(X=2) + 3 \cdot \Pr(X=3)}{\Pr(N<4)}$$

should be

$$\frac{1 \cdot \Pr(N=1) + 2 \cdot \Pr(N=2) + 3 \cdot \Pr(N=3)}{\Pr(N<4)}$$

Posted April 6, 2009
In the text of Problem 9 in Practice Examination 19, statement II should read:
II. $Y - X$ has a normal distribution if and only if $\rho > 0$.

Posted April 6, 2009
In the text of Problem 2 in Practice Examination 19, the expression
(these two but no all three)
should be
these two but not all three)

Posted April 6, 2009
In the solution of Problem 1 in Practice Examination 19, the second displayed formula should be

$$k = \frac{1}{2\ln 2 - 1}.$$

instead of

$$1 = k = \frac{1}{2\ln 2 - 1}.$$

Posted April 3, 2009
On page 25 of the manual, the expression
0.60 and 0.40, assigned to X_1 and X_2, respectively.
should be
0.40 and 0.60, assigned to X_1 and X_2, respectively.

Posted March 19, 2009
The solution of Problem 3 in Practice Examination 8 should be:
Let E be the event that a new insured is accident-free during the second policy year, and F be the event that a new insured is accident-free during the first policy year, and let G be the event that this new insured was accident-free the last year, before the policy was issued. Note that for any year only the previous year affects a given year, but not the year before that. Therefore

$$\begin{aligned}
\Pr(E) &= \Pr\Big((E\cap F\cap G)\cup\left(E\cap F\cap G^C\right)\cup\left(E\cap F^C\cap G\right)\cup\left(E\cap F\cap G^C\right)\Big) = \\
&= \Pr(E\cap F\cap G) + \Pr\left(E\cap F\cap G^C\right) + \Pr\left(E\cap F^C\cap G\right) + \Pr\left(E\cap F^C\cap G^C\right) = \\
&= \Pr(G)\cdot\Pr(F|G)\cdot\Pr(E|F\cap G) + \Pr\left(G^C\right)\cdot\Pr\left(F|G^C\right)\cdot\Pr\left(E|F\cap G^C\right) + \\
&\quad + \Pr(G)\cdot\Pr\left(F^C|G\right)\cdot\Pr\left(E|F^C\cap G\right) + \Pr\left(G^C\right)\cdot\Pr\left(F^C|G^C\right)\cdot\Pr\left(E|F^C\cap G^C\right) = \\
&= 0.7\cdot 0.8\cdot 0.8 + 0.3\cdot 0.6\cdot 0.8 + 0.7\cdot(1-0.8)\cdot 0.6 + 0.3\cdot(1-0.6)\cdot 0.6 = 0.748.
\end{aligned}$$

Answer E.

Posted March 5, 2009
The first sentence of Problem 16 in Practice Examination 6 should end with $t < 1$, instead of $t > 1$.

Posted March 2, 2009
The properties of the cumulant moment-generating function should be:
The cumulant generating function has the following properties:

$$\psi_X(0) = 0, \qquad \frac{d}{dt}\ln E\left(e^{tX}\right)\Big|_{t=0} = \frac{E\left(Xe^{tX}\right)}{E\left(e^{tX}\right)}\Bigg|_{t=0} = E(X),$$

$$\frac{d^2}{dt^2}\psi_X(t)\Big|_{t=0} = \frac{d}{dt}\frac{E\left(Xe^{tX}\right)}{E\left(e^{tX}\right)}\Bigg|_{t=0} = \frac{E\left(X^2e^{tX}\right)E\left(e^{tX}\right) - E\left(Xe^{tX}\right)E\left(Xe^{tX}\right)}{\left(E\left(e^{tX}\right)\right)^2}\Bigg|_{t=0} = \mathrm{Var}(X),$$

$$\left.\frac{d^3}{dt^3}\psi_X(t)\right|_{t=0} = E\left(\left(X-E(X)\right)^3\right),$$

but for $k>3$,

$$\left.\frac{d^k}{dt^k}\psi_X(t)\right|_{t=0} = \psi_X^{(k)}(0) < E\left(\left(X-E(X)\right)^k\right).$$

Also, if X and Y are independent (we will discuss this concept later), $\psi_{aX+b}(t)=\psi_X(at)+bt$, and $\psi_{X+Y}(t)=\psi_X(t)+\psi_Y(t)$.

Posted January 13, 2009

In Practice Examination 8, Problem 24, the calculation of the expected value had a typo, an extra, unnecessary p in the second line, and it instead should be:

$$\begin{aligned}
E(X) &= \sum_{k=1}^{+\infty} k\cdot\Pr(X=k) = 1\cdot\Pr(X=1)+\sum_{k=2}^{+\infty} k\cdot\Pr(X=k) = \\
&= p+\sum_{k=1}^{+\infty}(k+1)\cdot\Pr(X=k+1) = p+\sum_{k=1}^{+\infty} k\cdot\Pr(X=k+1)+\sum_{k=1}^{+\infty}\Pr(X=k+1) = \\
&= p+\sum_{k=1}^{+\infty} k\cdot\underbrace{(1-p)\cdot\Pr(X=k)}_{=\Pr(X=k+1)}+\left(\underbrace{\left(\sum_{k=0}^{+\infty}\Pr(X=k+1)\right)}_{\text{this sum is equal to 1}}-\Pr(X=0+1)\right) = \\
&= p+(1-p)\cdot\sum_{k=1}^{+\infty} k\cdot\Pr(X=k)+\left(1-\Pr(X=1)\right) = \\
&= p+(1-p)\cdot E(X)+(1-p) = 1+(1-p)\cdot E(X).
\end{aligned}$$

Posted January 12, 2009

In Problem 28 in Practice Examination 20, the answer choices should be:

A. 2.04 B. 2.55 C. 3.01 D. 3.27 E. 3.98

In the solution, the payment made variable should be:

$$W=\begin{cases} 0 & \text{when } Y<4, \\ Y-4 & \text{when } 4\le Y<9 \\ 5 & \text{when } Y\ge 9. \end{cases}$$

The expected value should be calculated as:

$$E(W)=\underbrace{\int_0^4 0\cdot\frac{1}{8}e^{-\frac{1}{8}y}\,dy}_{=0}+\int_4^9 (y-4)\cdot\frac{1}{8}e^{-\frac{1}{8}y}\,dy+\int_9^{+\infty}5\cdot\frac{1}{8}e^{-\frac{1}{8}y}\,dy=\underbrace{\left|\begin{matrix} u=y-4 & v=-e^{-\frac{1}{8}y} \\ du=dy & dv=\frac{1}{8}e^{-\frac{1}{8}y}dy\end{matrix}\right|}_{\text{Integration by parts in the second integral}}=$$

$$=\left(\left.(y-4)\left(-e^{-\frac{1}{8}y}\right)\right|_{y=4}^{y=9}\right)+\int_4^9 e^{-\frac{1}{8}y}\,dy+\frac{5}{8}\int_9^{+\infty}e^{-\frac{1}{8}y}\,dy=-5e^{-\frac{9}{8}}+\int_4^9 e^{-\frac{1}{8}y}\,dy+\frac{5}{8}\int_9^{+\infty}e^{-\frac{1}{8}y}\,dy=$$

$$=-5e^{-\frac{9}{8}}+\left(\left.-8e^{-\frac{1}{8}y}\right|_{y=4}^{y=9}\right)+\left(\left.-5e^{-\frac{1}{8}y}\right|_{y=9}^{y\to\infty}\right)=8e^{-\frac{1}{2}}-8e^{-\frac{9}{8}}\approx 2.2550.$$

The answer choice should be B.

Posted January 12, 2009

In Problem 29 in Practice Examination 20, the answer choices should be:

A. 0.19 B. 1.09 C. 1.49 D. 1.87 E. 2.00

and in the solution, the survival function should be: $s_X(x|Y>1)=1$ for $x<1$, $s_X(x|Y>1)=0$ for $x>3$, **and**

$$s_X(x|Y>1)=-\frac{1}{24}x^3+\frac{3}{28}x^2+\frac{27}{28}$$

for $1<x<3$. **The expected value calculation should be corrected to say:**

$$E(X|Y>1)=\int_0^1 1\cdot dx+\int_1^3\left(-\frac{1}{14}x^3+\frac{3}{28}x^2+\frac{27}{28}\right)dx=1+\left(\left.\left(-\frac{1}{56}x^4+\frac{1}{28}x^3+\frac{27}{28}x\right)\right|_{x=1}^{x=3}\right)=$$

$$=1+\left(-\frac{81}{56}+\frac{27}{28}+\frac{81}{28}\right)-\left(-\frac{1}{56}+\frac{1}{28}+\frac{27}{28}\right)=1-\frac{80}{56}+\frac{26}{28}+\frac{54}{28}=\frac{17}{7}.$$

The resulting variance is

$$\mathrm{Var}(X|Y>1)=\frac{213}{35}-\left(\frac{17}{7}\right)^2=\frac{213}{35}-\frac{289}{49}=\frac{1491-1445}{245}=\frac{46}{245}\approx 0.1878.$$

The answer choice should be A.

Posted January 9, 2009

In Problem 29 in Practice Examination 19, the last sentence of the statement of the problem should be:

Find the variance of the conditional distribution of Y, given $X=x$.

Instead of

Find the variance of the conditional distribution of X, given $Y=y$.

Posted November 8, 2008
Problem 11 in Practice Examination 13 had several typos, and it should be:
Mr. Warrick Beige is gambling at the newly opened *You Was Robbed* casino in Nevada. In the game he is playing, first he has to choose one of two coins: coin A or coin B. Both coins are unfair. Coin A has the probability of heads of 0.60, and coin B has the probability of heads of 0.40. Mr. Beige pays \$20 to enter the game. He chooses a coin randomly, but the chances of picking the coins are not equal. He has 40% chance of picking coin A and 60% chance of picking coin B. Then he tosses the coin chosen. If the result is heads, he is paid \$250. If the result is tails, he pays \$200. For an additional payment of x dollars, Mr. Beige can test a coin chosen: he can toss it once and based on the result, either walk away and get \$20 paid initially back (he will do this if the first toss results in tails), or toss the same coin again (he will do this if the first toss results in heads). Assuming that Mr. Beige values all gambles based on the expected value of the payoff (i.e., he is *risk-neutral*), calculate the value of x such that Mr. Beige is indifferent between testing a coin and not testing it.

A. \$6.40 B. \$4.00 C. \$2.50 D. \$0.00 E. –\$2.00

Solution.
We begin by labelling the events:

A: Coin A is picked,
B: Coin B is picked,
H_1: First toss results in heads,
T_1: First toss results in tails,
H_2: Second toss results in heads,
T_2: Second toss results in tails.

We know that $\Pr(A)=0.40$, $\Pr(B)=0.60$, $\Pr(H_1|A)=0.60$, and $\Pr(H_1|B)=0.40$.
Therefore, the probability of getting heads in the first toss is

$$\Pr(H_1)=\Pr(H_1|A)\cdot\Pr(A)+\Pr(H_1|B)\cdot\Pr(B)=$$
$$=0.40\cdot0.60+0.60\cdot0.40=0.24+0.24=0.48.$$

Therefore, Mr. Beige's expected gain on this game without testing first is

$$-\$20+0.48\cdot\$250+0.52\cdot(-\$200)=-\$4.00.$$

Note that the two coin tosses are independent, and hence

$$\Pr(H_2|H_1)=\frac{\Pr(H_1\cap H_2)}{\Pr(H_1)}=\frac{\Pr(H_1\cap H_2|A)\cdot\Pr(A)+\Pr(H_1\cap H_2|B)\cdot\Pr(B)}{\Pr(H_1)}=$$
$$=\frac{\Pr(H_1|A)\cdot\Pr(H_2|A)\cdot\Pr(A)+\Pr(H_1|B)\cdot\Pr(H_2|B)\cdot\Pr(B)}{\Pr(H_1)}=$$
$$=\frac{0.6\cdot0.6\cdot0.4+0.4\cdot0.4\cdot0.6}{0.48}=0.5.$$

For now, let us disregard the cost x of testing the coin, and calculate the expected net payoff of the game without that additional fee. If Mr. Beige's coin test results in tails, his payoff for the game will be zero. Thus the expected net payoff for the case when he tests the coin (disregarding the fee of x dollars) is

$$-\$20+\$20\cdot\Pr\left(H_1^C\right)+\$250\cdot\Pr\left(H_1\right)\cdot\Pr\left(H_2\middle|H_1\right)-\$200\cdot\Pr\left(H_1\right)\cdot\Pr\left(T_2\middle|H_1\right)=$$

$$=-\$20+\$20\cdot 0.52+\$250\cdot 0.48\cdot 0.50-\$200\cdot 0.48\cdot 0.50=\$2.40.$$

This means that Mr. Beige's expected payoff changes from –\$4 to \$2.40 as a result of testing the coin. In order for him to be indifferent between the two choices, the additional fee should be set at the difference of these two amounts, equal to his gain in the expected payoff of the game, i.e., \$6.40.
Answer A.

Posted November 6, 2008
The first sentence of Problem 17 in Practice Examination 17 should be:
You are given a discrete random variable N such that its only possible values are 0, 1, 2, 3, 4, and 5.
instead of
You are given a discrete random variable N such that its only possible values are 0, 1, 2, 3, 4, and 5, and $f_N(n+1)>f_N(n)$ for $n=0, 1, 2, 3, 4$.

Posted November 4, 2008
The answer choices in Problem 18, Practice Examination 16, should be:
A. 0.3333 B. 0.4875 C. 0.6075 D. 1.3333 E. 2.1251
They were mislabeled as A, B, B, C, E.